Syl.

(i) Direct Stiffness method
(ii) Moment distribution method
(iii) IL for indeterminate structures

Ref:

Structural analysis by Aslam Kassimali.

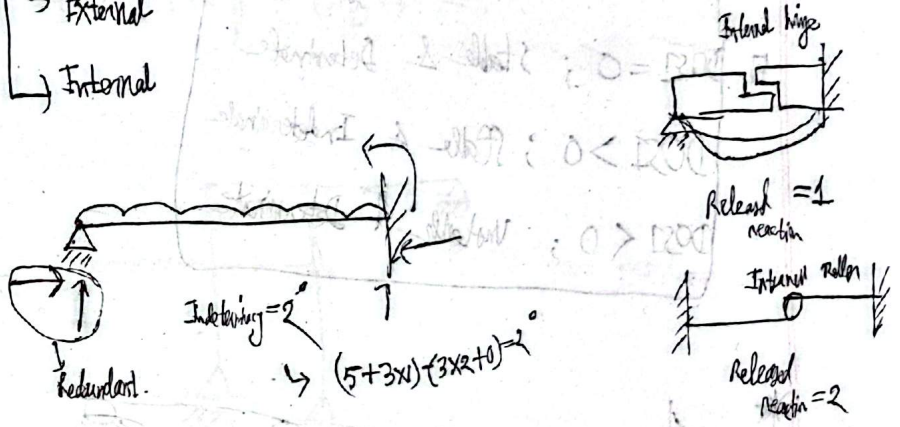
Lec Zihad Sin

Direct Stiffness method:

Exact method, Is used in computer program.

- Statically analysis → Force method (DOSI)
  → Displacement method (DOKI)

- Degree of Static Indeterminacy (DOSI):
  - External
  - Internal



$$I_s = I_{se} + I_{sI}$$

$$\Rightarrow I_s = (\text{External Reaction} - 3) + (3m - 3j + 3 - R_r)$$

R_r = Released Reaction

$$= R - 3 + 3m - 3j + 3 - R_r$$

$$\Rightarrow I_s = (R + 3m) - (3j + R_r)$$ [for 2D (beam, frame)]

$$I_s = (R + 6m) - (6j + R_r)$$ [in 3D]

Truss

$$I_s = m + R - 2j$$

- DOSI = 0 ; Stable & Determinate
- DOSI > 0 ; Stable & Indeterminate
- DOSI < 0 ; Unstable & Determinate

*



$$I_s = (R + 3m) - (3j + R_r)$$

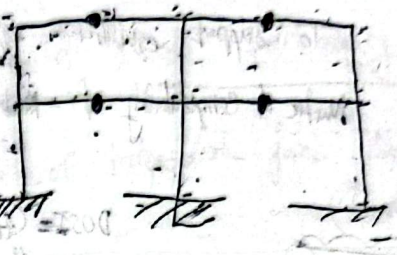
$$= (6 + 3\times4) - (3\times5 + 1)$$

$$= 2^\circ$$

$$I_{se} = (\text{Ext. reaction} - 3) = 6 - 3 = 3^\circ$$

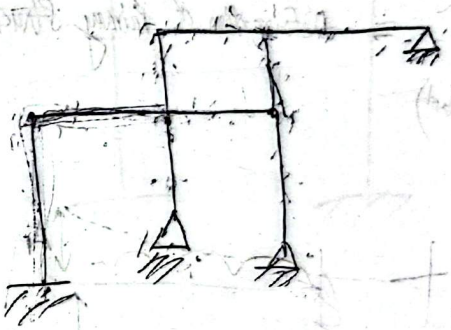
$$I_s - I_{se} = 2 - 3 = -1^\circ$$

*



$$I_s = (9 + 3\times4) - (3\times8 + 4) = 8^\circ$$

*

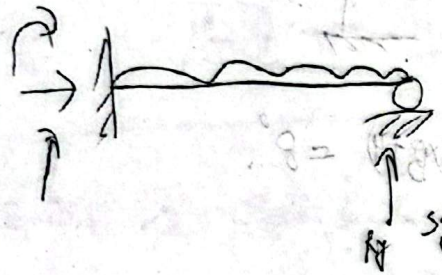


$$I_s = (9 + 3\times8) - (3\times9 + 6) = 6^\circ$$

Method of consistent deformation / Flexibility method

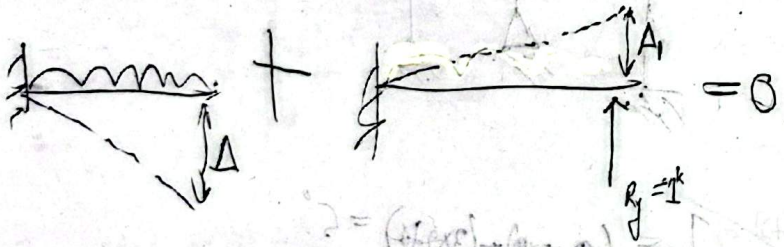
Compatibility eqn: To support equilibrium eqn's.

Number of compatibility eq's = Degree of Indeterminacy.



DOSI = (4+3×1) − (3×2+0)
= 1°

Deformation due to = Combined effect (Ext loading + Redundant) = Deformation of Primary Structure.

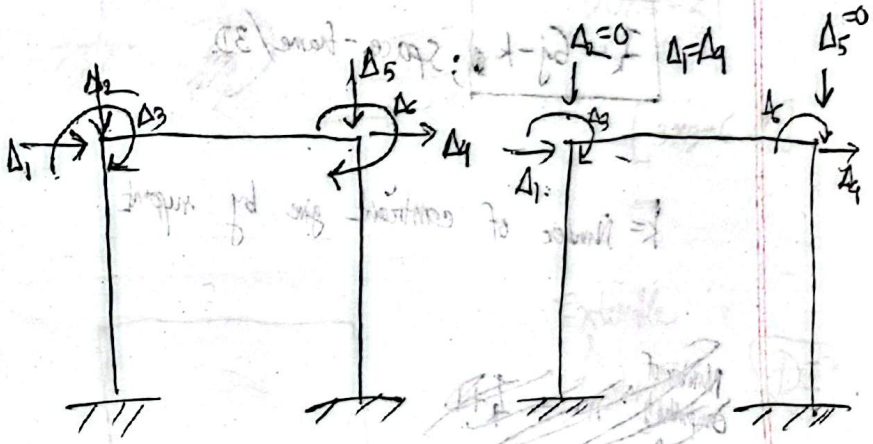


$\Rightarrow \Delta + R_y A_1 = 0$ [compatibility eqn]

$\Rightarrow R_y = ?$

Degree of Kinetic Indeterminacy: (DOKI) / Degree of Freedom (DOF):

Number of independent joint displacement to describe deformed shape. (Translational, Rotational)



If member is extensible

If member is inextensible

DOKI = 6.



DOKI = 0
DOSI = 3

k = 4
2j + k = 2×2 − 4 = 0

DOKI = 3



Extensible, DOKI = 2 [3×2 − 4 = 2]

Inextensible, DOKI = 1 [3×2 − (4+1) = 1]

$I_k = 2j - k$; for beam, truss

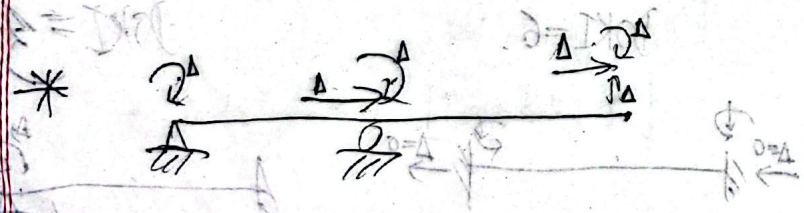
$I_k = 3j - k$; for plane-frame/2D

$I_k = 6j - k$; space-frame/3D.

k = number of constraint give by support.

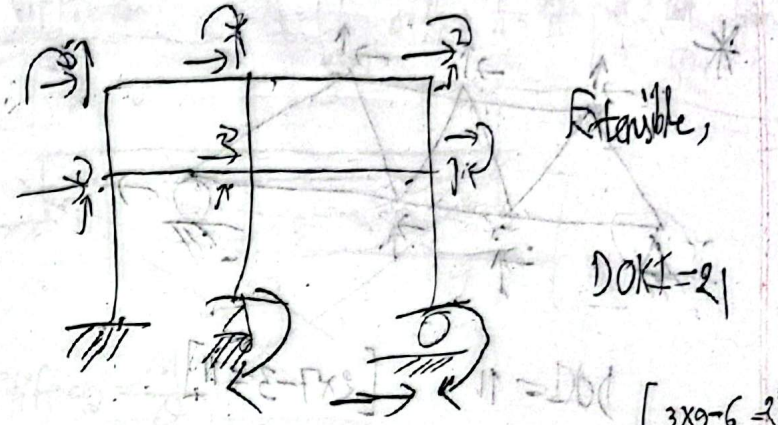


$I_k = 3j - (k+m)$ [Inextensible beam]



Extensible $\rightarrow I_k = 6$

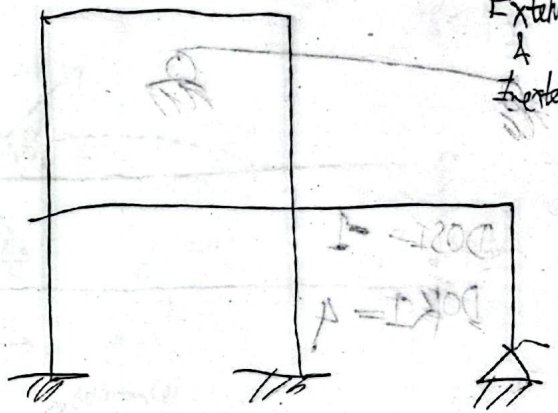
Inextensible $\rightarrow I_k = 4$



Extensible,

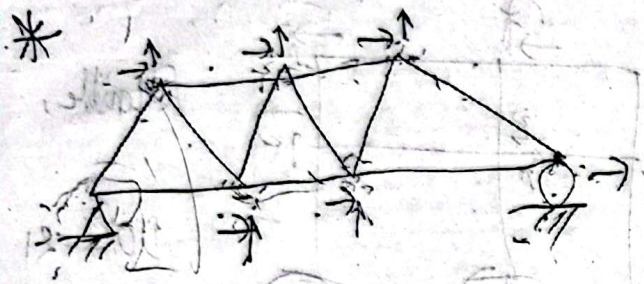
DOKI = 21

[3x9-6 = 21]

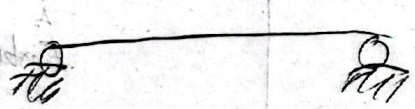


Extensible & Inextensible

DI?



DOI = 11 → [2×7−3=11]

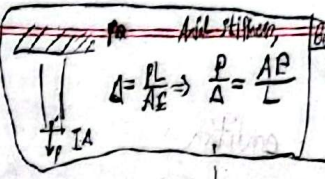


DOSI = −1

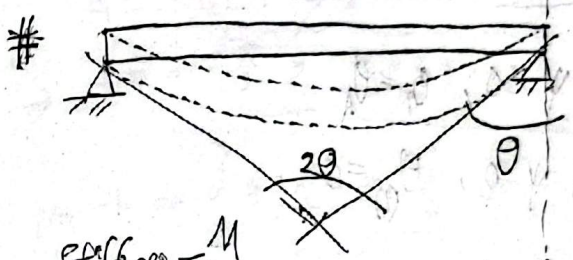
DOKI = 4

28-06-25

Stiffness :



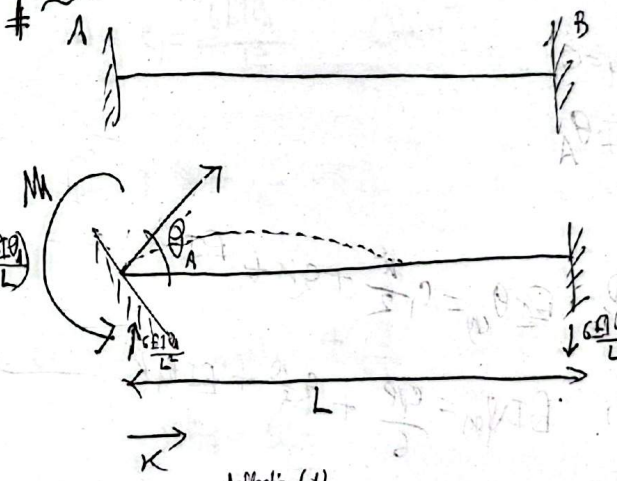
In 1 Unit rotation, the moment required is called stiffness



$\text{Stiffness} = \frac{M}{\theta}$

Deviation

Due to Rotation :



deflection (y)

$EI\frac{d^4y(x)}{dx^4} = P$

As no external loading, $P = 0$.

