GEPM 375

Project Planning and Construction Management

Linear Programming-2

Process/ Steps in Formulating Real Life Problems into LP Problems:

- **Step-1:** Identify the decision variables –which decides output (X and Y)
- Step-2: Define the objective functions – optimized solution. ($Z=6X+5Y$)
- Step-3: Write down constraints – Limitations on decision variables.
($X + Y \leq 5$ and $3X+2Y \leq 12$)
- State Non-negativity variables ($X \geq 0$ and $Y \geq 0$)

Methods of Solving LP Problems

- **Graphical Method**
- **Open Solver method**
- **Simplex Method**
- **North-West Corner Method**
- **Least Cost Method**

Graphical Method:

- A linear program can be solved by multiple methods.

- **One of the methods is Graphical method for solving a linear program.**

- Graphical method is ***used to solve a two variable linear program.***

- If there is **only two decision variables**, you should use the graphical method to find the optimal solution.

Question: What is graphical method of solving LP program?

Process of Graphical method

• A graphical method involves **formulating a set of linear inequalities subject to the constraints.**

• Then the **inequalities are plotted on a X-Y plane.**

• Once we have plotted all the inequalities on a graph the **intersecting region gives us a feasible region**.

•**Feasible region** explains **what all values our model can take**.

•And model also **gives us the optimal solution**

Question: Write down the process of solving LP problems by graphical method.








Let's understand Graphical methods with the help of an example:

Example: A farmer has recently acquired an **110 hectares piece of land.** He has decided to **grow Wheat and Barley** on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know **how to plant each variety in the 110 hectares,** given the **costs, net profits** and **labor requirements** according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

Farmer has a budget of **US$10,000** and an availability of **1,200 man-days** during the planning.

Find the optimal solution and the optimal value.

Solution: To solve this problem, first to formulate the linear program.

Formulation of Linear Problem

Step 1: Identify the decision variables

- Let, the total area for growing Wheat = X (in hectares)
- Let, the total area for growing Barley = Y (in hectares)

X and Y are the decision variables.

Step 2: Write the objective function:

- Since the production from the entire land can be sold in the market.
- Farmer would want to **maximize the profit** for his total produce.
- Here it is given net profit for both Wheat and Barley.
- Farmer earns a net profit of US$50 for each hectare of Wheat and US$120 for each Barley.

Objective function (given by Z) is, Max Z = 50X + 120Y

Step 3: Writing the constraints

- It is given that the farmer has a **total budget of US$10,000.**
- **Cost of producing** Wheat and Barley per hectare is also given to us.
- We have the **total cost spent** by the farmer. So our equation becomes,

1st constraint: $100X + 200Y \leq 10{,}000$

- Availability of total man-days is **1200 man-days.**
- Man-days for wheat and barley is 10 & 30 man-days respectively

2nd Constraints: $10X + 30Y \leq 1200$ **(man-days)**

3rd constraint is the **total area present for plantation**.

- Total available area is **110 hectares**.

- So the equation becomes,

3rd constraint: $X + Y \leq 110$

Step 4: The non-negativity restriction

- Values of X and Y will be greater than or equal to 0. This goes without saying.

$X \geq 0, Y \geq 0$

- Formulation of linear program is done.

- It's time to solve it.

Question: For the problem given, formulate the linear program.

Solving a LP through Graphical method

- Since we know that X, Y ≥ 0, **consider only the first quadrant.**

- To plot for the graph for the equations, first **simplify all the equations**.

- **100X + 200Y ≤ 10,000** can be simplified to **X + 2Y ≤ 100** by dividing by 100.

- **X + 2Y ≤ 100 (cost) --------- (1)**

10X + 30Y ≤ 1200 can be simplified to **X + 3Y ≤ 120** by dividing by 10.

