EWCE 411 – SA

Indeterminate structure :

① Direct stiffness method → SA
② moment distribution method
③ IL for indeterminate structure } SB

Ref. Book :

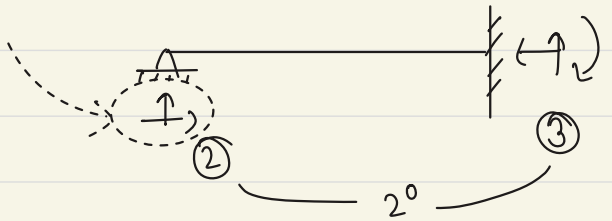
① Structural Analysis by Aslam Kassimali

DOSI & DOKI ← Kinetic

Degree of Static Indeterminacy (DOSI)

→ External static indeterminacy
→ Internal " "

Redundant →

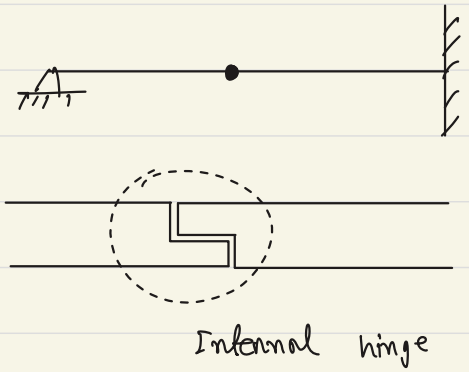


$I_s = I_{SE} + I_{SI}$

: (external reaction − 3) + (3m − 3; + 3 − R_n) ← Released reaction

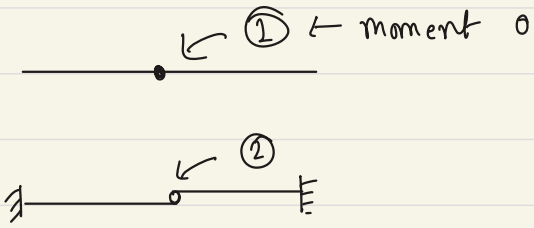
: (R − 3) + (3m − 3; + 3 − R_n)

= (R + 3m) − (3; + R_n) [2D; beam / frame]



Internal hinge

Released reaction



3D: $(R + 6m) - [6j] + R_n)$ ← check

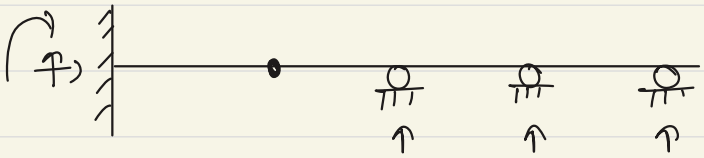
Truss: $m + R - 2j$

DOSI = 0 → stable & determinate

> 0 → " " indeterminate

< 0 → unstable & determinate

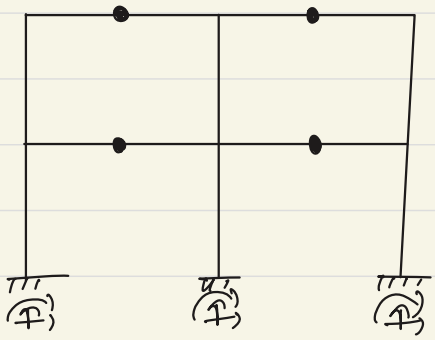
Practice:



$(R + 3m) - (3j + R_n)$

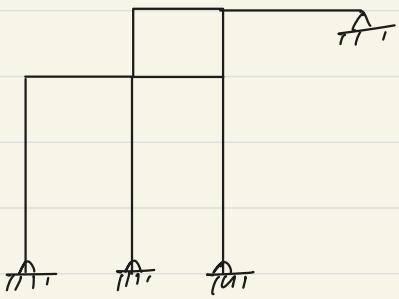
$(6 + 3 \times 4) - (3 \times 5 + 1) = 2°$