$\therefore EI\frac{d^4y(x)}{dx^4} = 0$ [Load]

$\Rightarrow EI\frac{d^3y(x)}{dx^3} = c_1$ [Shear] $\Rightarrow EI\frac{d^2y(x)}{dx^2} = c_1x + c_2$ [moment] $\Rightarrow EI\frac{dy(x)}{dx} = c_1\frac{x^2}{2} + c_2x + c_3$ [slope]

$\Rightarrow EIy(x) = \frac{1}{6}c_1x^3 + c_2\frac{x^2}{2} + c_3x + c_4$ [deflection]

Boundary condition:

- $x=0;\ \psi=0;\ \theta_{(x)}=\theta_A$

$x=L;\ \psi=0;\ \theta_{(L)}=0$

Putting $x=0, \psi=0$ in eqn (III) & eqn (IV),

$C_4=0$.

$C_3 = EI\theta_A$

From eqn (II), $EI\theta_{(x)} = C_1\frac{x^2}{2} + C_2 x + EI\theta_A$ ——— (V)

From eqn (IV), $EI\psi_{(x)} = \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + EI\theta_A x$ ——— (VI)

putting $x=L;\ \theta=0$ in eqn (III)

$0 = C_1\frac{L^2}{2} + C_2L + EI\theta_A$ ——— (VII)

putting $x=L;\ \psi=0$ in eqn (IV),

$0 = \frac{C_1L^3}{6} + C_2\frac{L^2}{2} + EI\theta_A L$ ——— (VIII)

(VII) × L − (VIII) →

$C_1\frac{L^3}{2} + C_2L^2 + EI\theta_A L - C_1\frac{L^3}{6} - C_2\frac{L^2}{2} - EI\theta_A L = 0$

$\Rightarrow C_2\frac{L^2}{2} = -C_1\frac{L^3}{3} \Rightarrow C_2 = -\frac{2}{3}C_1L$ ——— (IX)

Putting the value of C_2 in eqn (VII)

$0 = C_1\frac{L^2}{2} + \left(-\frac{2}{3}C_1L\right)L + EI\theta_A$

$\Rightarrow C_1 = \frac{6EI\theta_A}{L^2}$ [Shear].

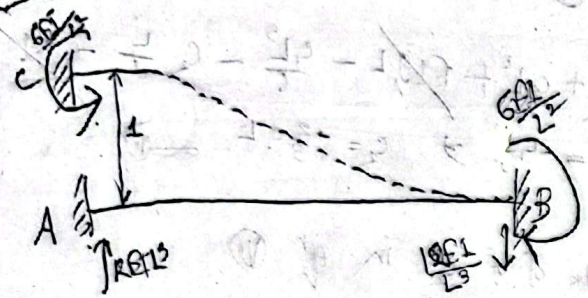
from eqn (IX), $C_2 = -\frac{4EI\theta_A}{L}$.

for $x=0 \rightarrow$ Moment $= \frac{4EI\theta_A}{L}$ [eqn (II)]

Stiffness $= \frac{4EI}{L}$.

for $x=L \rightarrow$ Moment $= \frac{2EI\theta_A}{L}$ [eqn (II)]

* Due to Deflection:



$$EI\frac{d^4y}{dx^4} = p = 0 \quad \text{[Load]}$$

$$EI\frac{d^3y}{dx^3} = c_1 \quad \text{[Shear]} \quad \text{——— (I)}$$

$$EI\frac{d^2y}{dx^2} = c_1x + c_2 \quad \text{[Moment]} \quad \text{——— (II)}$$

$$EI\frac{dy}{dx} = c_1\frac{x^2}{2} + c_2x + c_3 \quad \text{[Slope]} \quad \text{——— (III)}$$

$$EIy = c_1\frac{x^3}{6} + c_2\frac{x^2}{2} + c_3x + c_4 \quad \text{[Deflection]} \quad \text{——— (IV)}$$

BC

$x=0; y=1; \theta=0^\circ$ — From eqn (IV), $c_4 = EI$

$x=L; y=0; \theta=0^\circ$ — From eqn (III), $c_3 = 0$

Putting the values of c_3 & c_4 in eqn (III) & (IV),

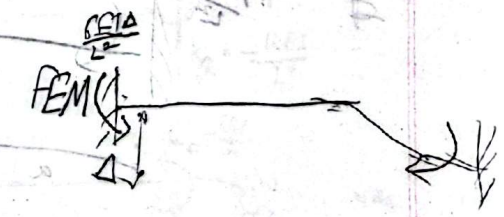
$$EI\frac{dy}{dx} = c_1\frac{x^2}{2} + c_2x \quad \text{——— (V)}$$

$$EIy = c_1\frac{x^3}{6} + c_2\frac{x^2}{2} + 0 + EI \quad \text{——— (VI)}$$

$x=L; y=0; \theta=0^\circ$ from eqn (V) & (VI),

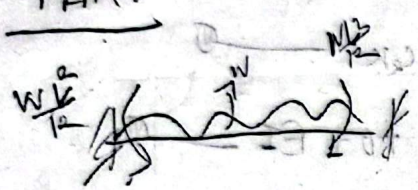
$$c_1 = \frac{12EI}{L^3}$$

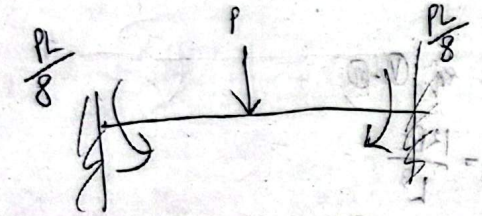
$$c_2 = -\frac{6EI}{L^2}$$

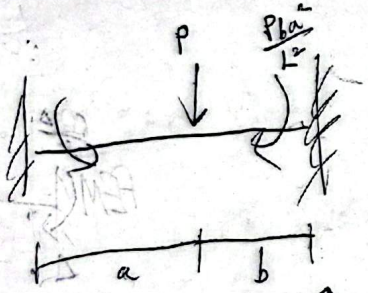


[12-09-26]

FEM:

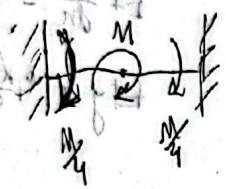


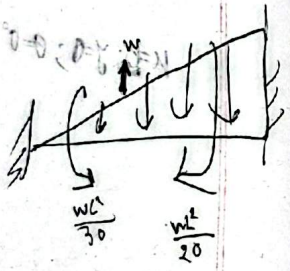




$\frac{Pab^2}{L^2}$ $\frac{Pba^2}{L^2}$ P a b

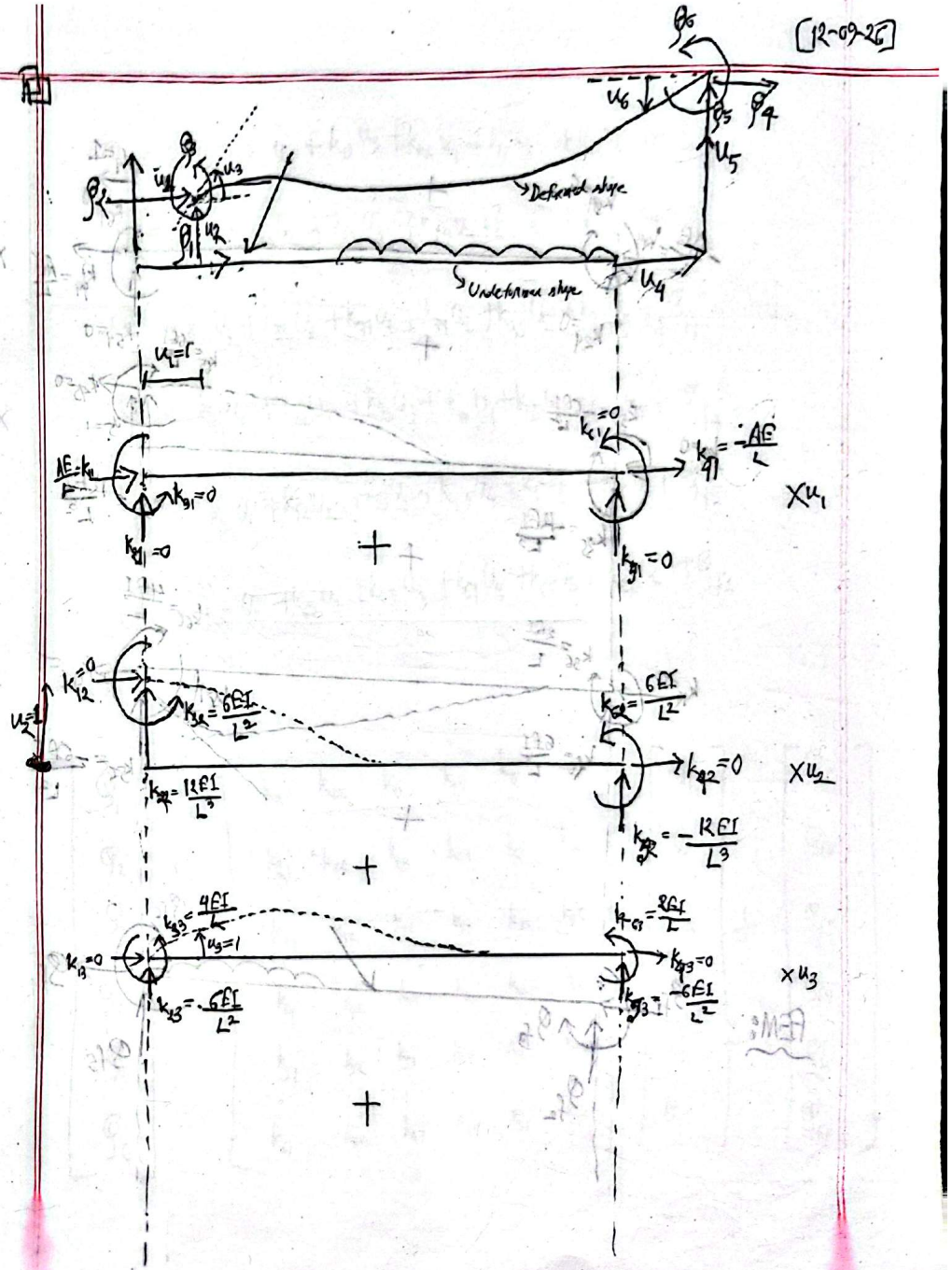
$\frac{Pb^2}{L^3}(3a+b)$ $\frac{Pa^2}{L^3}(3b+a)$

