14.06.26
Lec Zihad

Syllabus

1) Direct stiffness method
2) Moment dist method
3) IL for indeterminate structures

Book Structural Analysis by Aslam Kassimal

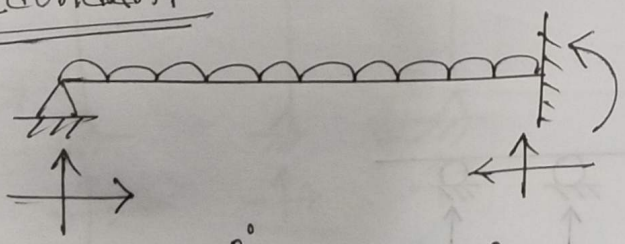
Direct Stiffness Method

DoSI and DoKI
static ← → kinetic

Degree of static indeterminancy ⇒

- External static indeterminancy
- Internal static indeterminacy

Redundant:



2°

DoSI

$$I_s = I_{se} + I_{sI}$$

I_{se} → external, I_{sI} → internal

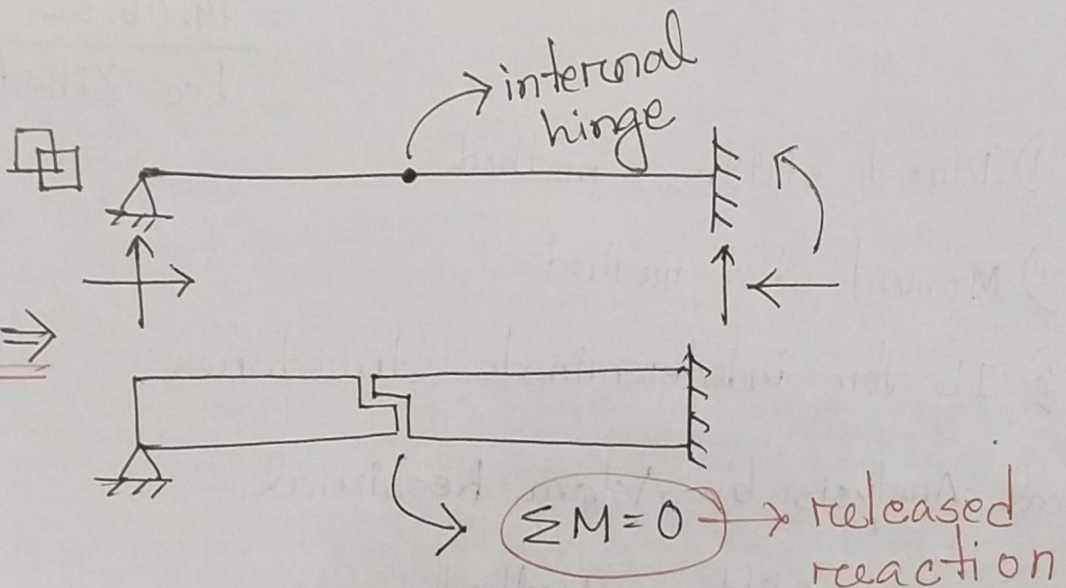
$$= (\text{External reaction} - 3) + (3m - 3J + 3 - R_r)$$