External: $6 - 3 = 3°$



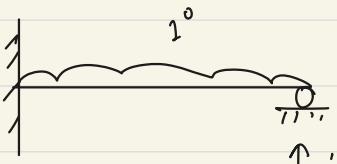
$(R+3m) - (3j + R_n)$

$= (9 + 3\times14) - (3\times13 + 4) = 8°$ ✓



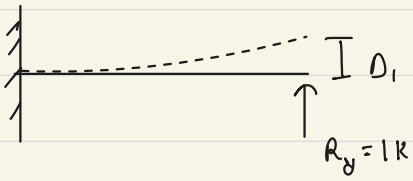
Ans.: 6

compatibility eqn



↑ :-- redundent (let) R_y





combined effect

↳ external loading deformation

+

redundent "

=

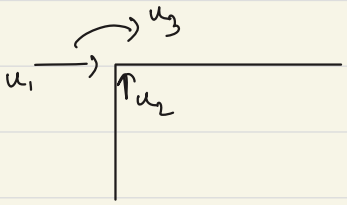
primary struct. deformation

main struct. def. = 0

$0 = \Delta + R_y \Delta_1$

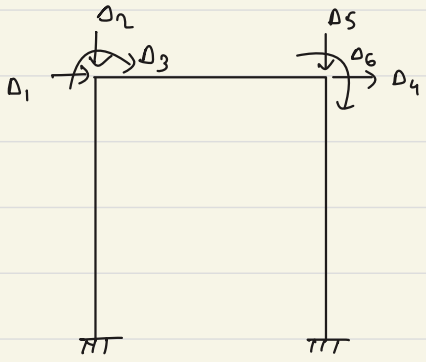
DOSI → degree → redundent → compat. eqn → solve

DOKI: Number of independent joint displacement (↗ translational, rotational) to describe the deformed shape

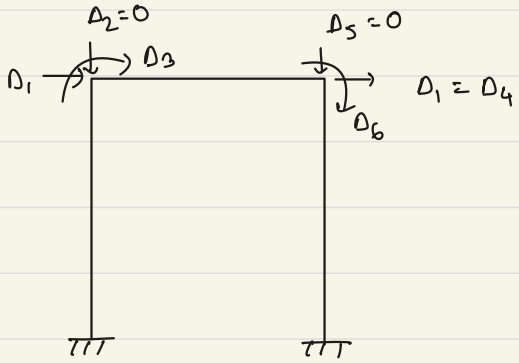


Degree of kinetic Indeterminacy (DOKI) / Degree of freedom (DOF)

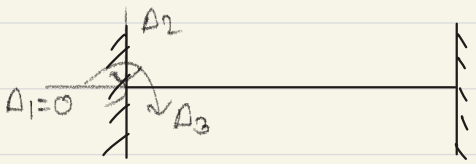
Number of independent joint displacement (translational & rotation) to describe the deformed shape.