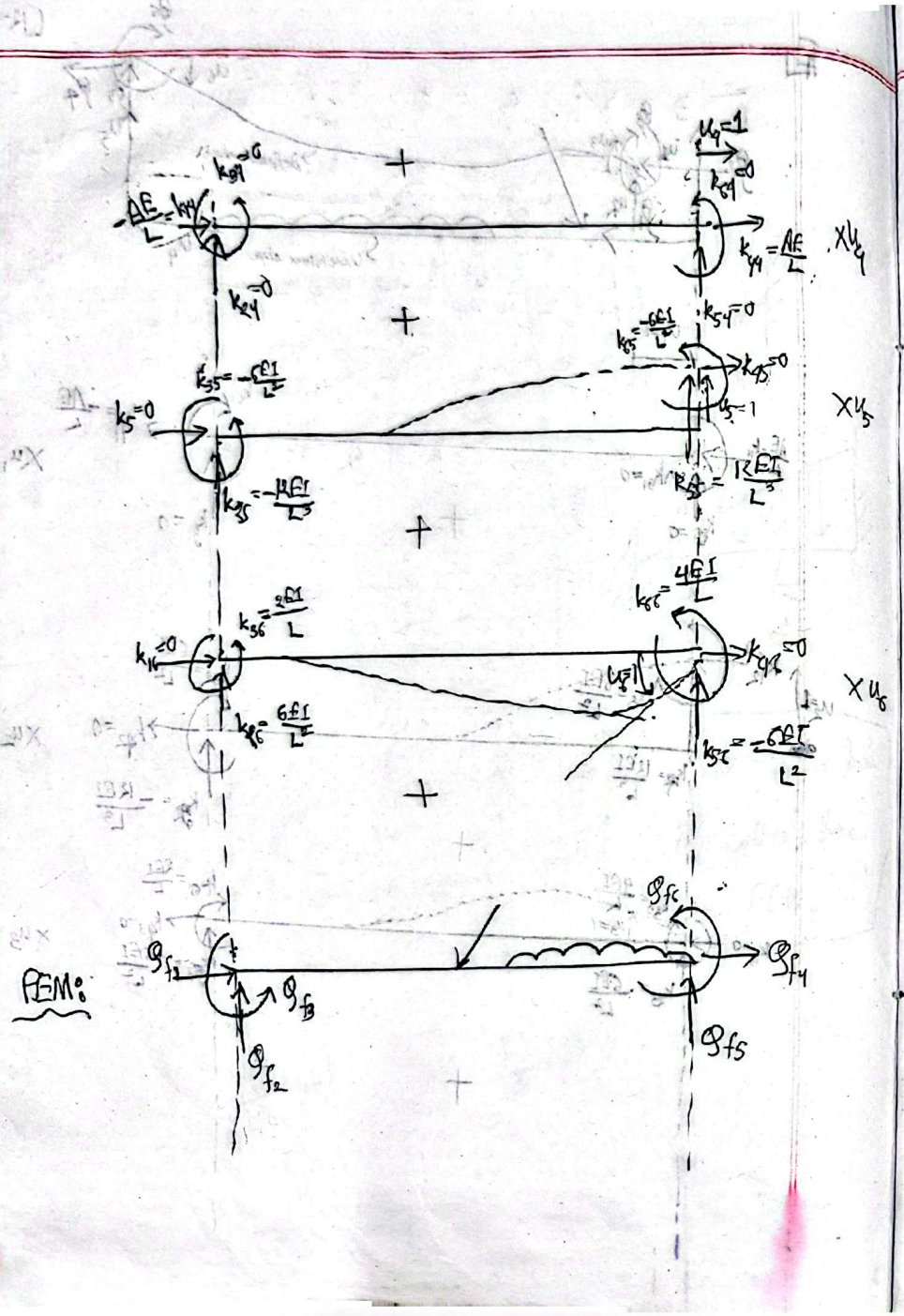




$\frac{wL^2}{30}$ $\frac{wL^2}{20}$

In joint where load is applied, there is no FEM.





$$Q_1 = k_{11}u_1 + k_{12}u_2 + k_{13}u_3 + k_{14}u_4 + k_{15}u_5 + k_{16}u_6 + Q_{f1}$$

$$Q_2 = k_{21}u_1 + k_{22}u_2 + k_{23}u_3 + k_{24}u_4 + k_{25}u_5 + k_{26}u_6 + Q_{f2}$$

$$Q_3 = k_{31}u_1 + k_{32}u_2 + k_{33}u_3 + k_{34}u_4 + k_{35}u_5 + k_{36}u_6 + Q_{f3}$$

$$Q_4 = k_{41}u_1 + k_{42}u_2 + k_{43}u_3 + k_{44}u_4 + k_{45}u_5 + k_{46}u_6 + Q_{f4}$$

$$Q_5 = k_{51}u_1 + k_{52}u_2 + k_{53}u_3 + k_{54}u_4 + k_{55}u_5 + k_{56}u_6 + Q_{f5}$$

$$Q_6 = k_{61}u_1 + k_{62}u_2 + k_{63}u_3 + k_{64}u_4 + k_{65}u_5 + k_{66}u_6 + Q_{f6}$$

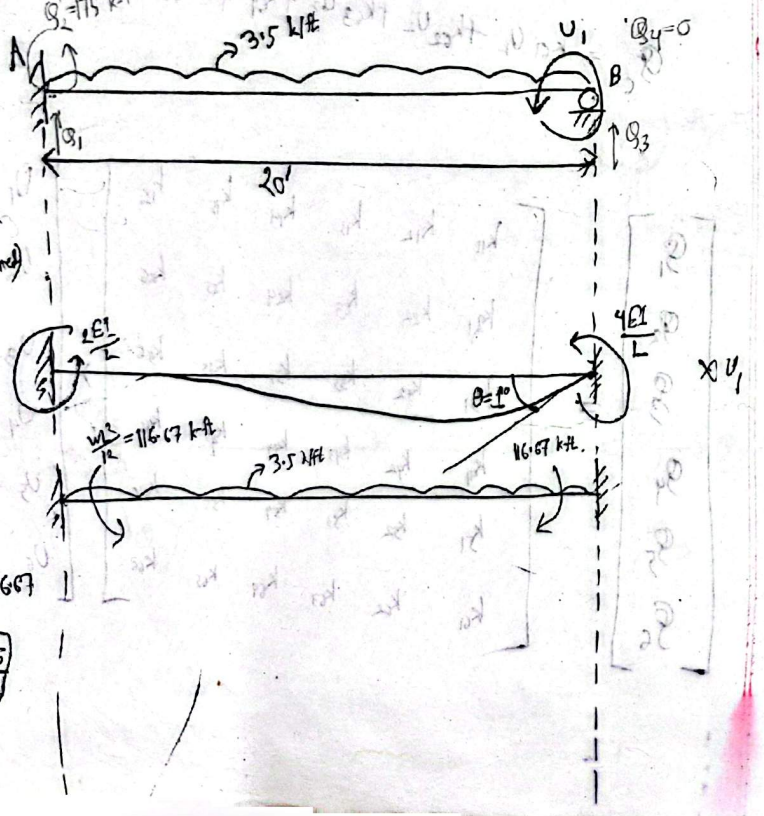
$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix}$$

$$Q = kU + Q_f$$

Here, Q → Member end force
k → Member stiffness
U → Member displacement (Translation, Rotation)
Q_f → Fixed end Moment for external loading

Ex-1:

Det SFD, BMD
EI = Constant
Inextensible beam
(If not mentioned)



Sol:

DOF = 1.

At B,

$Q_4 = \frac{4EI}{L} U_1 - 116.67$

$0 = \frac{4EI}{L} U_1 - 116.67$

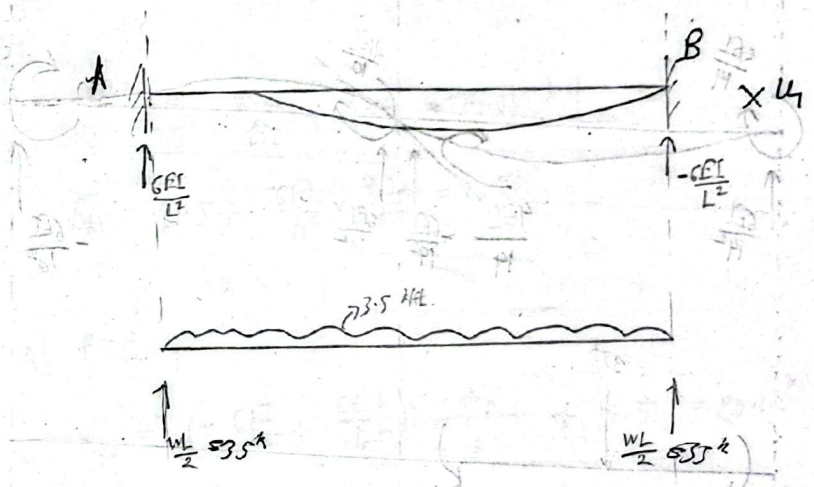
$U_1 = \frac{583.35}{EI}$

At A,

$Q_2 = \frac{2EI}{L} U_1 + 116.67$

$= \frac{2EI}{L} \times \frac{583.35}{EI} + 116.67$

$= 175$ k-ft.



At A,

$Q_1 = \frac{6EI}{L^2} \times U_1 + \frac{WL}{2}$

$= 43.75$ k

At B,

$Q = 16.25$ k

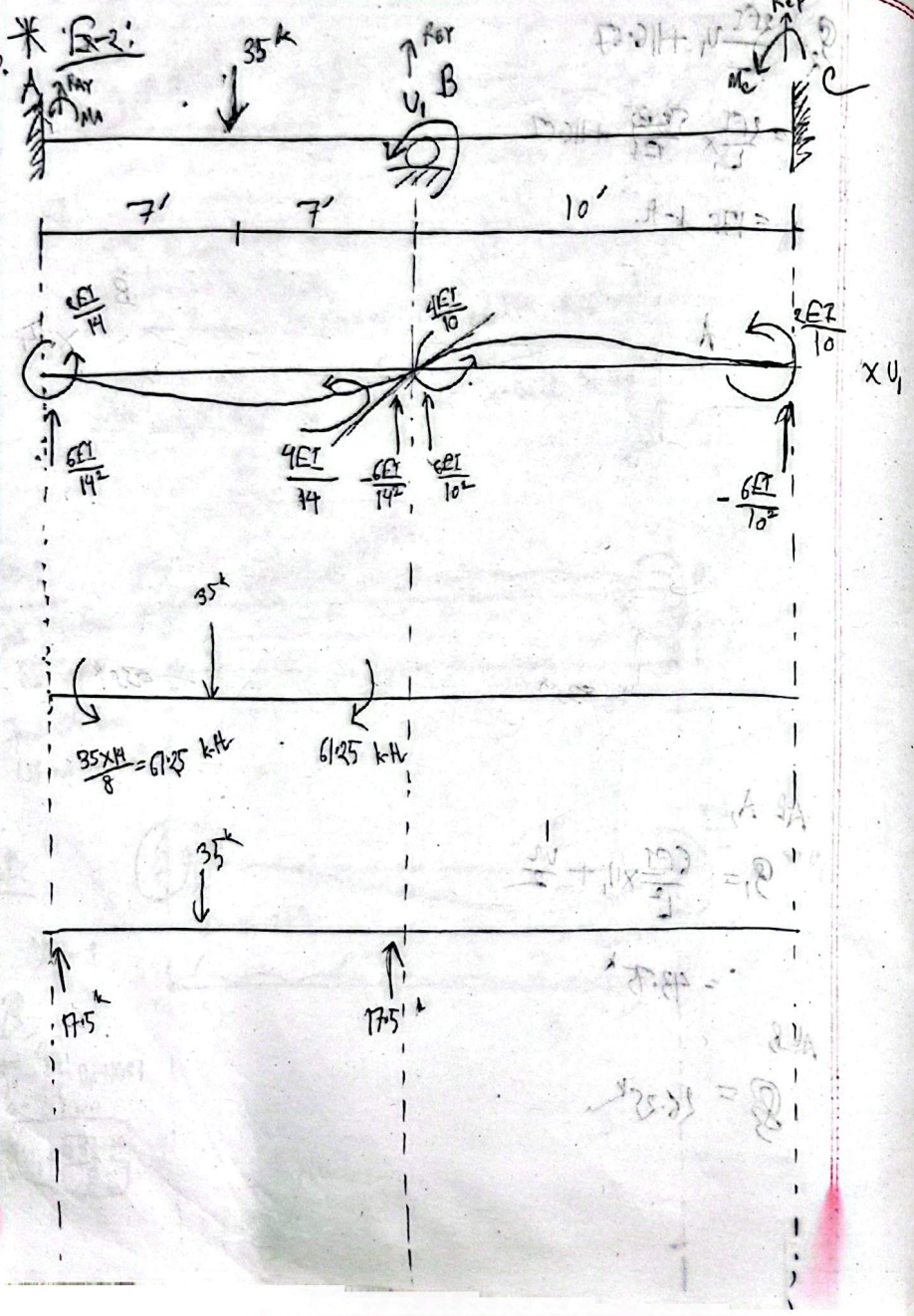
[25-07-16]