m → member, J → joints, R_r → released reaction

$$= R - 3 + 3m - 3J + 3 - R_r$$

$$I_s = (R + 3m) - (3J + R_r)$$

→ 2D



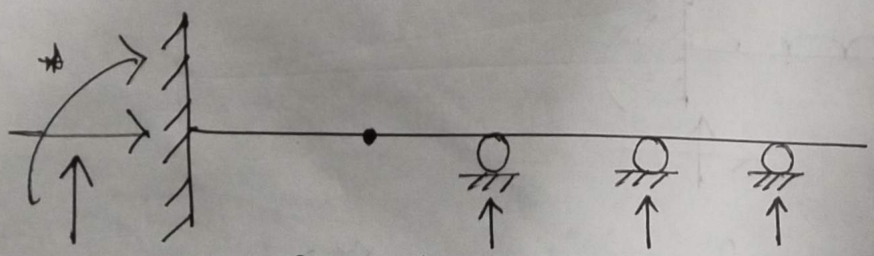
For 3D ; $I_s = (3R+6m) - (6j+R_p)$

Truss ⇒ $I_s = m+R-2j$

Do SI = 0 ; Stable n determinate

Do SI > 0 ; Stable n indeterminate

Do SI < 0 ; unstable n determinate

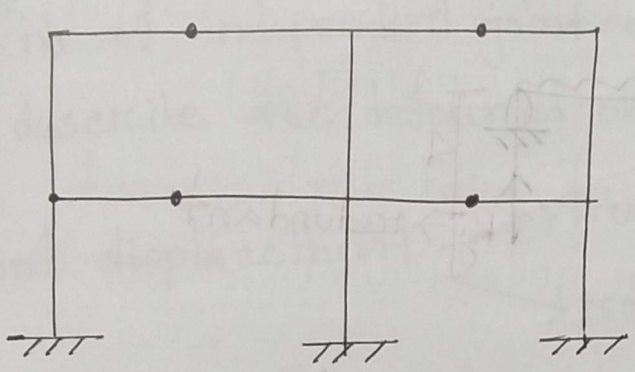


$I_s = (R+3m) - (3j+R_p)$

$= (6+3\times4) - (3\times5+1)$

$= 2^\circ$

*

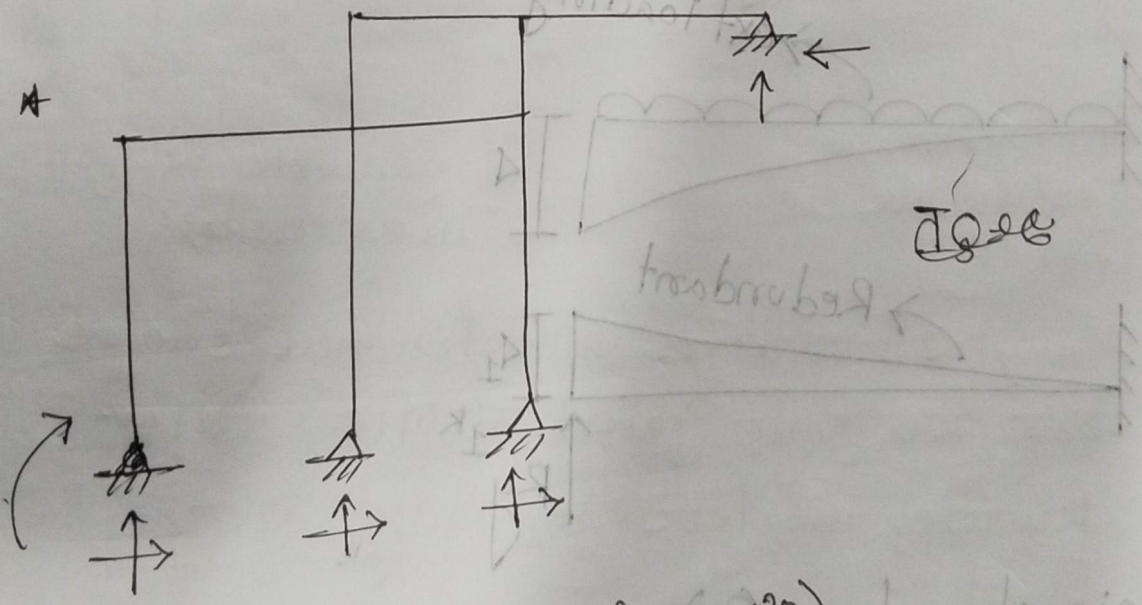


$I_s = (3R + 3m) - (3j + Rr)$

$= (9 + 3 \times 4) - (3 \times 3 + 4)$

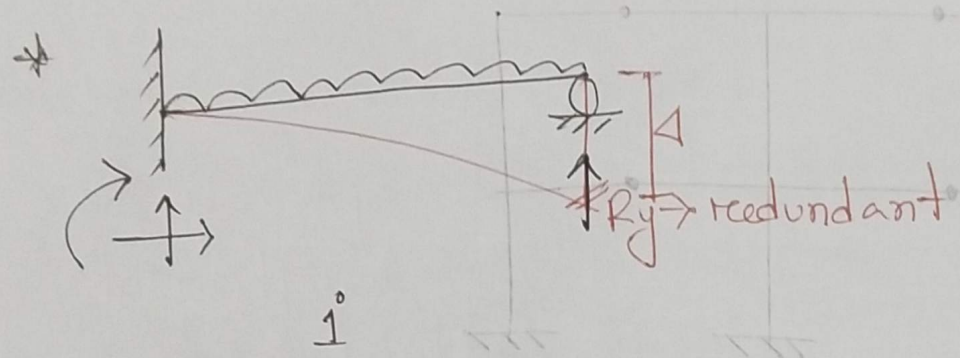
$= 8$

*



$I_s = (9 + 3 \times 9) - (3 \times 9 + 3)$

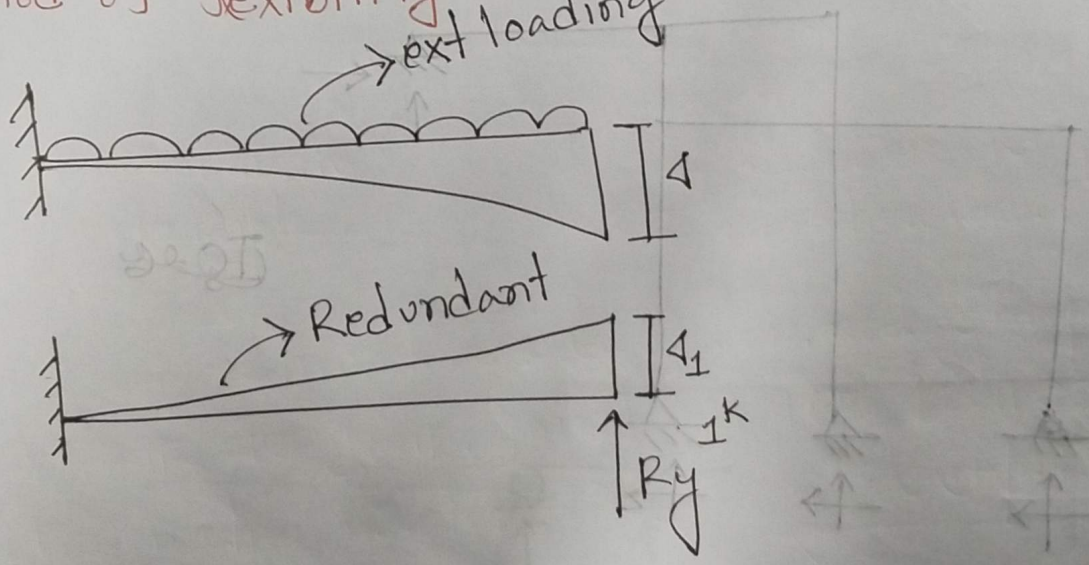
$= 6$



1°

Compactibility eqn → deformation due to combined effect (ext loading, redundant) must be equal to primary structures deformation.

method of flexibility ⇒



main structure ⇒ $0 = \Delta + R_y \Delta_1$

$\Rightarrow 0 = \Delta + R_y \Delta_1$

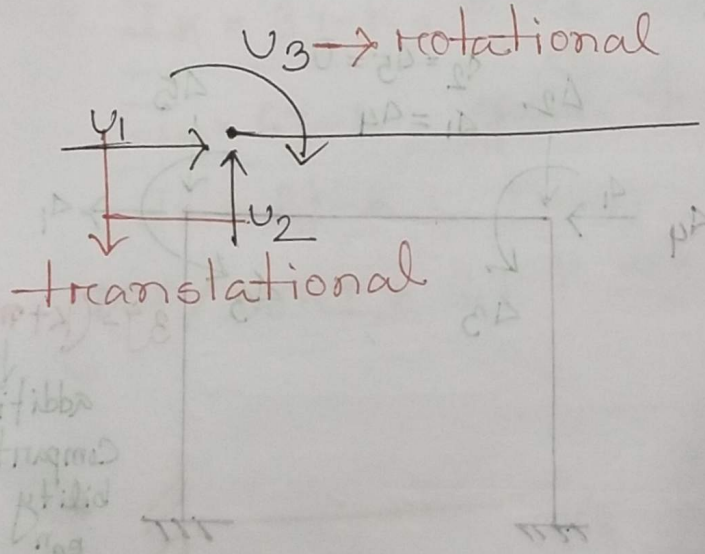
$\Rightarrow R_y =$

Truss example must

DoKI

no of independent joint displacement to describe the deformed shape

joint displacement → translational
→ rotational



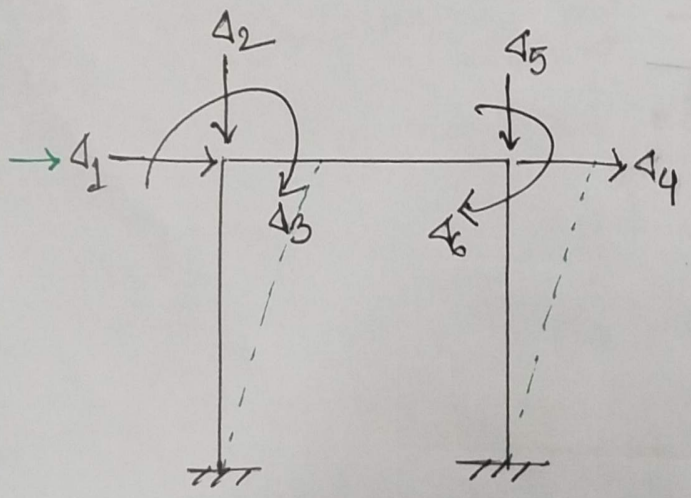
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DoKI (Imp)

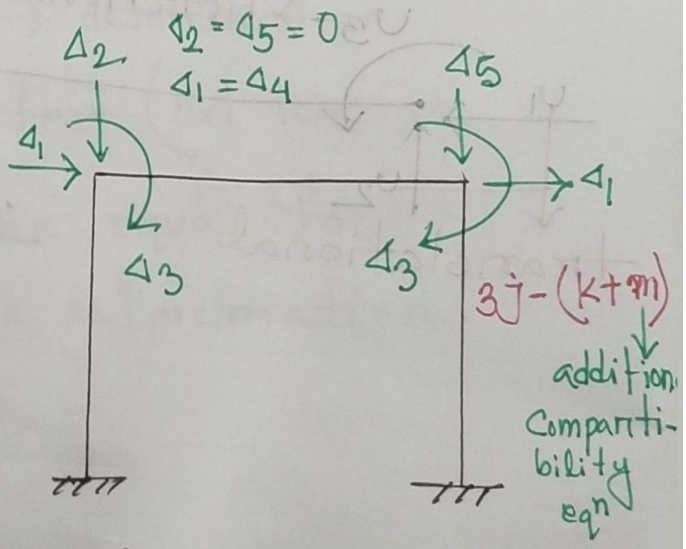
DoKI / Degree of freedom (DoF)

→ No of independent joint displacement (transitional ∩ rotation) to describe the deformed shape.



if member is extensible

DOF = 6



if member is inextensible

DOF = 3

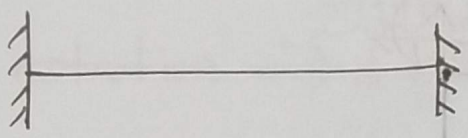
[For any structure ∪ deformed shape এর জন্য যতগুলো joint লাগবে সেটাই তার Kinetic indeterminancy তত]

* For inextensible → member সরবে না, load এর জন্য shape সরবে
Kinetic indeterminancy = 3

* For Beam axial load is zero

* Redundant ↑ displacement ↓

*



DOF = 0 [fixed]

DOSI = 3

no of constraint given by support

$I_K = 2j - K$, for Beam (inextensible)

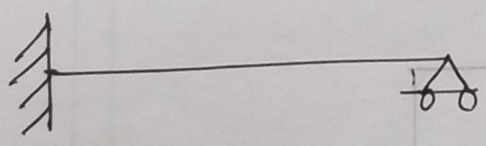
$I_K = 3j - K$, plane frame (2D)

$I_K = 6j - K$, space fram (3D)