- **X + 3Y ≤ 120 (man-days) ----------(2)**

Third equation is in its simplified form, **X + Y ≤ 110**

- **X + Y ≤ 110 (total land) ---------- (3)**

Plot the equation (1) and (2) on a graph in **first quadrant (like shown below)**

•Optimal feasible solution is achieved at the point of **intersection** where the budget & man-days **constraints are active**.

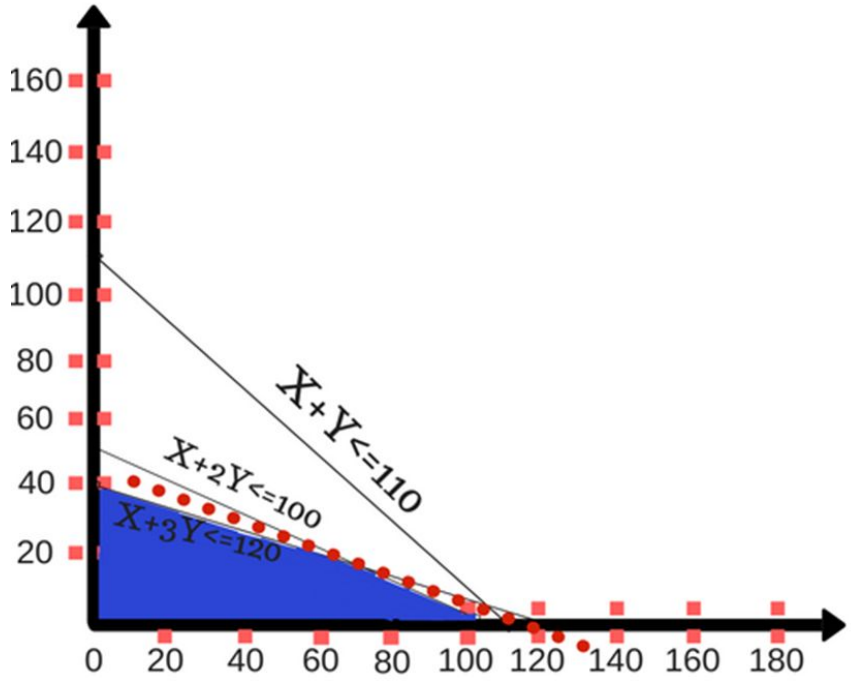
•This means the point at which the equations $X + 2Y \leq 100$ and $X + 3Y \leq 120$ **intersect gives** us the optimal solution.

•Values for X and Y which gives the optimal solution is at (60,20).

•Maximize profit the farmer should produce Wheat and Barley in **60 hectares and 20 hectares** of land respectively

Maximum profit the company will gain is,

Max Z = 50 * (60) + 120 * (20)
= US$5400



Other Methods of solving LP problems

- Open Solver method.
- Simplex Method
- Northwest Corner Method and
- Least Cost Method

Solve Linear Program using Open-Solver

- In reality, a linear program can **contain 30 to 1000 variables** and solving it either Graphically or Algebraically is next to impossible.

- Companies generally use Open-Solver to tackle these real-world problems.

- Open-Solver is an **open source linear and optimizer for Microsoft Excel.** It is an advanced version of built-in excel Solver

Simplex Method:

• Simplex Method is one of the **most powerful & popular methods** for linear programming.

•**Standard technique** in linear programming for solving an optimization problem, typically one involving **a function and several constraints** expressed as inequalities.

Northwest Corner Method:

•Adopted to compute the initial feasible solution of the **transportation problem**.

•Name is given to this method because the basic variables are selected from **the extreme left corner**.

Northwest Corner Method:

- Concept of North-West Corner can be well understood through **a transportation problem** given below:

Source \ To	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150

Northwest Corner Method:

In the table:

- Three **sources A, B and C** with the production **capacity of 50 units, 40 units, 60 units** of product respectively is given.

- Every day the **demand of three retailers** D, E, F is to be furnished with at least 20 units, 95 units and 35 units of product respectively.

- **Transportation costs** are also given in the matrix.