Det SFD, BMD. EI = Const. Inextensible bar

Sol^n:

DOKI = 1

* Ex-2:



At B,

$$0 = -61.25 + \left(\frac{4EI}{14} + \frac{4EI}{10}\right) U_1$$

$$\Rightarrow U_1 = \frac{89.32}{EI}$$

At A,

$$M_A = \frac{2EI}{14} \times \frac{89.32}{EI} + 61.25 = 74.01 \text{ k-ft}$$

$$R_{AY} = 17.5 + \frac{6EI}{14^2} \times \frac{89.32}{EI} = 20.23^{k}$$

At B,

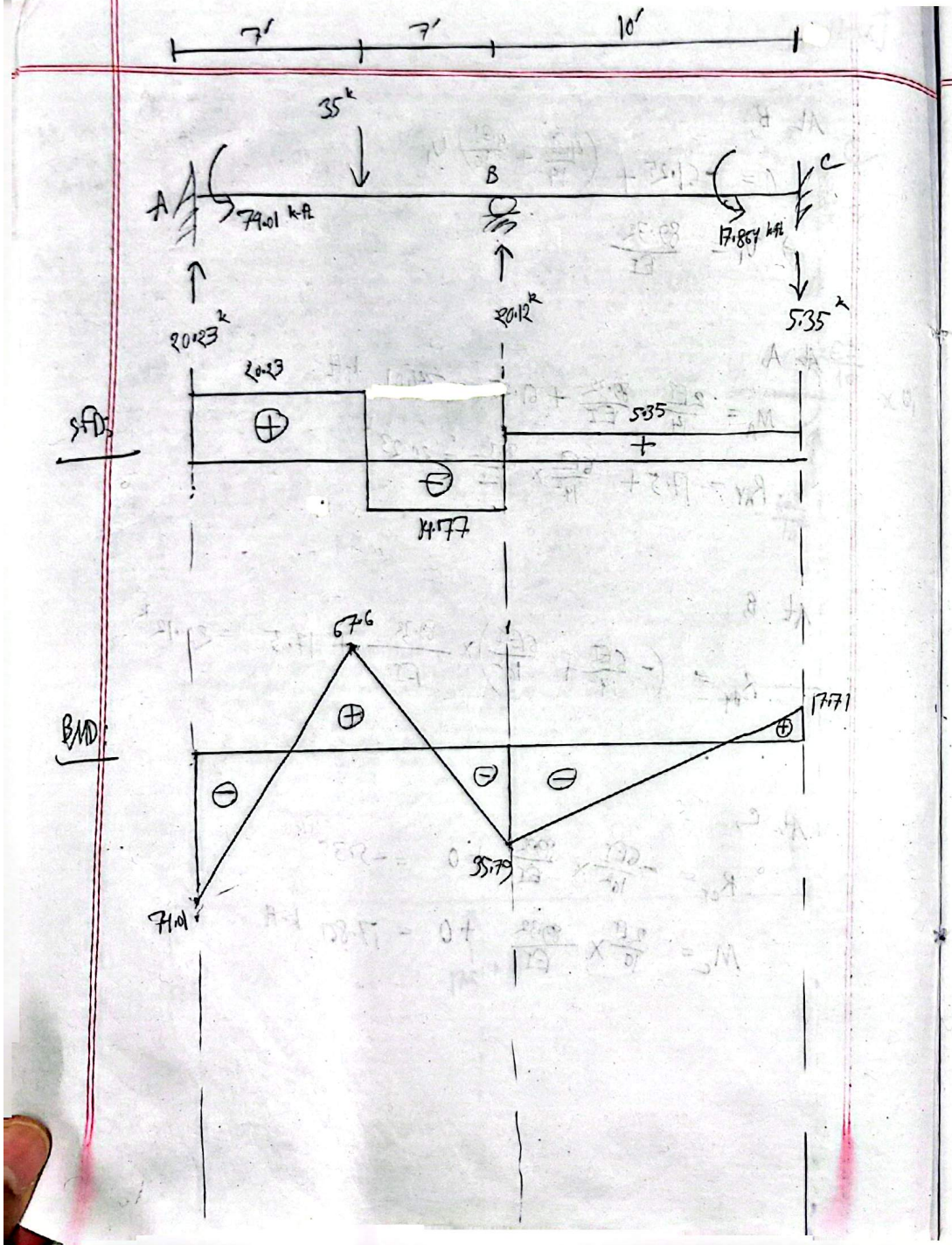
$$R_{BY} = \left(-\frac{6EI}{14^2} + \frac{6EI}{10^2}\right) \times \frac{89.32}{EI} + 17.5 = 20.12^{k}.$$

At C,

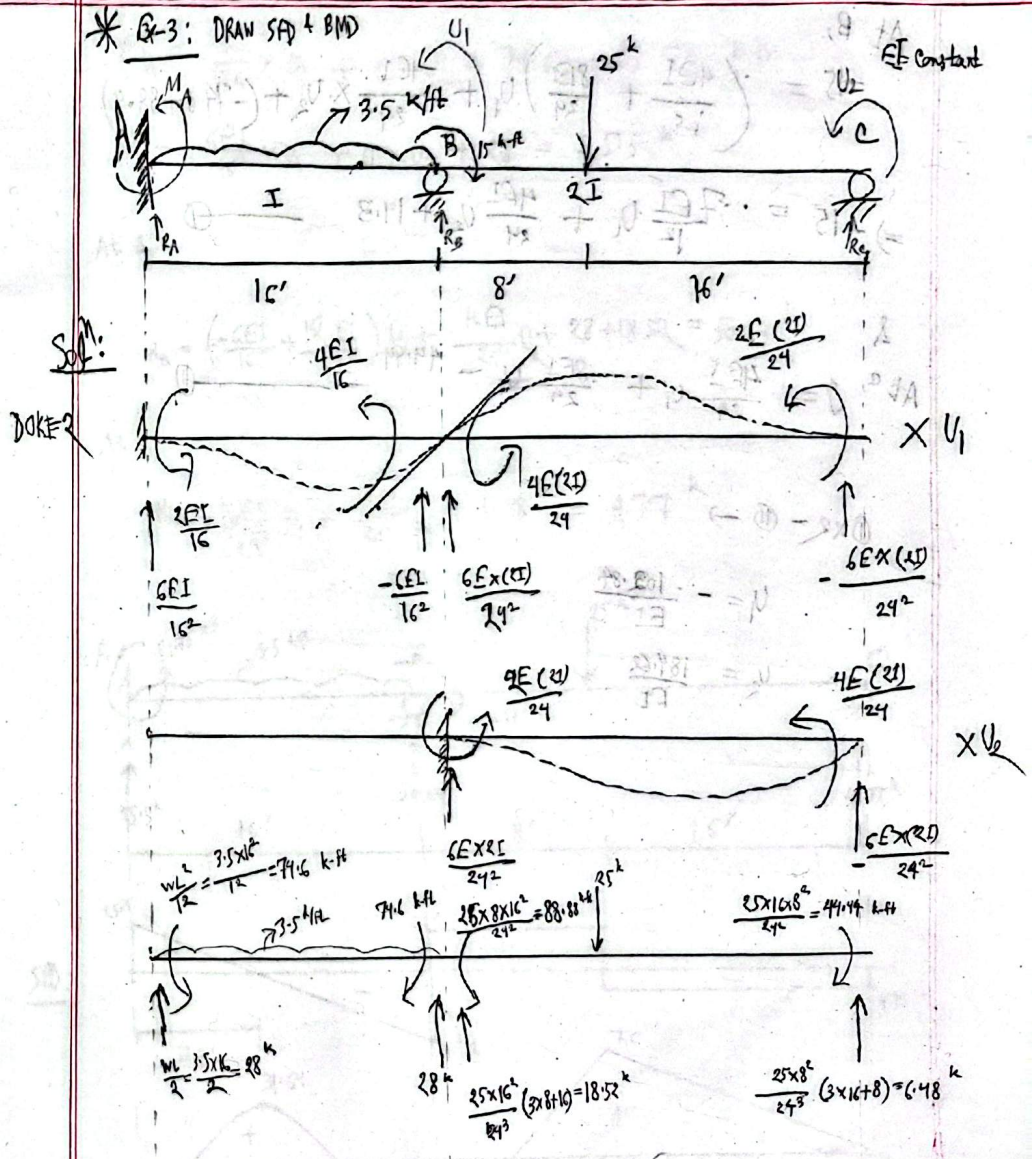
$$R_{CY} = -\frac{6EI}{10^2} \times \frac{89.32}{EI} + 0 = -5.35^{k}$$

$$M_C = \frac{2EI}{10} \times \frac{89.32}{EI} + 0 = 17.864 \text{ k-ft}$$

CamScanner



Ex-3: DRAW SFD & BMD



At B,

$$-15 = \left(\frac{4EI}{16} + \frac{8EI}{24}\right)U_1 + \frac{4EI}{24} \times U_2 + (-74.6 + 88.9)$$

$$\Rightarrow -15 = \frac{7EI}{12}U_1 + \frac{4EI}{24}U_2 + 14.3 \quad \text{——} (I)$$

&

At C, $0 = \frac{4EI}{24}U_1 + \frac{8EI}{24}U_2 - 44.44$ ——(II)

(I) × 2 − (II) →

$$U_1 = -\frac{102.84}{EI}$$

$$U_2 = \frac{184.62}{EI}$$

At A,

$$M_A = \frac{2EI}{16} \times U_1 + 0 \times U_2 + 74.6 = 61.73 \text{ k-ft}$$

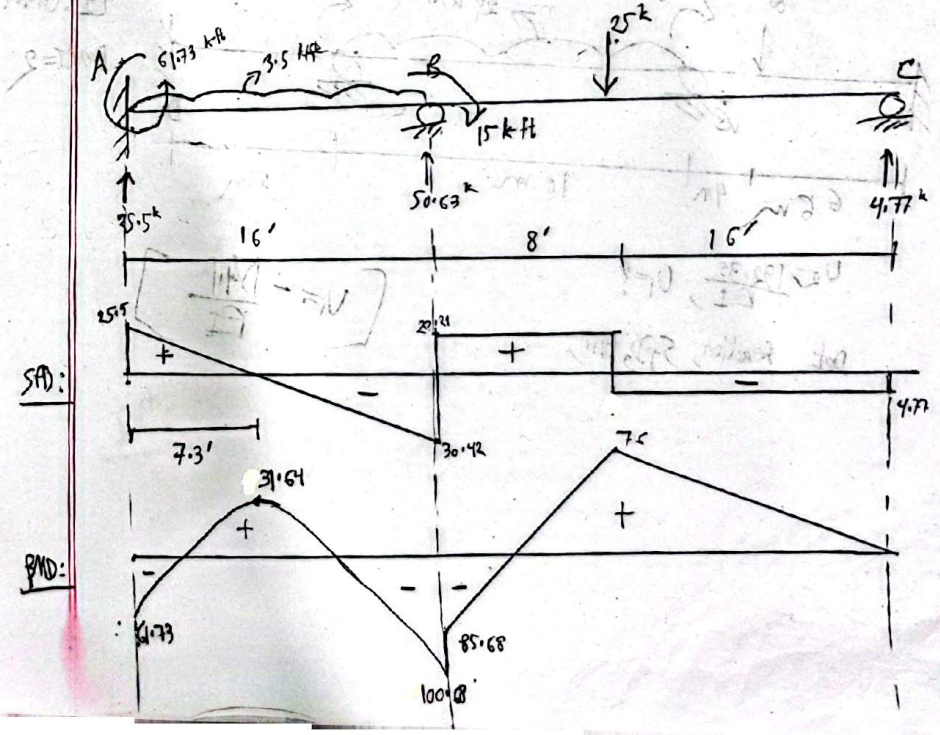
$$R_A = \frac{6EI}{16^2} \times U_1 + 0 \times U_2 + 28 = 25.5^{k}$$

At B,

$$R_B = \left(\frac{-6EI}{16^2} + \frac{12EI}{24^2}\right)U_1 + \frac{12EI}{24^2}U_2 + 28 + 18.52 = 50.63^{k}$$