$I_K = 2j - K$, truss

$DOF / DOKI = 2j - K = (2 \times 2) - 4 = 0$

*



$I_K = 2j - K = 2 \times 2 - 4 = 0$

$I_K = 2j - (k+m)$

$= 2 \times 2 - (2+2)$

~~$2j - (k+m) = 3 \times 4 -$~~

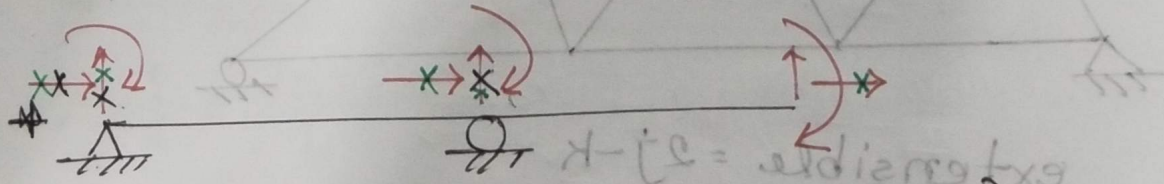
for extensible, $I_K = 2j - K$

$= 2 \times 2 - 4 = 0$

inextensible, $I_K = 3j - (K+m)$

$= 3 \times 2 - (2+2)$

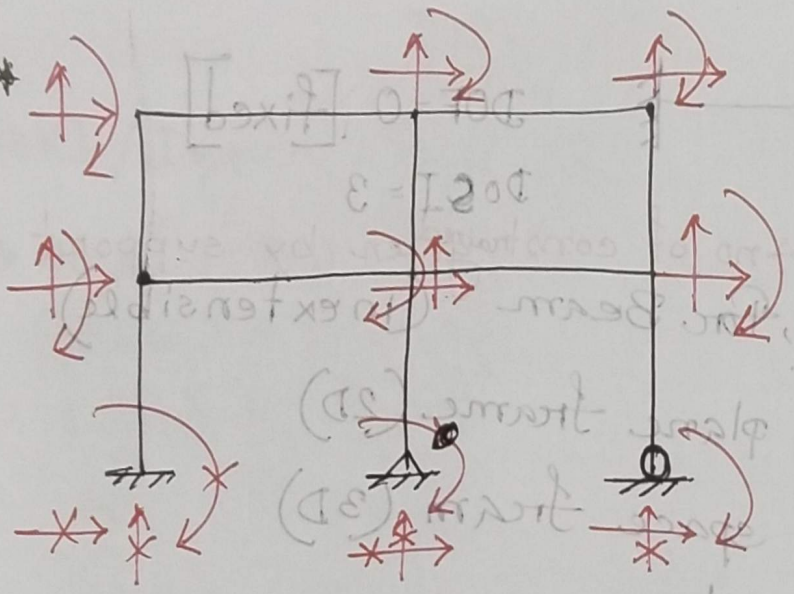
$= 2$



for extensible, $I_K = 6$ (Black ink)

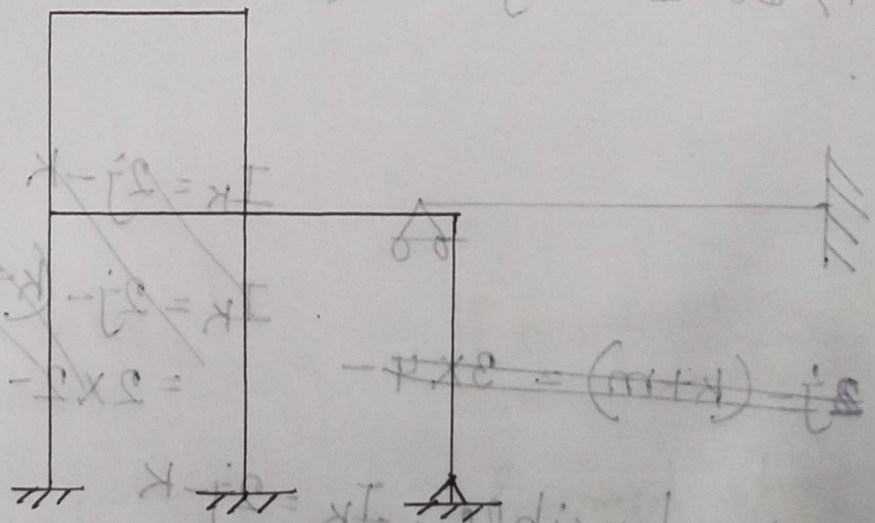
inextensible, $I_K = 4$ (Green ink)

*



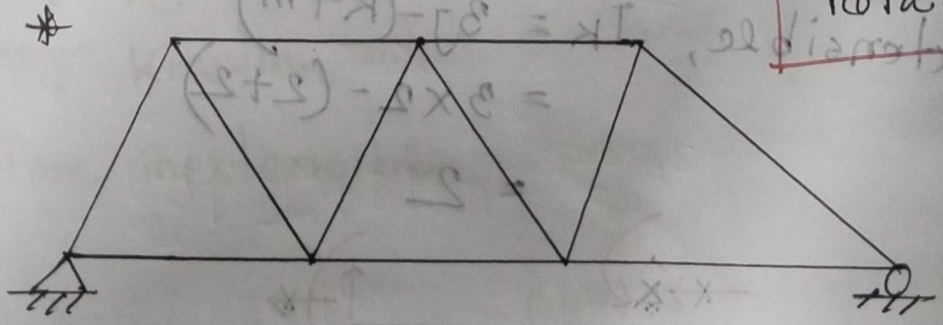
$I_k = 21$

*



truss ला rotation नाही

*



extensible $= 2j - k$

$= 2 \times 7 - 3 = 11$

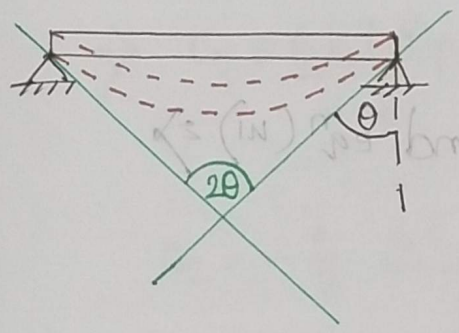
* For simply supported beam, DoSI = 0

DoKI = 2

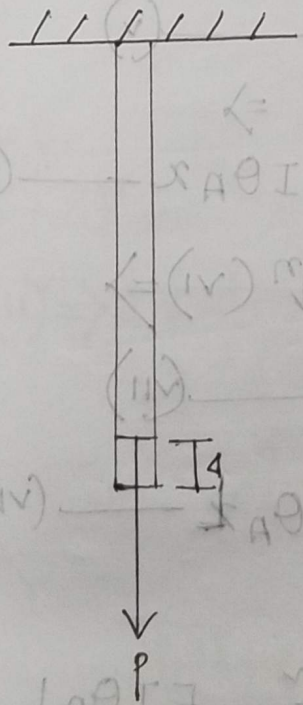
28.06.26

Stiffness — Fixed End Moment (FEM)

For rotation ⇒ ***

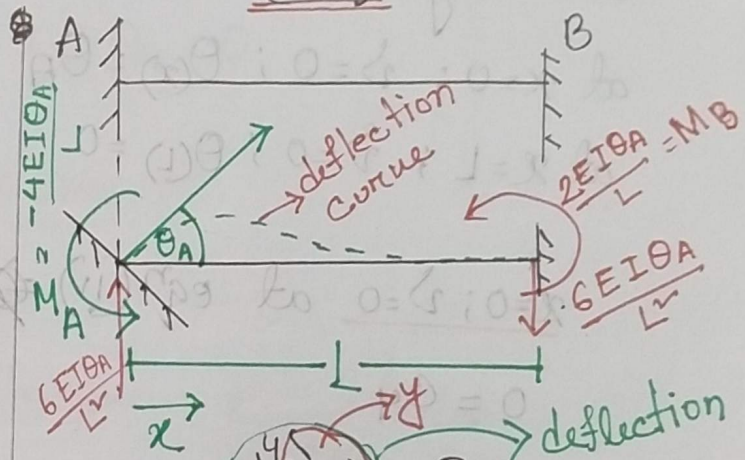


$$\text{stiffness} = \frac{M}{\theta}$$



$$\Delta = \frac{PL}{AE}$$

$$\Rightarrow \frac{P}{\Delta} = \frac{AE}{L}$$



$$EI\frac{d^4v(x)}{dx^4} = P$$

as no ext loading, $P=0$

$$EI\frac{d^4v(x)}{dx^4} = 0$$

$$\Rightarrow EI\frac{d^3v(x)}{dx^3} = C_1 \quad \{\text{Shear}\} \quad \text{(I)}$$

$$\Rightarrow EI\frac{d^2v(x)}{dx^2} = C_1x + C_2 \quad \{\text{moment}\} \quad \text{(II)}$$

$$\Rightarrow EI\frac{dv(x)}{dx} = C_1\frac{x^2}{2} + C_2x + C_3 \quad \{\text{slope}\} \quad \text{(III)}$$

$$\Rightarrow EIv(x) = \frac{1}{6}C_1x^3 + C_2\frac{x^2}{2} + C_3x + C_4 \quad \{\text{deflection}\} \quad \text{(IV)}$$