If member is extensible



If member is inextensible



DOSI = 3

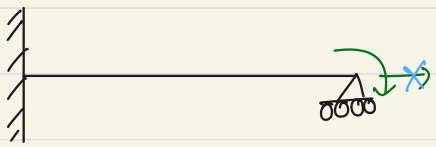
DOKI = 0

no axial ld/ deform

$2\partial - k = 2 \times 2 - 4$

Extensible: $3 \times 2 - 4 = 2$

Inextensible: $3 \times 2 - 5 = 1$



const displacement

$I_k =$ $2\partial - k$; for beam

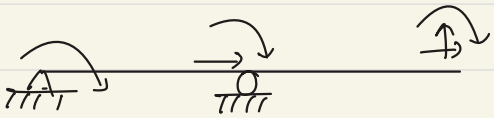
$3\partial - k$; for plane frame / 2D

$6\partial - k$; for frame / 3D

$3\partial - (k+m)$

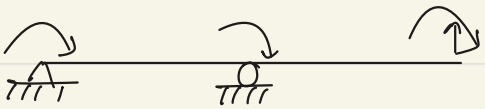
k = number of cons. given by support

m = num of compatibility eqn



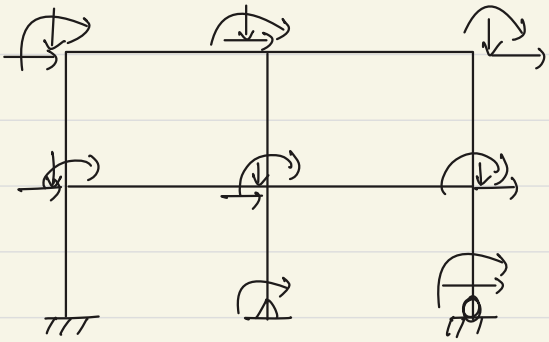
Extensible

$I_K = 6$



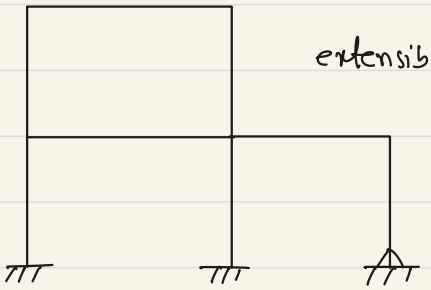
Inextensible

$I_K = 4$

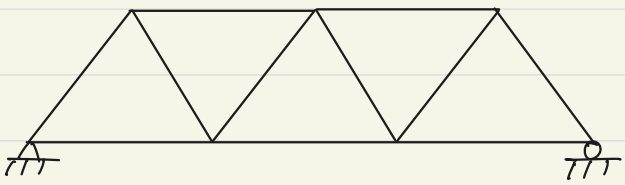


Extensible

$I_K = 21$ or, $3 \times 9 - (3+2+1) = 21$



extensible & inextensible

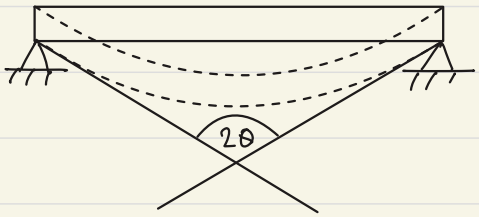


For truss, $I_K = 2j - k$

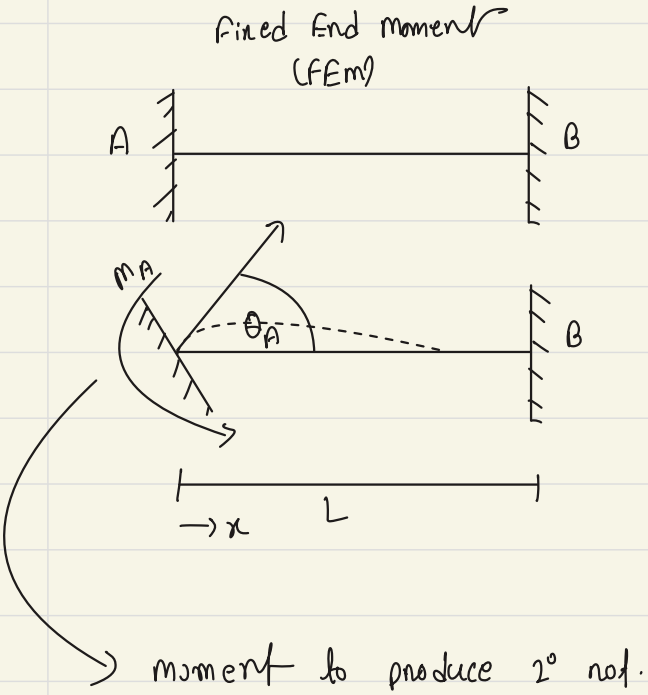
$2 \times 7 - 3 = 11$

L03 28/06

Stiffness (derivation) (Rotation)