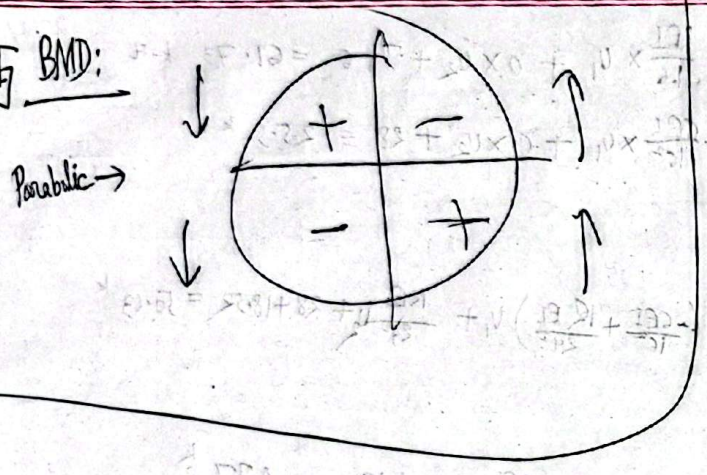
At C,

$$R_C = -\frac{12EI}{24^2}U_1 - \frac{12EI}{24^2}U_2 + 6.48 = 4.77^{k}$$

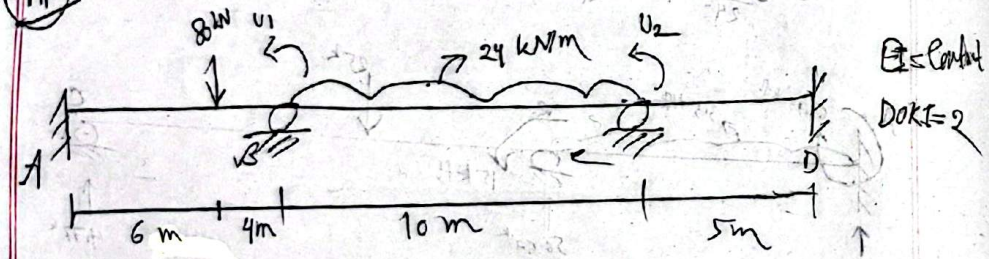


BMD:

Parabolic →



H/W



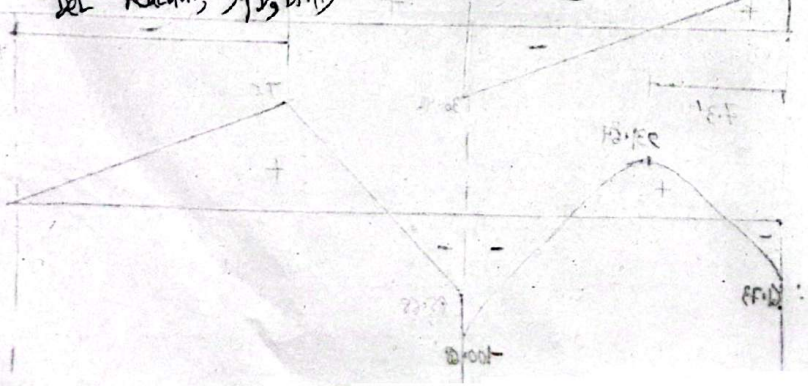
EI = Constant

DOKI = 2

$U_2 = \frac{192.35}{EI}$, U_1 = ?

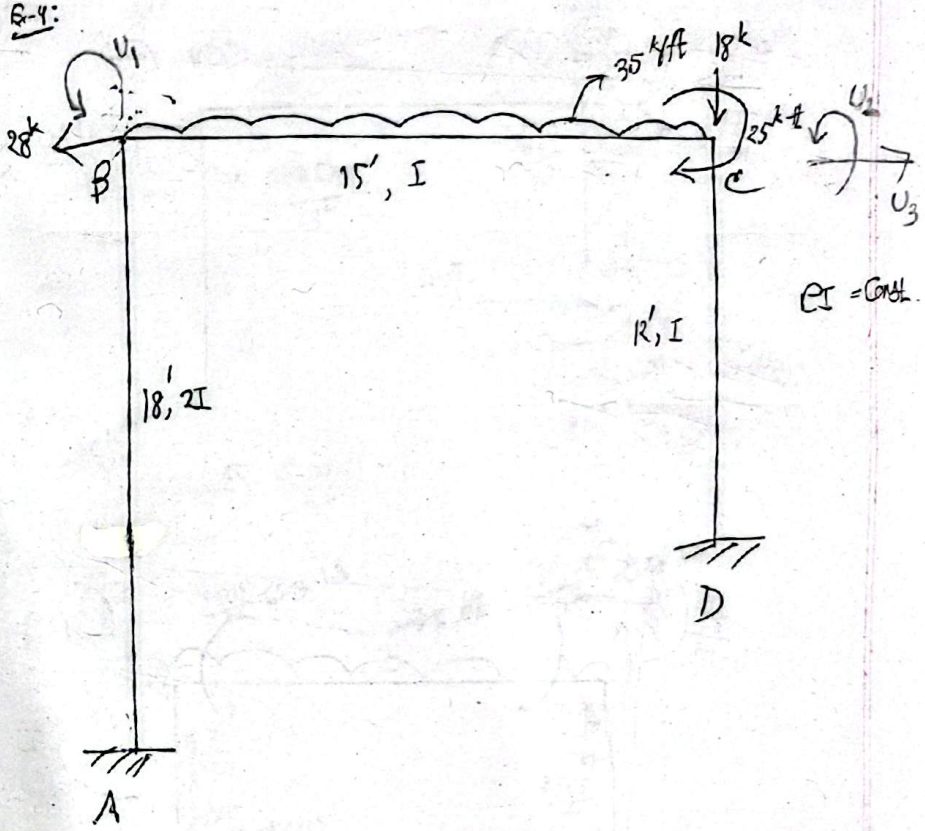
Det Reaction, SFD, BMD

$\left[U_1 = -\frac{154.1}{EI} \right]$



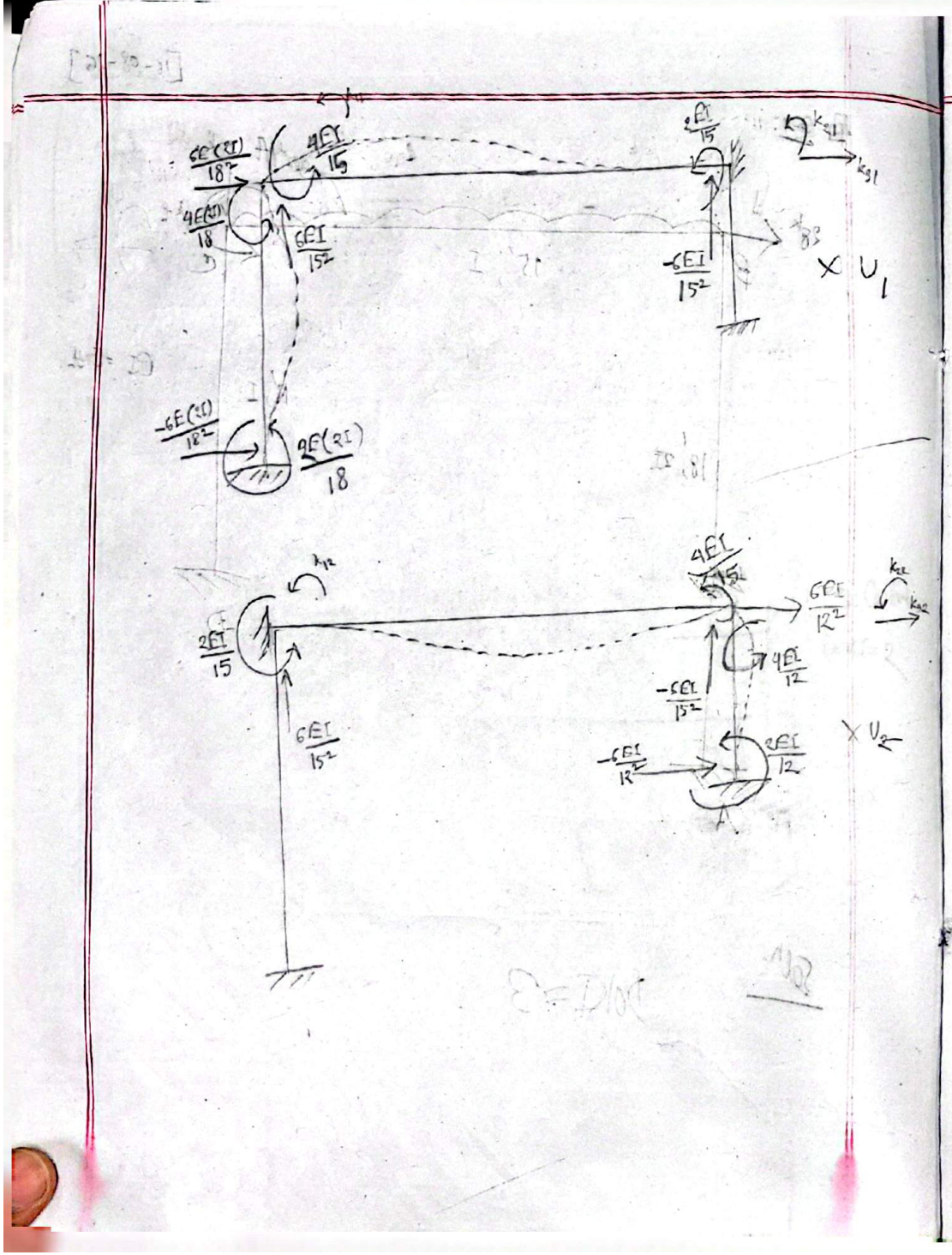
[16-08-26]

Q-4:



Soln:

DOKI = 3


6E(2I)/18²
4E(2I)/18
4EI/15
2EI/15
6EI/15²
-6EI/15²
4EI/15
6EI/12²
2EI/15
6EI/15²
-6EI/15²
4EI/12
2EI/12
-6E(2I)/18²
9E(2I)/18
12E(2I)/18³
6E(2I)/18²
12EI/12³
6EI/12²
-12E(2I)/18³
6E(2I)/18²
-12EI/12³
6EI/12²
3.5X15²/12 = 65.63 k-ft
3.5 k/ft
3.5X15/2 = 26.25 k
3.5X15/2 = 26.25 k

At B,

$$0 = 65.63 + \left(\frac{4EI}{15} + \frac{8EI}{18}\right)U_1 + \frac{2EI}{15}U_2 + \frac{12EI}{18^2}U_3 \quad \text{(I)}$$

At C,

$$-25 = -65.63 + \frac{2EI}{15}U_1 + \left(\frac{4EI}{15} + \frac{4EI}{12}\right)U_2 + \frac{6EI}{12^2}U_3 \quad \text{(II)}$$

&

$$-28 = 0 + \frac{12EI}{18^2}U_1 + \frac{6EI}{12^2}U_2 + \left(\frac{24EI}{18^3} + \frac{4EI}{12^3}\right)U_3 \quad \text{(III)}$$