{1 unit rotation এর জন্য যে পরিমাণ moment লাগবে সেটাই Stiffness

Boundary conditions $\Rightarrow$

at $x=0$; $v=0$; $\theta(x)=\theta_A$

at $x=L$; $v=0$; $\theta(L)=0$

$x=0; v=0$ at eqn (iv) and eqn (iii) $\Rightarrow$

$0 = C_4$

$EI\theta_A = C_3$

Substituting value of C_3 in eqn (iii) $\Rightarrow$

$$EI\theta x = C_1\frac{x^2}{2} + C_2 x + EI\theta_A \quad \text{(v)}$$

Subs value of C_3, C_4 in eqn (iv) $\Rightarrow$

$$EIv(x) = \frac{1}{6}C_1x^3 + C_2\frac{x^2}{2} + EI\theta_A x \quad \text{(vi)}$$

$x=L$, $v=0$ at eqn (v) and eqn (vi) $\Rightarrow$

$$0 = C_1\frac{L^2}{2} + C_2 L + EI\theta_A \quad \text{(vii)}$$

$$0 = \frac{1}{6}C_1L^3 + C_2\frac{L^2}{2} + EI\theta_A L \quad \text{(viii)}$$

$L \times (vii) - (viii) \Rightarrow$

$$C_1\frac{L^3}{2} + C_2L^2 + EI\theta_A L - \frac{1}{6}C_1L^3 - C_2\frac{L^2}{2} - EI\theta_A L$$

$$\Rightarrow C_1\left(\frac{L^3}{2} - \frac{L^3}{6}\right) + C_2\left(L^2 - \frac{L^2}{2}\right) = 0$$

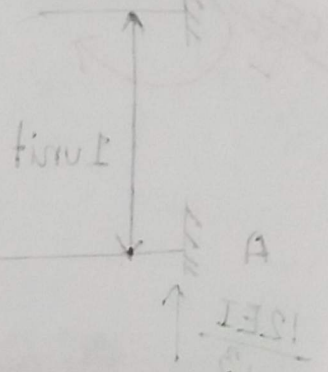
$$\Rightarrow C_1 = -C_2\frac{\left(L_2 - \frac{L^2}{2}\right)}{\left(\frac{L^3}{2} - \frac{L^3}{6}\right)}$$

$$\Rightarrow C_2 = -\frac{2}{3}C_1L$$

value of C_2 in eq^n (vii) $\Rightarrow$

$$C_1\frac{L^2}{2} - \frac{2}{3}C_1L^2 + EI\theta_A = 0$$

$$\Rightarrow C_1 = \frac{6EI\theta_A}{L^2} \quad \text{(shear)}$$



$$C_2 = -\frac{2}{3}\frac{6EI\theta_A}{L^2} \times L$$

$$\Rightarrow C_2 = -\frac{2}{3}\frac{6EI\theta_A}{L}$$

$$\Rightarrow C_2 = -\frac{4EI\theta_A}{L} \quad \text{(Moment)}$$

top $\rightarrow$ tension

bottom $\rightarrow$ compression

(ii) $\Rightarrow$ at $x = L$;

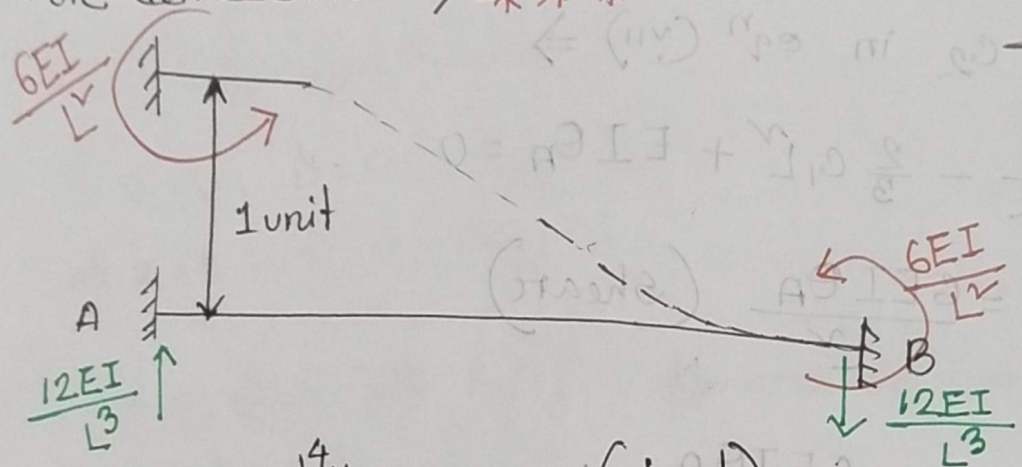
$$EI\frac{d^2y(x)}{dx^2} = \frac{6EI\theta_A}{L^2} \times L - \frac{4EI\theta_A}{L}$$

$$= \frac{2EI\theta_A}{L}$$

$$\text{Stiffness} = \frac{M_A}{\theta_A} = \frac{4EI}{L} = K$$

For deflection ⇒ ***

05.07.26



here,
AB → Beam

$$EI\frac{d^4y}{dx^4} = P = 0 \quad (\text{load})$$

$$EI\frac{d^3y}{dx^3} = C_1 \quad (\text{Sheare}) \text{ —— (i)}$$

$$EI\frac{d^2y}{dx^2} = C_1x + C_2 \quad (\text{Moment}) \text{ —— (ii)}$$

$$EI\frac{dy}{dx} = C_1\frac{x^2}{2} + C_2x + C_3 \quad (\text{Slope}) \text{ —— (iii)}$$

$$EIy = C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4 \quad (\text{deflection}) \text{ —— (iv)}$$

for BC ⇒

$x=0$; $y=1$; $\theta=0^\circ$
$x=L$; $y=0$; $\theta=0^\circ$
} Boundary condition

$x=0$; $y=1$ putting eqⁿ (iv) ⇒

$$C_4 = EI$$

$x=0$; $\theta=0$ putting eqⁿ (iii) ⇒

$$C_3 = 0$$

Sub C_3, C_4 ~~values~~ values,

$$EI\frac{dy}{dx} = C_1\frac{x_1^2}{2} + C_2x \text{ —— (v)}$$

$$EIy = C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + EI \text{ —— (vi)}$$