Source \ To	D	E	F	Supply
A	5 (20)	8 (30)	4	50
B	6	6 (40)	3	40
C	3	9 (25)	6 (35)	60
Demand	20	95	35	150

Northwest Corner Method:

- Select the **north-west or extreme left corner** of the matrix, assign as many units as possible to cell AD, within the supply and demand constraints. Such as **20 units are assigned to the first cell,** that satisfies the demand of destination D while the supply is in surplus.

- **Now move horizontally** and **assign 30 units to the cell AE**. Since 30 units are available with the source A, the **supply gets fully saturated.**

- **Now move vertically** in the matrix and **assign 40 units** to Cell BE. The supply of **source B also gets fully saturated**.

Northwest Corner Method:

- **Again move vertically**, and **assign 25 units** to cell CE, the **demand of destination E is fulfilled**.

- **Move horizontally** in the matrix and **assign 35 units** to cell CF, both the demand and supply of origin and destination **gets saturated**.

Now the total cost can be computed.

- The Total cost can be computed by multiplying the units assigned to each cell with the concerned transportation cost. Therefore,
- **Total Cost** = 20*5+ 30*8+ 40*6+ 25*9+ 35*6 = Tk 1015.00

Least Cost Method:

•Least Cost Method is another method used to obtain the **initial feasible solution for the transportation problem**.

•Here, the allocation **begins with the cell which has the minimum cost.**

•Lower cost cells are chosen over the higher-cost cell with the **objective to have the least cost of transportation**.

Let's understand the concept of Least Cost method through a problem given below:

Source \ To	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150

Least Cost Method:

Minimum transportation cost can be obtained by following the steps given below:

Source \ To	D	E	F	Supply
A	5	8 (50)	4	50
B	6	6 (5)	3 (35)	40
C	3 (20)	9 (40)	6	60
Demand	20	95	35	150

Least Cost Method:

•**In the given matrix**, the supply of each source A, B, C is given Viz. 50units, 40 units, and 60 units respectively.

•Weekly demand for three retailers D, E, F i.e. 20 units, 95 units and 35 units is given respectively.

•Shipping cost is given for all the routes.

- **Minimum cost in the matrix is Rs** 3, but there is a **tie in the cell BF, and CD**, now the question arises in which cell we shall allocate.

- Generally, the **cost where maximum quantity** can be assigned should be chosen to obtain the better initial solution.

- Therefore, **35 units shall be assigned** to the cell BF. With this, the **demand for retailer F gets fulfilled**, and only **5 units are left with the source B**.

•Again the **minimum cost in the matrix is Rs 3**.

•Therefore**, 20 units shall be assigned** to the cell CD.

•With this, the demand of retailer D gets fulfilled. Only 40 units are left with the source C.

- Next minimum cost **is Tk 4**, but however, the demand for F is completed, we will move to the next minimum cost which **is Tk. 5**.

- Again, the demand of D is completed.

- Next minimum cost is **Tk. 6**, and there is a **tie between** three cells. But however, no units can be assigned to the cells BD and CF as the demand for both the retailers D and F are saturated.

- So, we shall assign **5 units to Cell BE**. With this, the supply of source B gets saturated.

Least Cost Method:

- Next minimum cost is **Tk. 8, assign 50 units** to the cell AE.

- Supply of source A gets saturated.

- Next **minimum cost is Tk. 9**; we shall **assign 40 units** to the cell CE.

- With his both the demand and supply of all the sources and origins gets saturated.

Total cost can be calculated by multiplying the assigned quantity with the concerned cost of the cell.
Therefore,

- **Total Cost** = 50*8 + 5*6 + 35*3 +20*3 +40*9 = Tk. 955.00

Difference between Least Cost method and North west corner method:

- **Least Cost Method** is considered to produce more optimal results than the **North-west Corner because**:

-it **considers the cost** while making the allocation, basing on the availability and supply requirement.

-whereas the North-West corner method **only considers the availability and supply requirement** and allocation **begin with the extreme left corner, irrespective of the shipping cost.**

Thank you