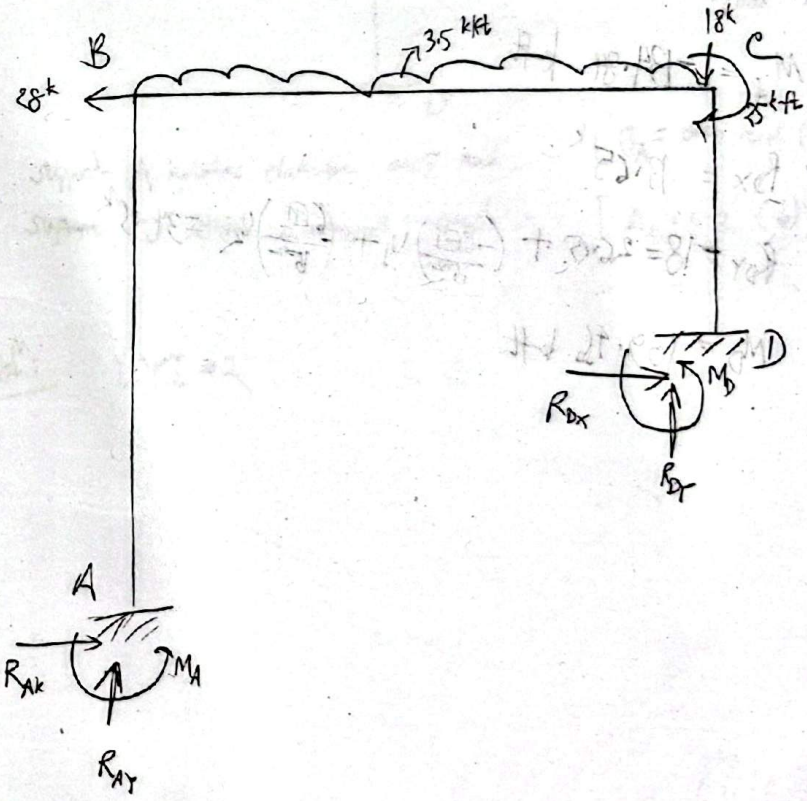
$$\begin{bmatrix} 0 \\ -25 \\ -28 \end{bmatrix} = \begin{bmatrix} 65.63 \\ -65.63 \\ 0 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \times \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -65.63 \\ 40.63 \\ -28 \end{bmatrix} = k \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$U_1 = \frac{51.42}{EI}$$

$$U_2 = \frac{830.57}{EI}$$

$$U_3 = -\frac{3949.21}{EI}$$



At B,

$$R_{AY} = 26.25 + \frac{6EI}{15^2}\theta_1 + \frac{6EI}{15^2}\theta_2 + 0 = 36.44^{k}$$

$R_{AX} = 14.35^{k}$

$M_A = -134.81$ k-ft

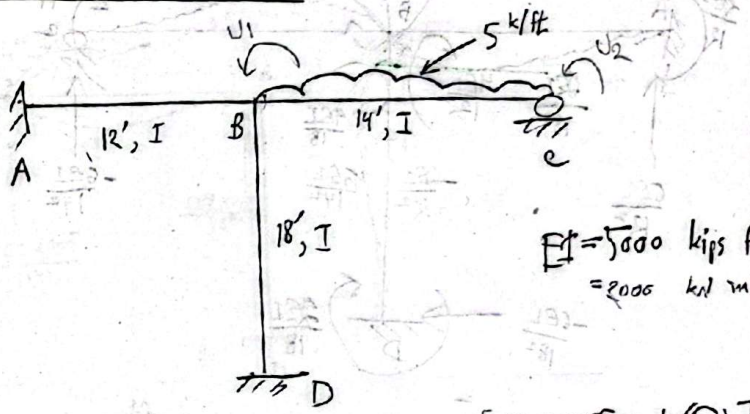
At C, $R_{DX} = 13.65^{k}$

$$R_{DY} = 18 = 26.25 + \left(\frac{-6EI}{15^2}\right)\theta_1 + \left(\frac{6EI}{15^2}\right)\theta_2 = 34.05^{k}$$

$M_D = -109.46$ k-ft

Support Settlement & Rotation:

Ex-1:



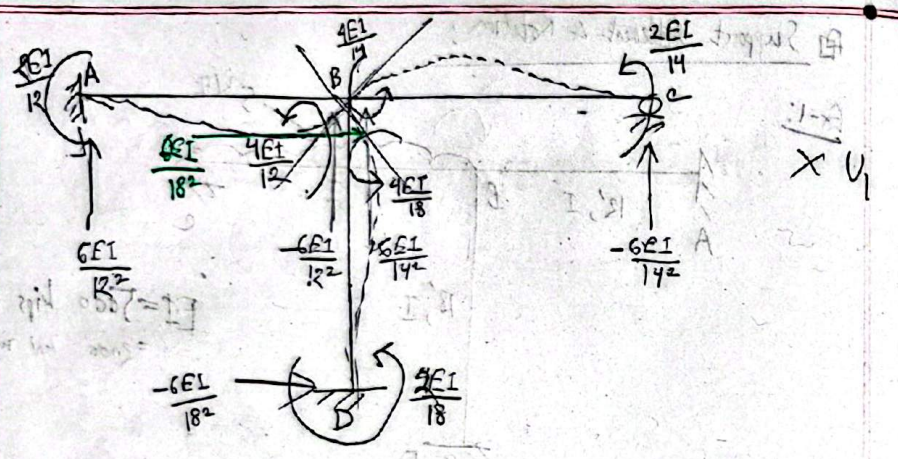
$EI = 5000$ kips ft^2
$= 2000$ kN m^2

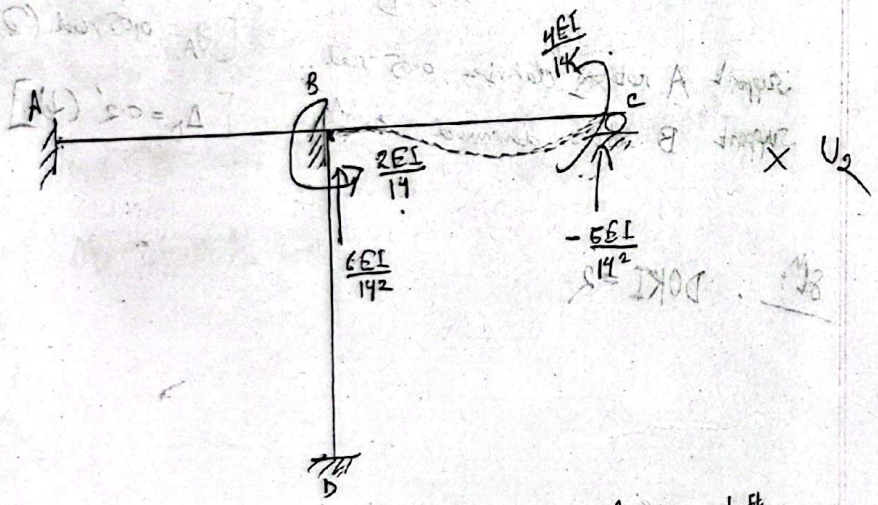
support A rotates clockwise 0.05 rad. [$\theta_A = 0.05$ rad (↻)]

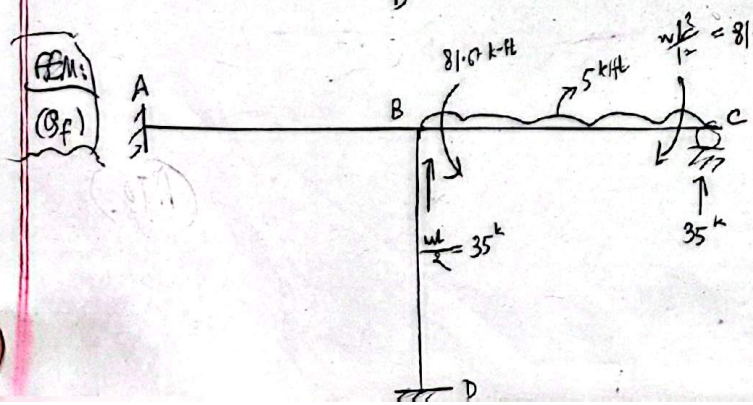
support D settles downward 0.2'. [$\Delta_D = 0.2'$ (↓)]

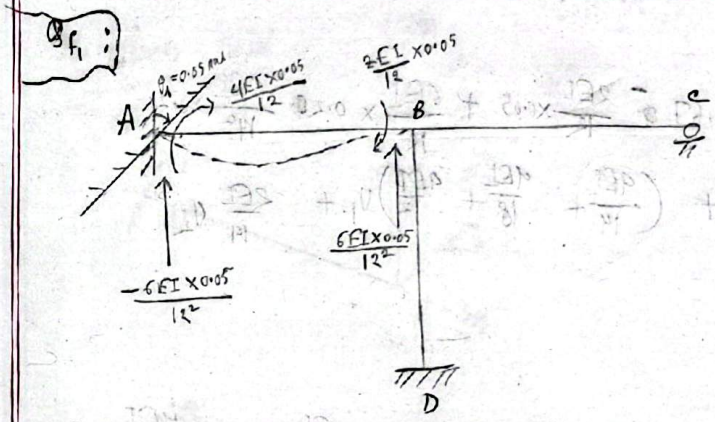
Sol'n: DOKI = 2

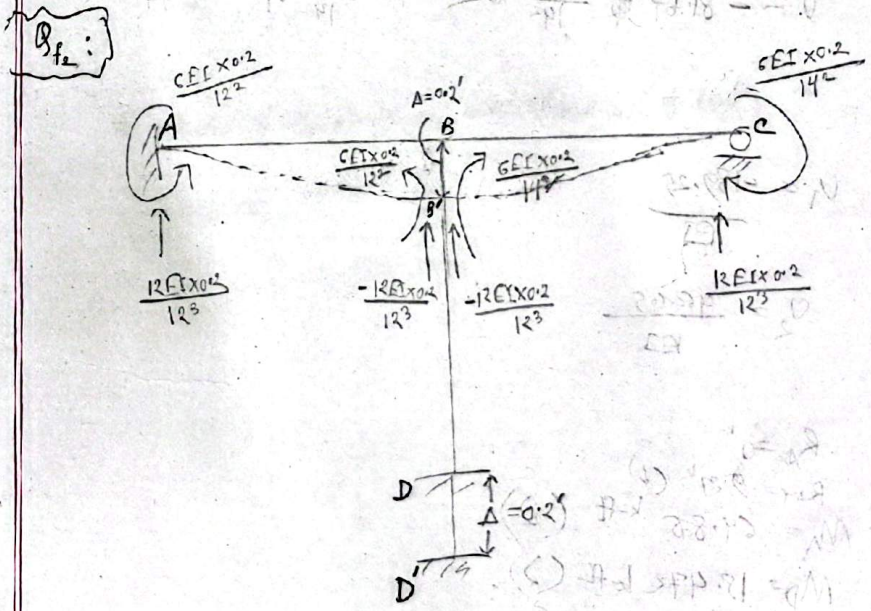
(To be continued)


4EI/14
2EI/14
6EI/12
A
B
C
4EI/12
4EI/18
6EI/12²
6EI/14²
-6EI/14²
-6EI/18²
B'
4EI/18
× U1


4EI/14
A
B
C
2EI/14
-6EI/14²
6EI/14²
D
× U2


FEM:
(Bf)
A
B
C
81.67 k-ft
5 k/ft
wL²/12 = 81.67 k-ft
wL/2 = 35k
35k
D


Bf1 :
θA = 0.05 rad
4EI×0.05/12
2EI/12 ×0.05
A
B
C
-6EI×0.05/12²
6EI×0.05/12²
D


Bf2 :
6EI×0.2/12²
A=0.2'
A
B
C
6EI×0.2/14²
6EI×0.2/12²
6EI×0.2/14²
B'
12EI×0.2/12³
-12EI×0.2/12³
-12EI×0.2/12³
12EI×0.2/12³
D
Δ=0.2'
D'

At B,

$$0 = +81.67 - \frac{2EI}{12}\times 0.05 + \frac{6EI}{12^2}\times 0.2 - \frac{6EI}{14^2}\times 0.2 + \left(\frac{4EI}{14} + \frac{4EI}{18} + \frac{4EI}{12}\right)U_1 + \frac{2EI}{14}U_2$$

At e,

$$0 = -81.67 - \frac{6EI}{14^2}\times 0.2 + \frac{2EI}{14}U_1 + \frac{4EI}{14}U_2$$

$$U_1 = \frac{-139.25}{EI}$$

$$U_2 = \frac{496.05}{EI}$$

$R_{Ax} = 0^k$

$R_{Ay} = 9.27^k$ (↓)

$M_A = 64.85$ k-ft (↺)

$M_D = 18.472$ k-ft (↻)

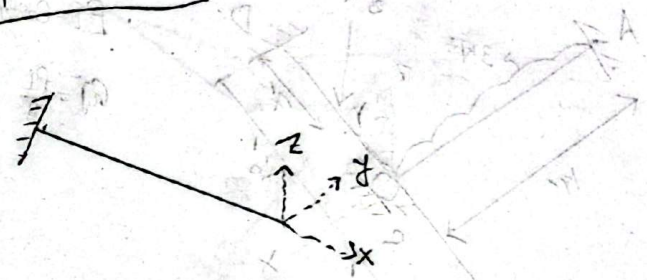
$R_{Dx} = 2.58^k$ (→)