CamScanner

$x=L, y=0$ in (vi) $\Rightarrow$

$$0 = C_1\frac{L^2}{6} + C_2\frac{L^2}{2} + EI$$

$$\Rightarrow 0 = L^2\left(\frac{C_1}{6} + \frac{C_2}{2}\right)EI$$

$$\Rightarrow C_1 = \frac{12EI}{L^3}$$

$x=L, \theta=0$ in (v) $\Rightarrow$ $0 = C_1\frac{L^2}{2} + C_2 L$

$$\Rightarrow 0 = \frac{12EI}{L^3}\cdot\frac{L^2}{2} + C_2 L$$

$$\Rightarrow -C_2 L = \frac{12EI}{2L}$$

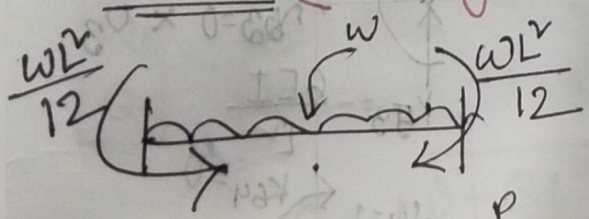
$$\Rightarrow -C_2 = \frac{6EI}{L}$$

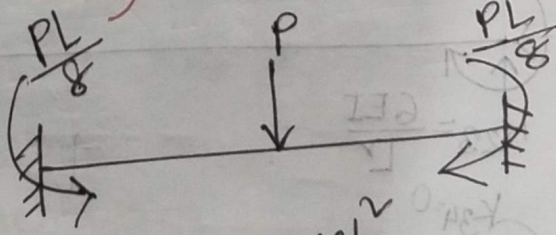
$$\Rightarrow C_2 = -\frac{6EI}{L^2}$$

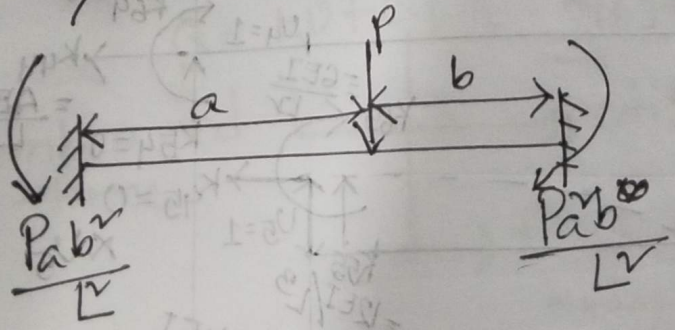
xm ও মোমেন্ট এন্টারে

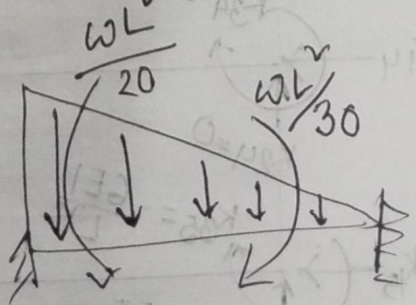
therefore, stiffness, $K = \frac{6EI}{L^2}$

FEM (Last pg of ref book)

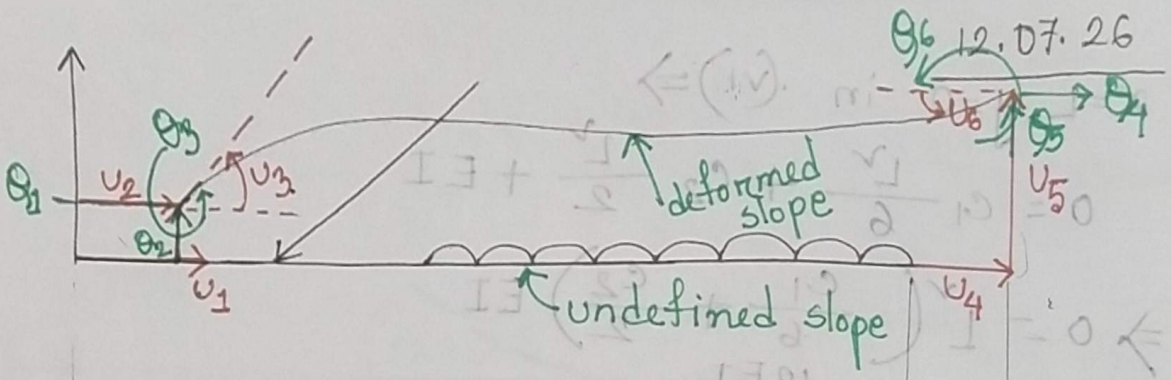








12.07.26



$\Delta = \frac{PL}{AE} \Rightarrow \frac{P}{\Delta} = \frac{AE}{L}$

$\frac{AE}{L} = K_{11}$ $K_{31}=0$ $K_{61}=0$ $K_{41} = \frac{-AE}{L} * U_1$ $K_{51}=0$ $K_{21}=0$

$0 = K_{12}$ $1 = U_2$ $K_{32} = \frac{6EI}{L^2}$ $K_{22} = \frac{12EI}{L^3}$ $K_{62} = \frac{6EI}{L^2}$ $K_{42}=0 \times U_2$ $K_{52} = -\frac{12EI}{L^3}$

$0 = K_{13}$ $K_{33} = \frac{4EI}{L}$ $K_{63} = \frac{2EI}{L}$ $K_{43}=0 \times U_3$ $K_{23} = \frac{6EI}{L^2}$ $K_{53} = -\frac{6EI}{L^2}$

$\frac{-AE}{L} = K_{14}$ $K_{34}=0$ $K_{64}=0$ $U_4=1$ $K_{44} = \frac{AE}{L} \times U_4$ $K_{24}=0$ $K_{54}=0$

$0 = K_{15}$ $K_{35} = -\frac{6EI}{L^2}$ $K_{65} = \frac{6EI}{L^2}$ $K_{45}=0$ $U_5=1$ $\times U_5$ $K_{25} = \frac{-12EI}{L^3}$ $K_{55} = 12EI/L^3$

$K_{16}=0$ $K_{36} = \frac{2EI}{L}$ $K_{66} = \frac{4EI}{L}$ $K_{46}=0 \times U_6$ $K_{26} = \frac{6EI}{L^2}$ $K_{56} = -\frac{6EI}{L^2}$

Of_1 $Of_3 = \frac{wL^2}{12}$ $Of_6 = -\frac{wL^2}{12}$ Of_4 $Of_2 = \frac{wL}{2}$ $Of_5 = \frac{wL}{2}$

$$Q_1 = k_{11} \times v_1 + k_{12} \times v_2 + k_{13} \times v_3 \cdots\cdots + Q_{f_1}$$

exam এ এর eqn solve এর জন্য দিবে

19.07.26

$$Q_2 = k_{21} v_1 + k_{22} v_2 + k_{23} \times v_3 + k_{24} v_4 + k_{25} v_5 + k_{26} v_6 + Q_{f_2}$$

$$Q_3 = k_{31} v_1 + k_{32} v_2 + k_{33} v_3 + k_{34} v_4 + k_{35} v_5 + k_{36} v_6 + Q_{f_3}$$

$$Q_4 = k_{41} v_1 + k_{42} v_2 + k_{43} v_3 + k_{44} v_4 + k_{45} v_5 + k_{46} v_6 + Q_{f_4}$$

$$Q_5 = k_{51} v_1 + k_{52} v_2 + k_{53} v_3 + k_{54} v_4 + k_{55} v_5 + k_{56} v_6 + Q_{f_5}$$

$$Q_6 = k_{61} v_1 + k_{62} v_2 + k_{63} v_3 + k_{64} v_4 + k_{65} v_5 + k_{66} v_6 + Q_{f_6}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} + \begin{bmatrix} Q_{f_1} \\ Q_{f_2} \\ Q_{f_3} \\ Q_{f_4} \\ Q_{f_5} \\ Q_{f_6} \end{bmatrix}$$

$Q = KV + Q_f$

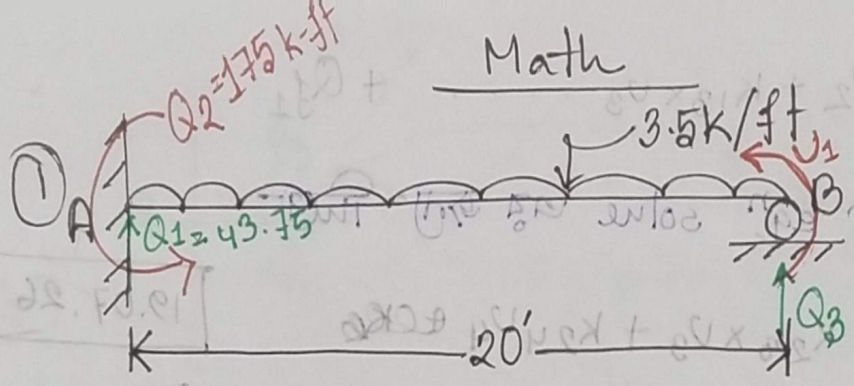
- Q → member end forces
- K → stiffness
- V → member displacement (translation, rotation)
- Q_f → fixed end moment for external loading