$$\text{stiffness} = \frac{m}{\theta}$$



moment to produce 2° rot.
→ stiffness

$$EI\frac{d^4v(x)}{dx^4} = P \quad \{\text{load}\}$$

As no external loading, $P = 0$

$$EI\frac{d^4v(x)}{dx^4} = 0$$ ← on y to indicate deflection

(i) $$\Rightarrow EI\frac{d^3v(x)}{dx^3} = C_1 \quad \{\text{shear}\}$$

(ii) $$\Rightarrow EI\frac{d^2v(x)}{dx^2} = C_1x + C_2 \quad \{\text{moment}\}$$

(iii) $$\Rightarrow EI\frac{dv(x)}{dx} = C_1\frac{x^2}{2} + C_2x + C_3 \quad \{\text{slope}\}$$

(iv) $$\Rightarrow EIv(x) = \frac{1}{6}C_1x^3 + C_2\frac{x^2}{2} + C_3x + C_4$$

$\{\text{deflection}\}$

Boundary conditions:

$x=0, \quad v = 0, \quad \theta_{(x)} = \theta_A$

$x=L, \quad v = 0, \quad \theta_{(L)} = 0$

Putting $x=0, v=0$, in eq (iv)

$C_4 = 0$

in eq (iii),

$$EI\theta_A = 0 + 0 + C_3 \Rightarrow C_3 = EI\theta_A$$

$$EI\,\theta_{(x)} = C_1 \frac{x^2}{2} + C_2 x + EI\,\theta_A \quad -(v)$$

$$EI\,v_{(L)} = \frac{C_1 x^3}{6} + C_2 \frac{x^2}{2} + EI\theta_A x \quad -(vi)$$

$x=L, \theta=0$

$$0 = C_1 \frac{L^2}{2} + C_2 L + EI\theta_A \quad -(vii)$$

$x=L, v=0$

$$0 = \frac{C_1 L^3}{6} + C_2 \frac{L^2}{2} + EI\,\theta_A L \quad -(viii)$$

moment to produce 1° rotation



$$0 = \frac{PL}{AE} \Rightarrow \frac{P}{\Delta} = \frac{AE}{L}$$

(vii) $\times L$ − (viii) $\Rightarrow$

$$C_1 \frac{L^3}{2} + C_2 L^2 + EI\theta_A L = 0$$

$$(-)\; C_1 \frac{L^3}{6} - C_2 \frac{L^2}{2} + EI\theta_A L = 0$$

$$\frac{1}{3} C_1 L^3 - \frac{1}{2} C_2 L^2 = 0$$

$$\Rightarrow C_2 = \frac{1}{3} C_1 L^3 \times 2 \frac{1}{L^2}$$

$$= \frac{2}{3} C_1 L$$

C_2 in eq (vii):

$$0 = C_1 \frac{L^2}{2} - \frac{2}{3} C_1 L^2 + EI\theta_A$$

$$\Rightarrow 0 = C_1 \left[\frac{1}{2}L^2 - \frac{2}{3}L^2\right] + EI\theta_A$$