$R_{Dy} = 49.8^k$ (↑)

$R_{Cy} = 29.47^k$ (↑)

[30-08-26]

Plane Coild: (31)



$$\theta = \frac{TL}{GJ}$$

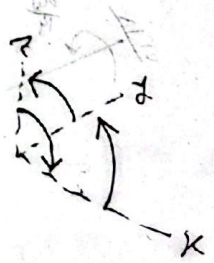
G = Modulus of Torsion/Shear modulus/Modulus of Rigidity

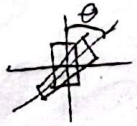
J = Torsional constant

GJ = Torsional Rigidity

Sign convention:

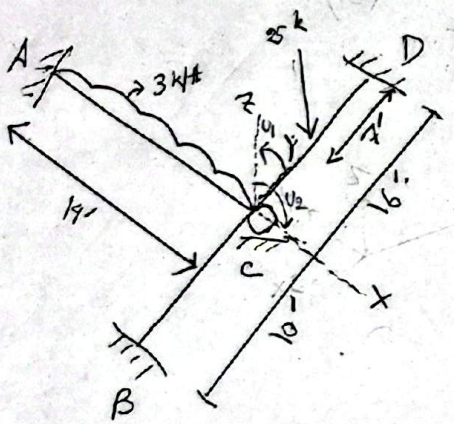
x to y
y to z
z to x
} ⊕ve





Torsional stiffness, $\boxed{\frac{T}{\theta} = \frac{GJ}{L}}$

Ex-1:

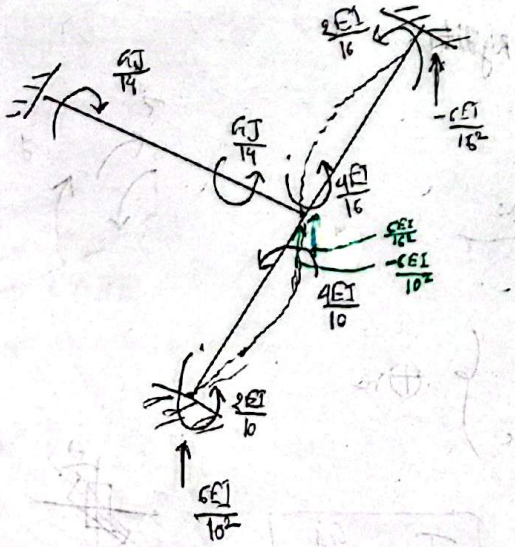


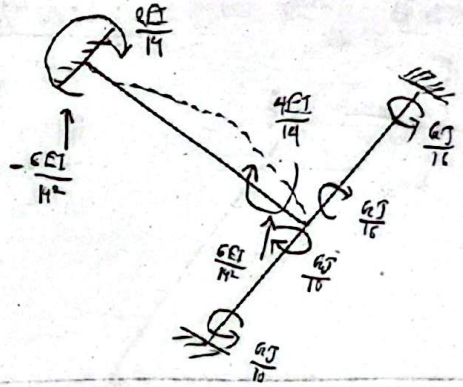
EI = Constant

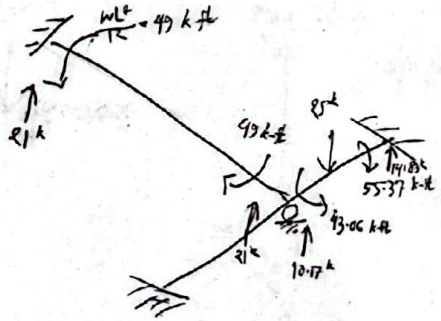
GJ = EI/3

Sol^n:

DOKI = 2







At C,

$$0 = \left(\frac{4EI}{16} + \frac{4EI}{10} + \frac{GJ}{14}\right)U_1 + 0 \times U_2 + 43.06 \quad \text{— (I)}$$

[For U_1] [Against X]

$$0 = 0 \times U_1 + \left(\frac{4EI}{14} + \frac{GJ}{16} + \frac{GJ}{10}\right)U_2 + 49 \quad \text{— (II)}$$

[For U_2] [Against Y]

Solving eq^n (I) & (II),

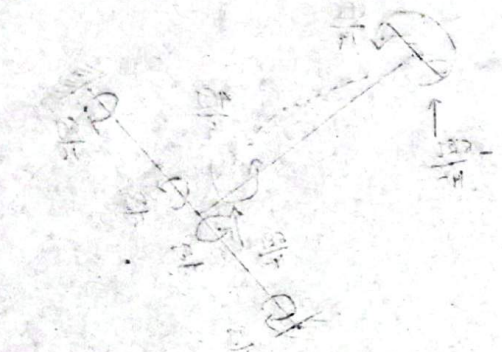
$$U_1 = \frac{-52.93}{EI}$$

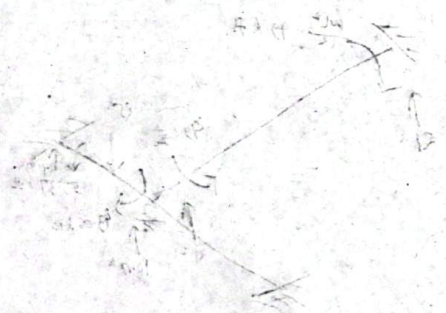
$$U_2 = \frac{-144.17}{EI}$$

$R_E = 29.094^{k}$

$M_D = -63.3\%\ k\text{-ft}$

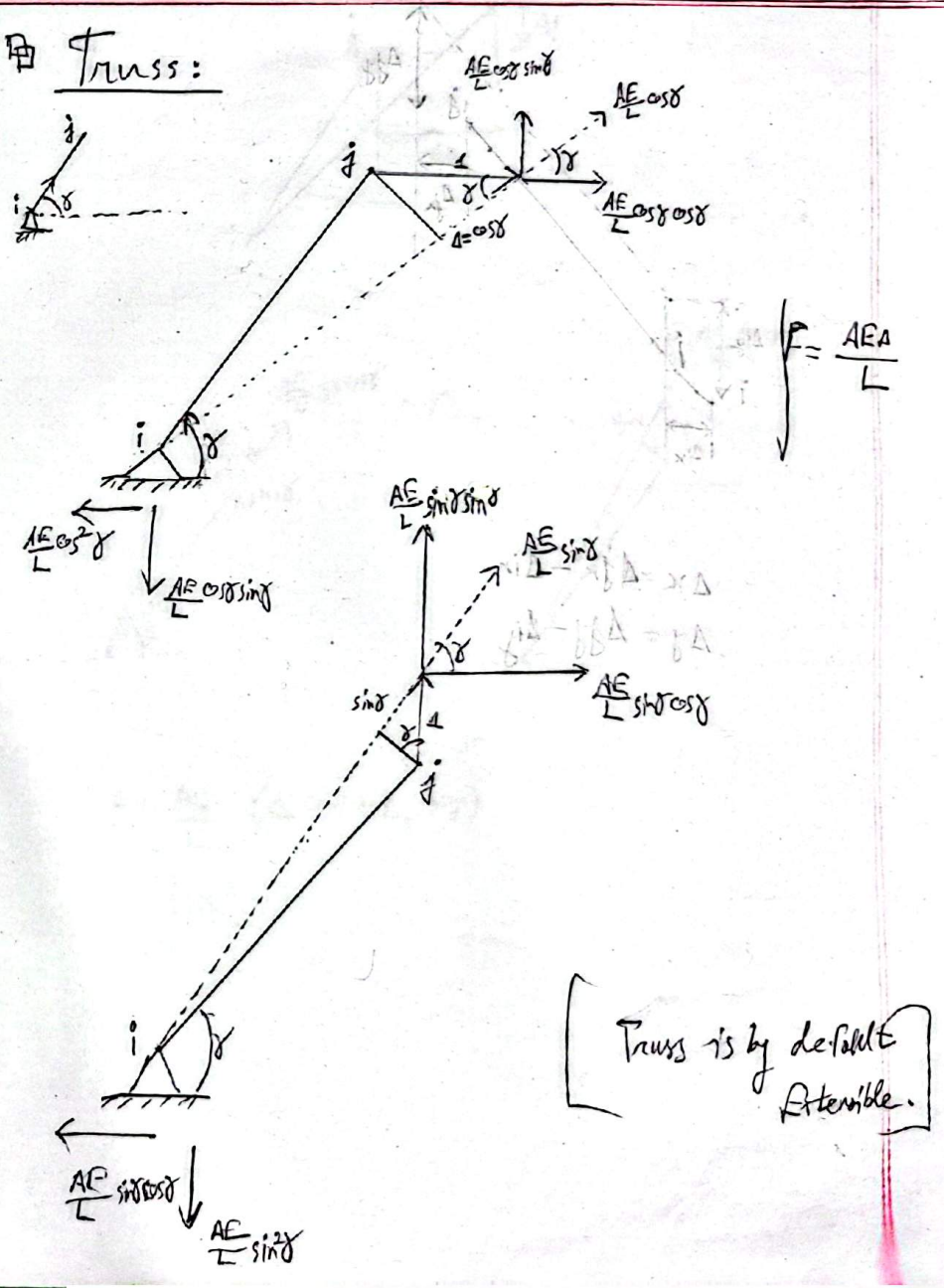
$T_D = -3^{U}\ k\text{-ft}$



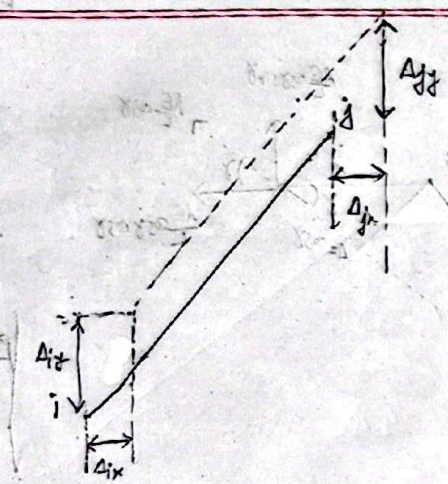


[06-09-25]

Truss:

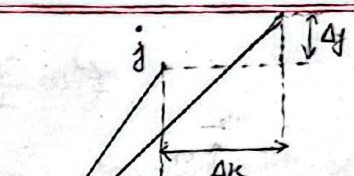


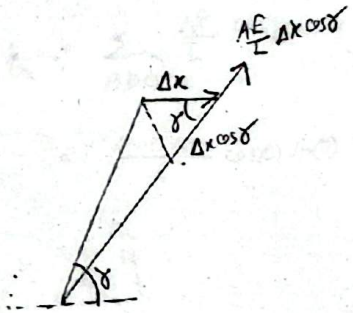
[Truss is by default stable.]