Inextensible beam

A ——— B → 1 force (rotation), no Horizontal

Math

Inextensible beam

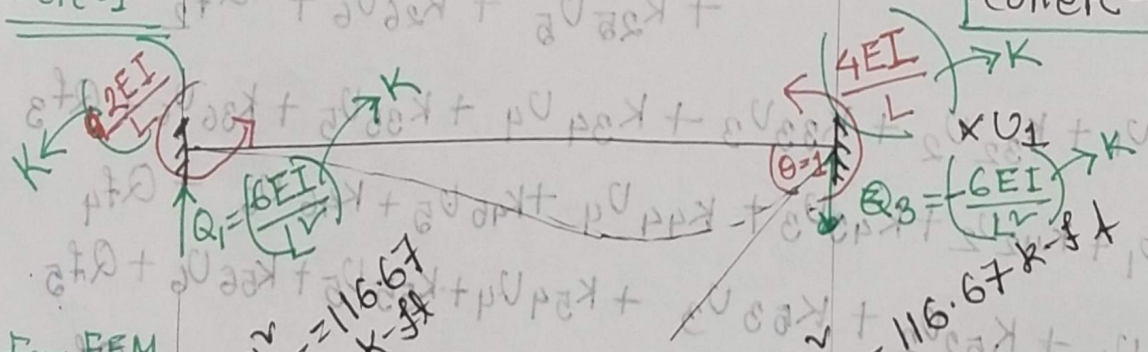


EI = Const.

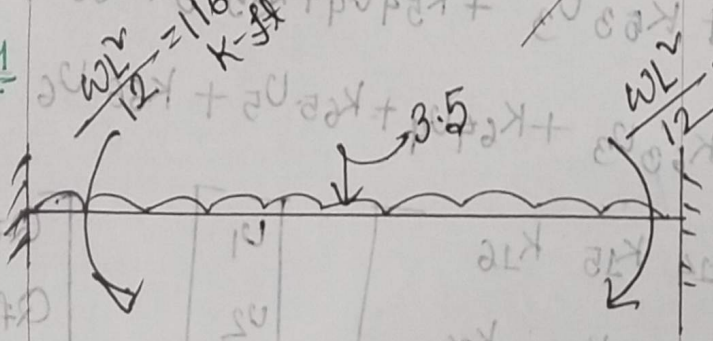
∴ DoKI = 1

External Cs
roller → Moment = 0

For U_1



For FEM



as at B there is ext roller support thus

$Q = 0$

$Q = KU + Q_f$

$\Rightarrow 0 = -\frac{4EI}{L} U_1 - \frac{WL^2}{12}$

$\Rightarrow 0 = \frac{4}{L} EI U_1 - 116.67$

$\Rightarrow U_1 = \frac{583.35}{EI}$

at point A $\Rightarrow$

$Q_2 = \frac{2EI}{L} U_1 + 116.67$

$= \frac{2EI}{L} \times \frac{583.35}{EI} + 116.67$

$= 175.005$ K-ft

for $Q_1 \Rightarrow$ $Q_1 = \frac{6EI}{L^2} \times U_1 + \frac{\omega L}{2}$

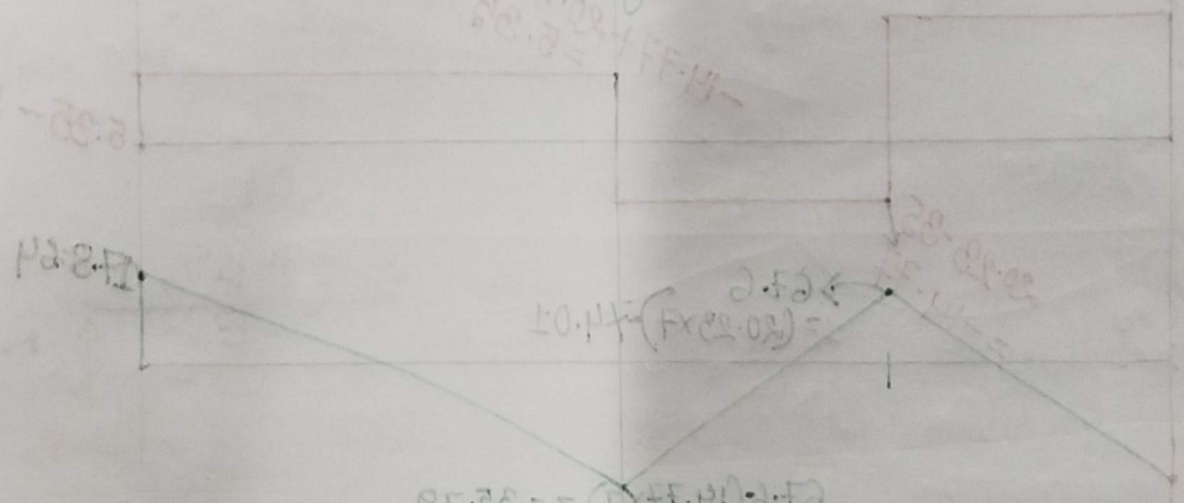
$= \frac{6EI}{L^2} \times \frac{583.35}{EI} + \frac{\omega L}{2}$

$= 43.75$ K-ft

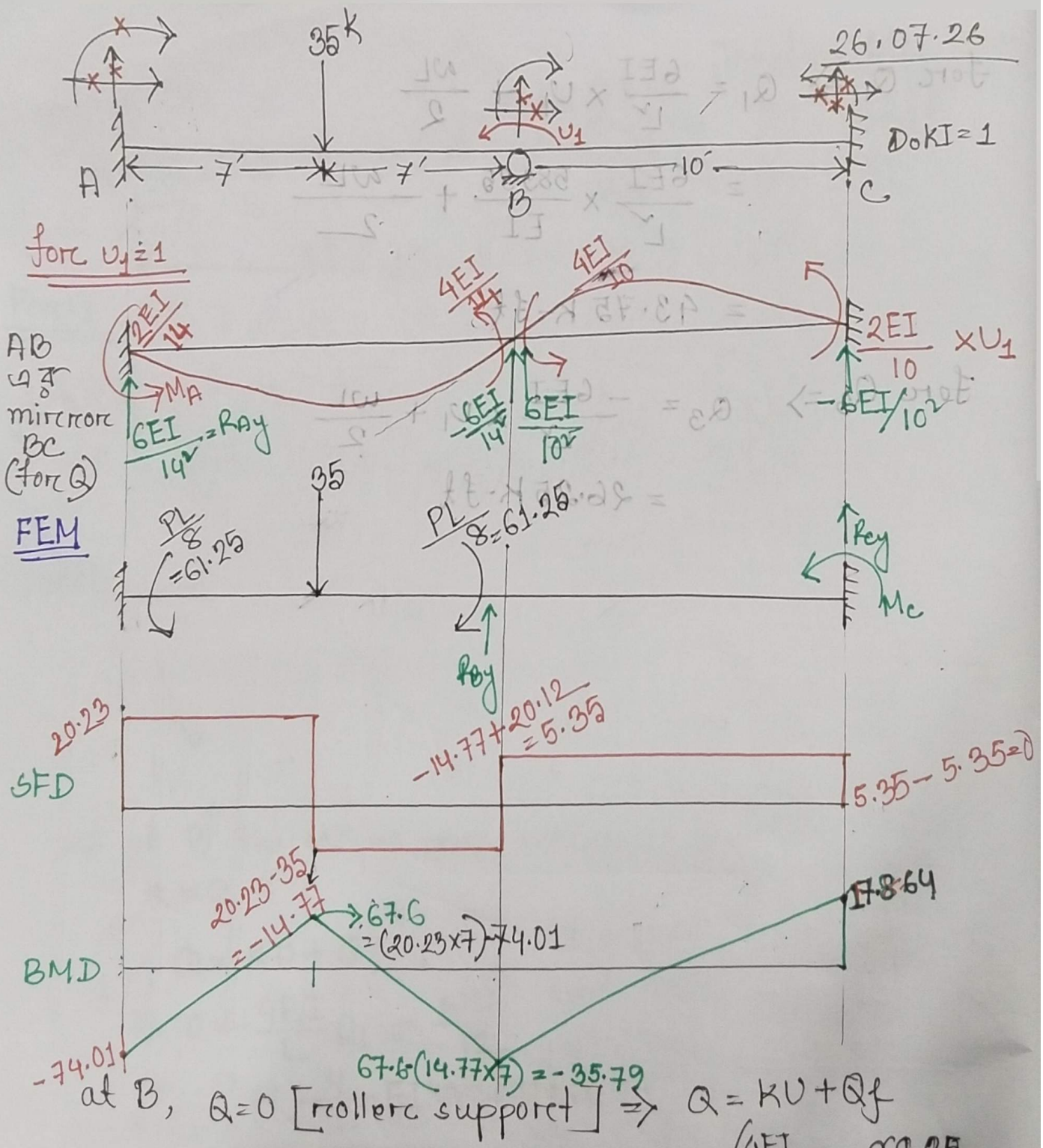
for $Q_3 \Rightarrow$ $Q_3 = \frac{-6EI}{L^2} \times U_1 + \frac{\omega L}{2}$