$$= C_1 \left[-\frac{1}{6}L^2\right] + EI\theta_A$$

$$\Rightarrow C_1 = -\frac{\quad}{\quad}$$

$$C_1 = \frac{6EI\theta_A}{L^2}$$

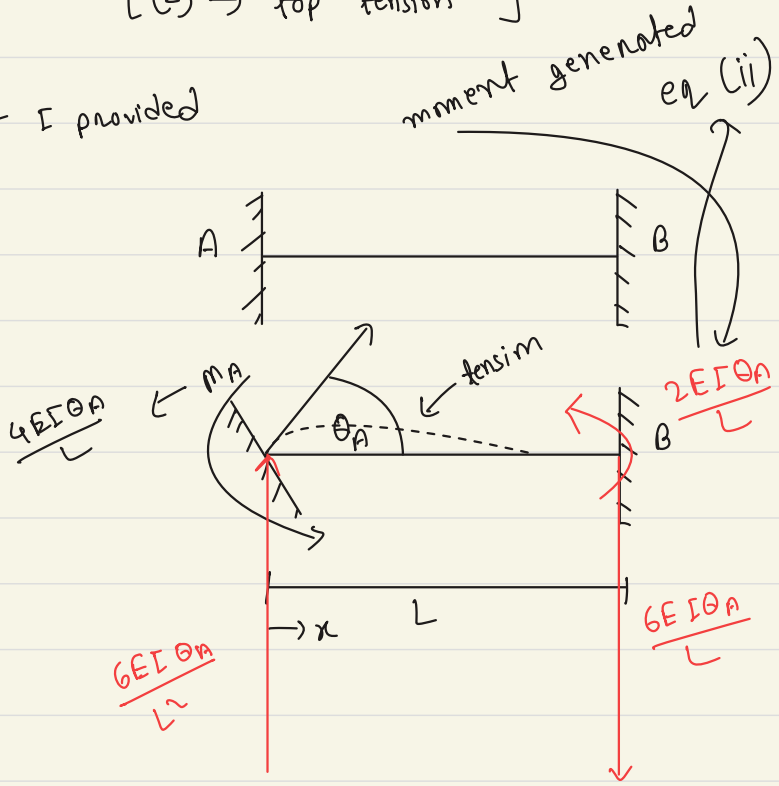
$C_2 = \cdots$

$$= -\frac{4EI\theta_A}{L}$$

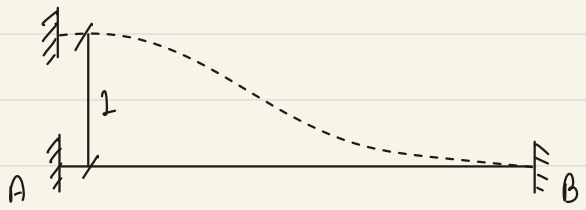
[(-) → top tension]

$$M = \frac{4EI\theta_A}{L} \leftarrow \text{moment I provided}$$

$$\Rightarrow \frac{M}{\theta_A} = \frac{4EI}{L}$$



(Deflection)



$EI \frac{d^4y}{dx^4} = p = 0$ ← no ext. load

$EI \frac{d^3y}{dx^3} = C_1$ (shear) — (i)

$EI \frac{d^2y}{dx^2} = C_1 x + C_2$ (moment) — (ii)

$EI \frac{dy}{dx} = C_1 \frac{x^2}{2} + C_2 x + C_3$ (slope) — (iii)

$EIy = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$ (deflection) — (iv)

Boundary conditions:

$x=0, \quad y=1, \quad \theta = 0^\circ$

$x=L, \quad y=0, \quad \theta = 0^\circ$

Putting them into (iv),

$EI \times 1 = 0 + 0 + 0 + C_4$

$\Rightarrow C_4 = EI$

Into (iii),

$0 = 0 + 0 + C_3$

$\Rightarrow C_3 = 0$

Rewriting (iii) & (iv)

$EI \frac{dy}{dx} = C_1 \frac{x^2}{2} + C_2 x$ — (v)

$EIy = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + EI$ — (vi)

Putting $x=L$, $y=0$, $\theta=0^\circ$ into (v), (vi):

(v): $0 = C_1 \frac{L^2}{2} + C_2 L \Rightarrow C_1 = -\frac{C_2 L}{L^2} 2$

$= -\frac{2C_2}{L}$

(vi): $0 = -\frac{2C_2}{L}\frac{L^3}{6} + C_2 \frac{L^2}{2} + EI$

$\Rightarrow 0 = C_2\left[-\frac{L^2}{3} + \frac{L^2}{2}\right] + EI \Rightarrow EI = C_2\left[\frac{L^2}{6}\right]$