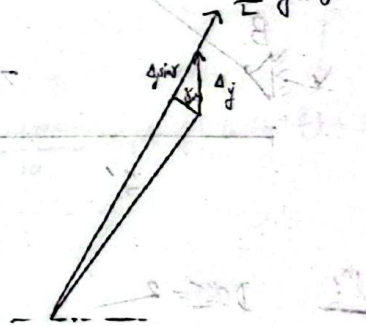


$\Delta x = \Delta_{jx} - \Delta_{ix}$

$\Delta y = \Delta_{jy} - \Delta_{iy}$

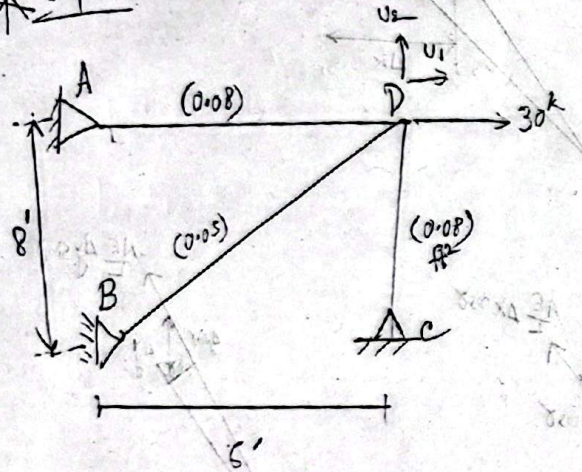






$$F = \frac{AE}{L}\left(\Delta_x \cos\gamma + \Delta_y \sin\gamma\right)$$

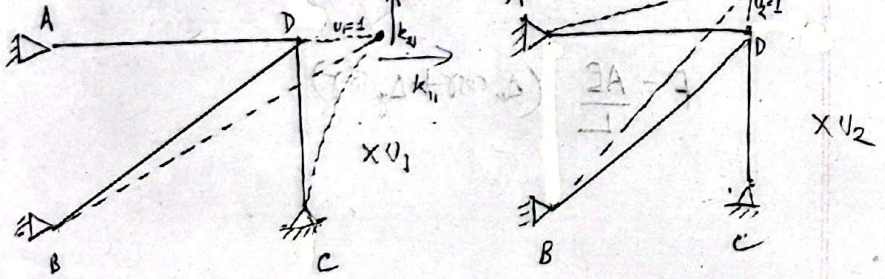
Exp-1



$E = 1000\ ^k/ft^2$

Sol^n:

DOI = 2



$$k_{11} = \sum_{AD,BD,CD} \frac{AE}{L}\cos^2\gamma$$

$$= \frac{0.08\times1000}{6}\cos(0)^2 + \frac{0.05\times1000}{10}\cos\left(\tan^{-1}\frac{8}{6}\right)^2 + \frac{0.08\times1000}{8}\cos(90)^2$$

$$= 15.13$$

$$k_{21} = \sum_{AD,BD,CD} \frac{AE}{L}\cos\gamma\sin\gamma$$

$$= \frac{0.08\times1000}{6}\cos(0)\sin(0) + \frac{0.05\times1000}{10}\cos\left(\tan^{-1}\frac{8}{6}\right)\sin\left(\tan^{-1}\frac{8}{6}\right) + \frac{0.08\times1000}{8}\cos(90)\sin(90)$$

$$= 2.4$$

$$k_{12} = \sum_{AD,BD,CD} \frac{AE}{L}\cos\gamma\sin\gamma = 2.4$$

$$k_{22} = \sum_{AD,BD,CD} \frac{AE}{L}\sin^2\gamma$$

$$= \frac{0.08\times1000}{6}\sin(0)^2 + \frac{0.05\times1000}{10}\sin\left(\tan^{-1}\frac{8}{6}\right)^2 + \frac{0.08\times1000}{8}\sin(90)^2$$

$$= 132$$

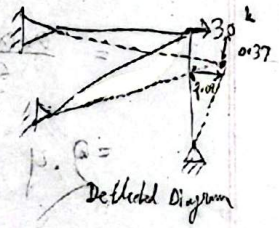
$P_j = KU$

$$30 = k_{11} U_1 + k_{12} U_2$$
$$0 = k_{21} U_1 + k_{22} U_2$$

$$\begin{bmatrix} 30 \\ 0 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$U_1 = 2.04$

$U_2 = -0.37$

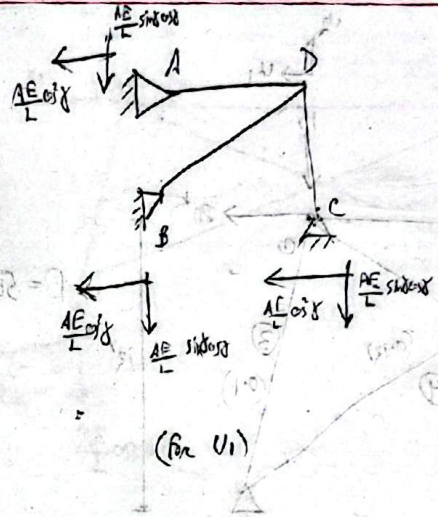


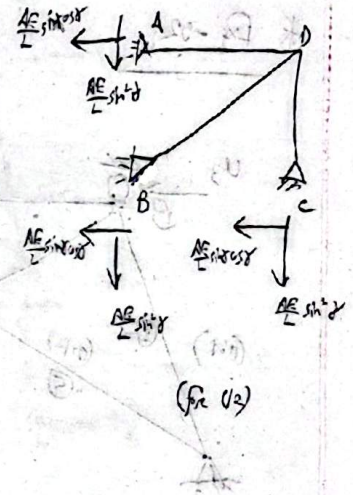
$$F = \frac{AE}{L}(\Delta x \cos\gamma + \Delta y \sin\gamma)$$

$\therefore F_{AD} = 27.2^k$ (T)

$F_{BD} = 4.64^k$ (T)

$F_{CD} = -3.7^k$ (C)





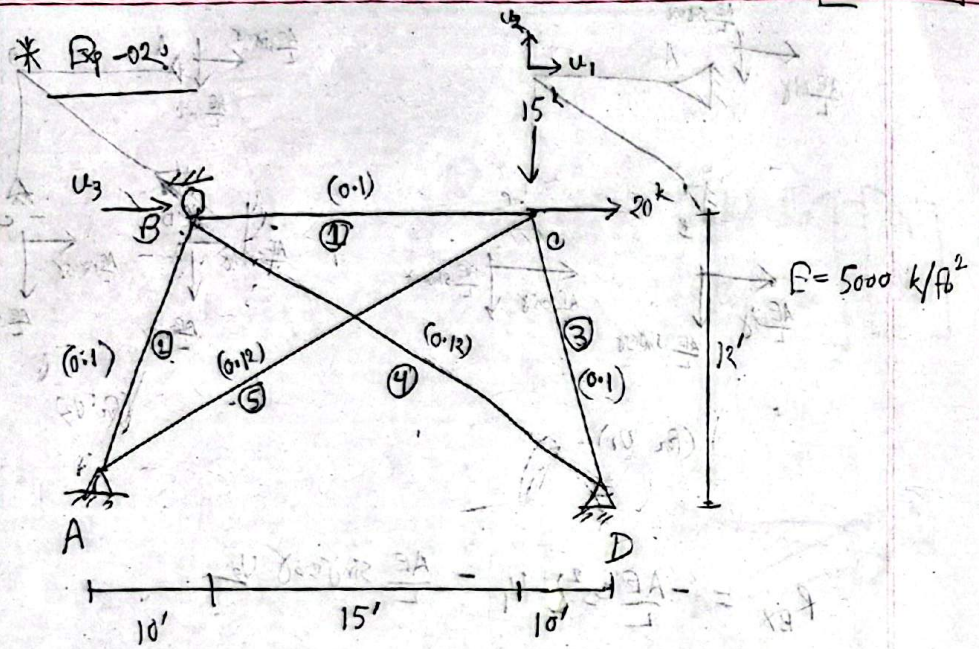
$$R_{BX} = -\frac{AE}{L}\cos^2\gamma\, U_1 - \frac{AE}{L}\sin\gamma\cos\gamma\, U_2$$

$= -2.784^k$ ($\leftarrow$)

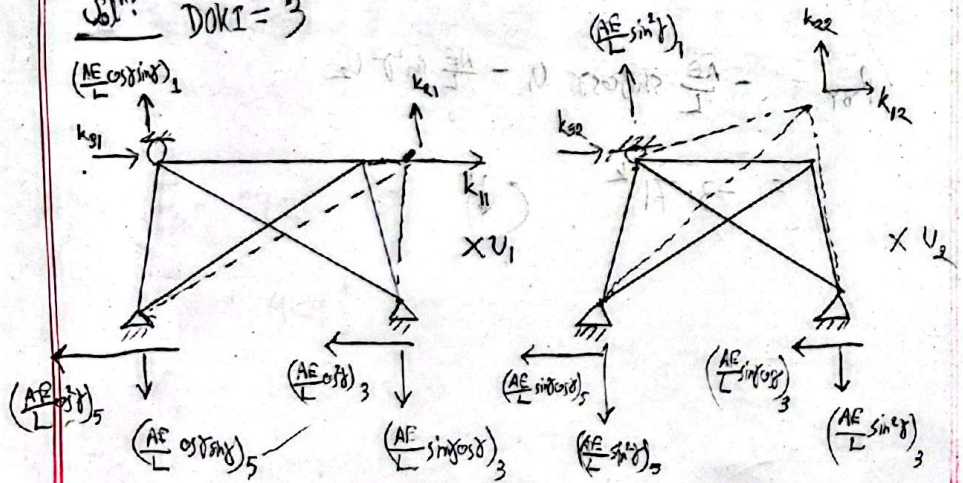
$$R_{BY} = -\frac{AE}{L}\sin\gamma\cos\gamma\, U_1 - \frac{AE}{L}\sin^2\gamma\, U_2$$

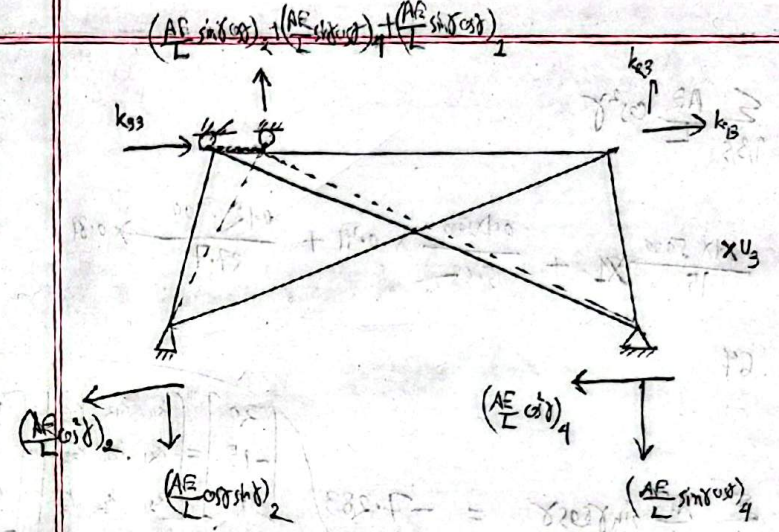
$= -3.71^k$ ($\downarrow$)

* Eg-02:



Sol^n: DOKI = 3





Member Name	Member End point	i	j	L (ft)	A (ft²)	γ	$\cos^2\gamma$	$\sin^2\gamma$	$\cos\gamma\sin\gamma$
1	BC	B	C	15	0.1	0	1	0	0
2	AB	A	B	15.62	0.1	50.19	0.41	0.59	0.49
3	CD	D	C	15.62	0.1	129.81	0.41	0.59	-0.49
4	BD	D	B	27.7	0.12	127.6	0.81	0.19	-0.39
5	AC	A	C	27.7	0.12	52.69	0.81	0.19	0.39

$$k_{11} = \sum_{135} \frac{AE}{L} \cos^2\gamma$$

$$= \frac{0.4 \times 5000}{15} \times 1 + \frac{0.1 \times 5000}{15.62} \times 0.41 + \frac{0.12 \times 5000}{27.7} \times 0.81$$

$$= 64$$

$$k_{21} = k_{12} = \sum \frac{AE}{L} \sin\gamma \cos\gamma = -7.283$$

$$k_{31} = k_{13} = -33.33$$

$$k_{22} = 22.937$$

$$k_{32} = k_{23} = 0$$

$$k_{33} = 64.04$$

$$\begin{bmatrix} 20 \\ -15 \\ 0 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$R_{AX} = 3.78^k$ (←)

$R_{BX} = 16.22^k$ (←)

$R_{AY} = 3.50^k$ (↓)

$R_{BY} = 1.30^k$ (↑)

$R_{DY} = 17.20^k$ (↑)

Member forces:

$F_{AB} = 3.67^k$ (T)

$F_{AC} = 1.59^k$ (T)

$F_{CD} = 20.43^k$ (C)

$F_{BC} = 5.47^k$ (T)

$F_{BD} = -3.49^k$ (C)

1 set Computer

Formalization of Stiffness Method

& Direct Stiffness method

[20-09-26]

* Member End's shear force & moment determination.

* Qualitative deflective shape.

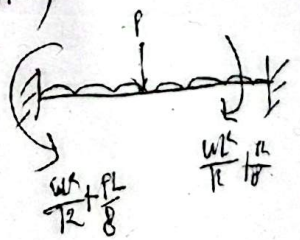
Manual

Formalization/Matrix

Brief

* UVL,

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