$= 26.25$ K-ft

FEM



26.07.26



at B, $Q = 0$ [roller c support] $\Rightarrow Q = KU + Qf$

$\Rightarrow 0 = \left(\frac{4EI}{14} + \frac{4EI}{10}\right) \times U_1 - 61.25$

$\Rightarrow 0.6857 U_1 - 61.25 = 0$

$\Rightarrow U_1 = \frac{89.32}{EI}$

$\left(\sum M_{ext} = \left(\sum M_{int}\right.\right.$

at A,

$M_A = KU + Qf$

$= \frac{2EI}{14} U_1 + 61.25$

$= 74.01$ k-ft

for R_{Ay} ⇒ $R_{Ay} = \frac{6EI}{L^2} \times v_1 + \frac{P}{2}$

$= \frac{6}{14^2} \times 89.32 + 68.25 \frac{35}{2}$

$= 20.23$

for R_{By} ⇒ $\frac{-6EI}{L^2} \times v_1 + \frac{6EI}{10^2} \times v_1 + \frac{P}{2}$

$L \rightarrow 14^2$

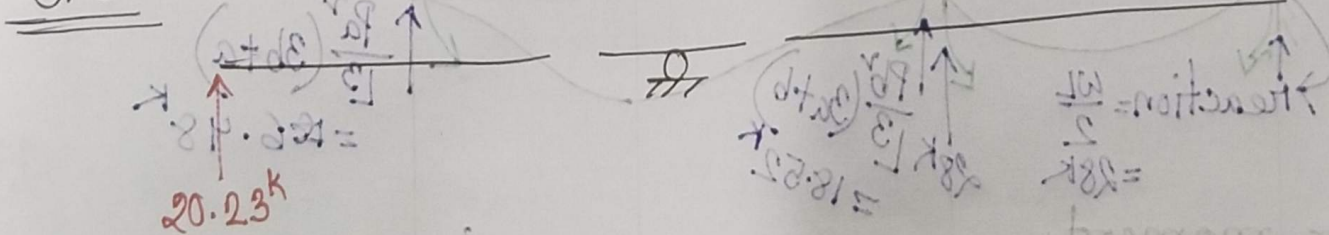
⇒ 20.12

for R_{Cy} ⇒ $\frac{-6EI}{10^2} \times v_1$

⇒ -5.35

for $M_C = \frac{2EI}{10} \times v_1 = 17.864$ K-ft

SFD



if Left side up → (+)ve shear
down → (+)ve shear
Right side up → (−)ve shear
down → (+)ve shear

BMD → tension → (−)ve

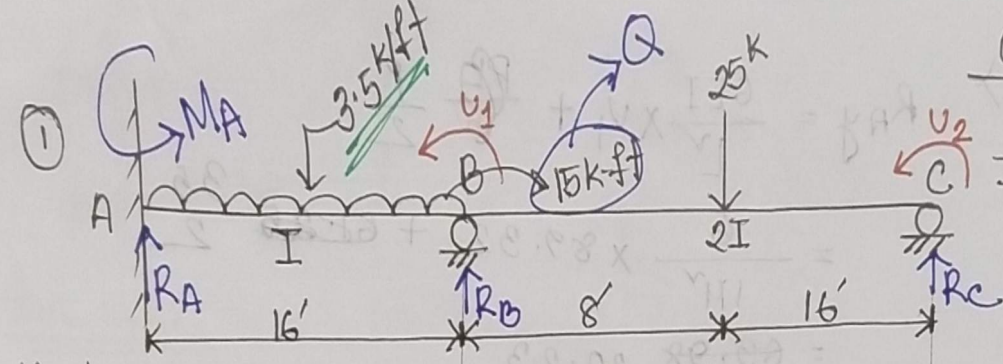
09.08.26

Zihad

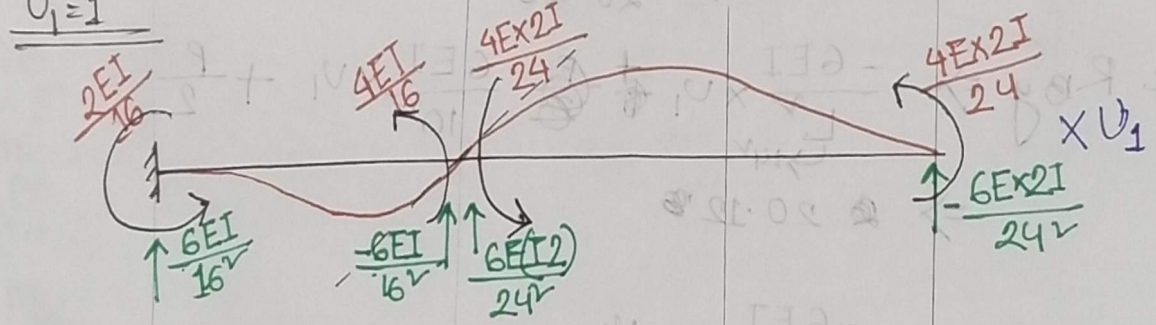
Inextensible

DoKI = 2

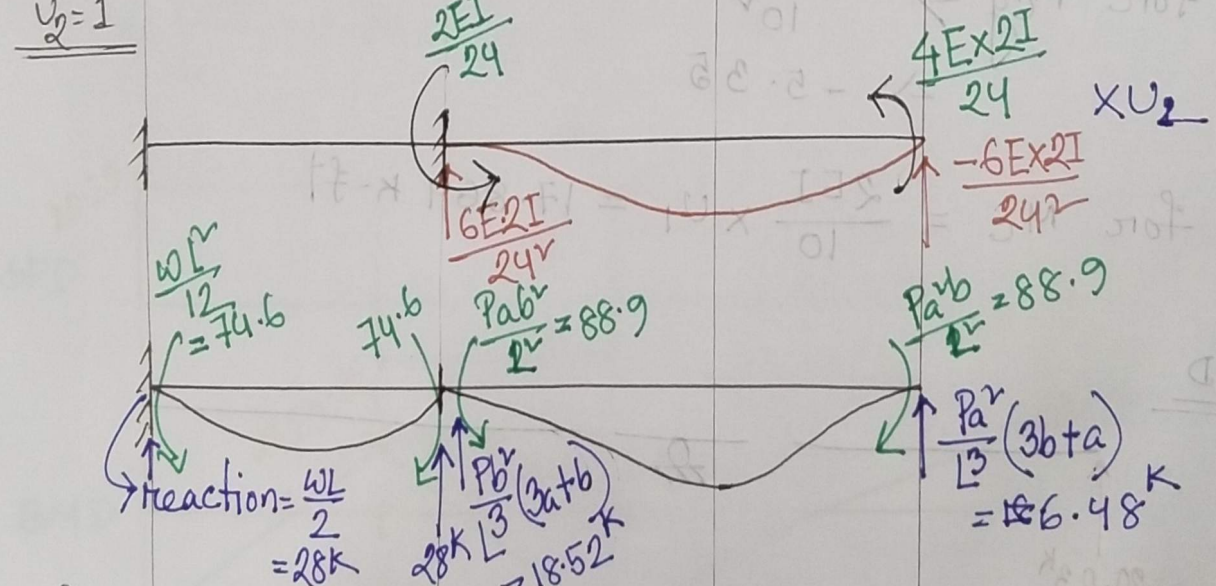
E = Const



$U_1 = 1$



$U_2 = 1$



for moment,

$Q = KU + Q_f$ {anticlock (+ve)}

at B,

$\Rightarrow -15 = \left(\frac{4EI}{16} + \frac{8EI}{24}\right)U_1 + \left(\frac{4EI}{24} \times U_2\right) + (74.6 + 88.9)$

$\Rightarrow -15 = \frac{7EI}{12}U_1 + \frac{4EI}{24} \times U_2 + 14.3$ ——— (i)

at C, $Q = KU + Q_f$

$\Rightarrow 0 = \frac{4EI}{24}U_1 + \frac{8EI}{24}U_2 - 44.4$ ——— (ii)

~~(i) − (ii) × 2 ⇒~~ (i), (ii) ⇒

$-30 = \left(\frac{14}{12}EI + \frac{4EI}{24}\right)U_1 - 44.4 - 28.6 = 0$

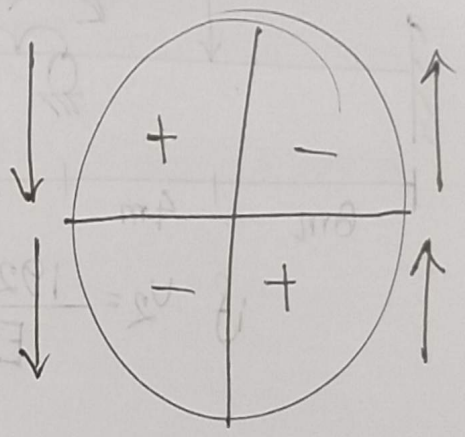
$\Rightarrow U_1 = \frac{-183}{EI}$ $U_2 = \frac{1847}{EI}$

$$M_A = \frac{2EI}{16}\left(\frac{-103}{EI}\right) + 74.6$$

$$= 61.73 \text{ k-ft}$$

$$R_A = \frac{6EI}{16^2}U_1 + 28$$

$$= 25.58^{K}$$

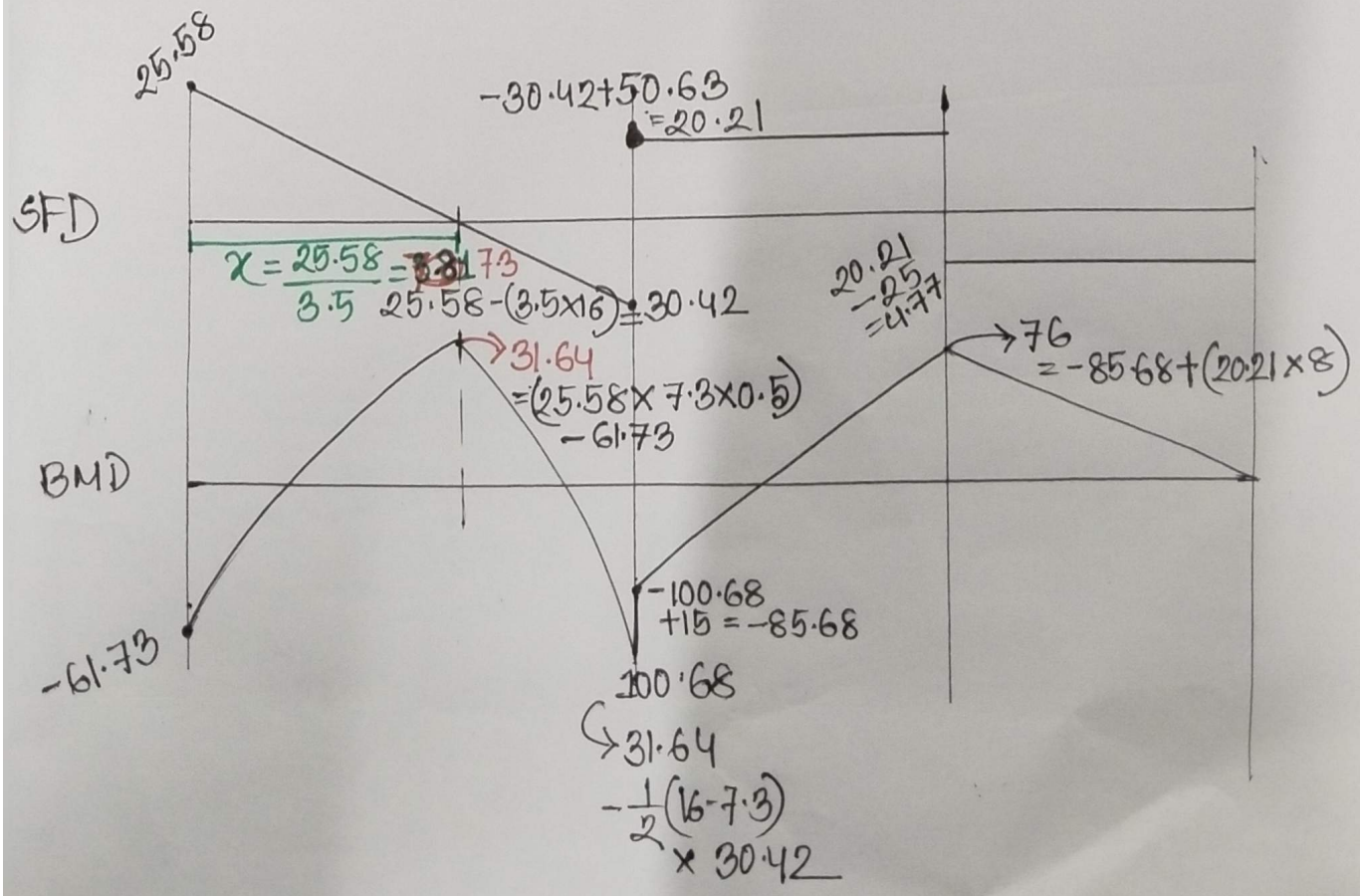


$$R_B = \frac{-6EI}{16^2} \times \frac{-103}{EI} + \frac{12}{24^2}(EI) \times U_2 + 16.52 + 28$$

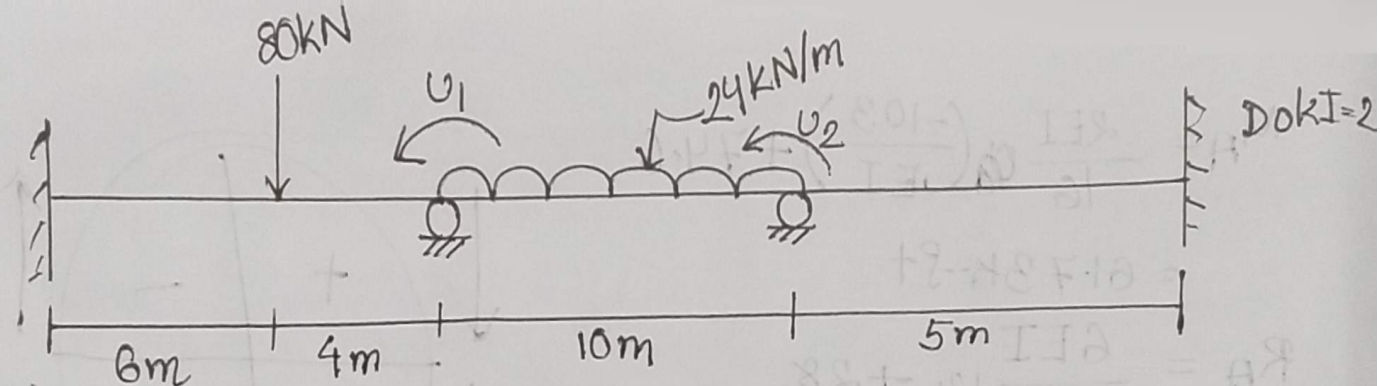
$$= \cancel{48.95}\ 50.63^{K}$$

$$R_C = \frac{-12EI}{24^2} \times \frac{-103}{EI} - \frac{12EI}{24^2}\left(\frac{184.7}{EI}\right) + 6.48$$

$$= 4.77^{K}$$



*



if $U_2 = \frac{192.35}{EI}$, $U_1 = ?$

c