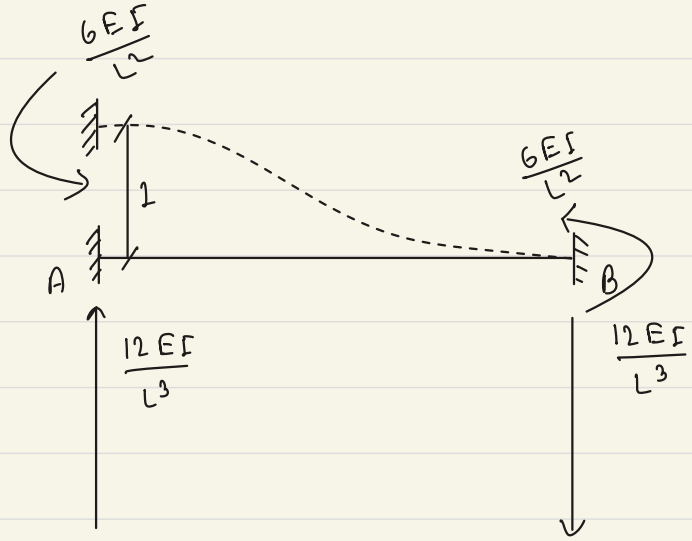
$\Rightarrow C_2 = \frac{6EI}{L^2}$

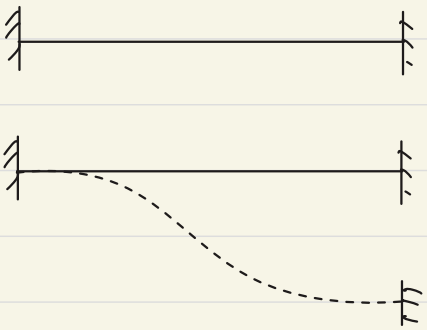
(vi): $0 = C_1 \frac{L^3}{6} + C_2 \frac{L^2}{2} + EI$

$\Rightarrow \ldots$

$\Rightarrow C_1 = \frac{12EI}{L^3}$

$C_2 = \frac{6EI}{L^2}$

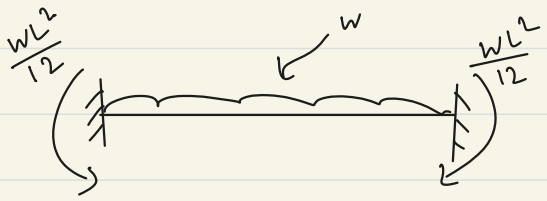


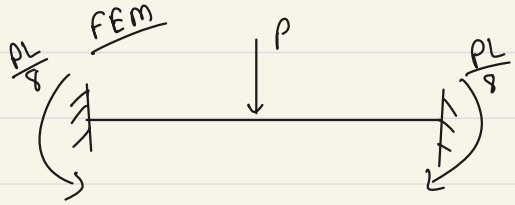


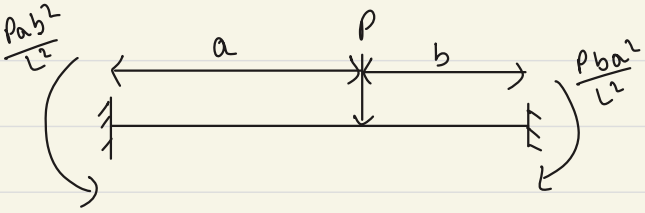
FEM

Fixed End moment

~ 25:00







$$Q_1 = K_{11}U_1 + K_{12}U_2 + K_{13}U_3 + K_{14}U_4 + K_{15}U_5 + K_{16}U_6 + Q_{f1}$$
$$Q_2 = K_{21}U_1 + K_{22}U_2 + K_{23}U_3 + K_{24}U_4 + K_{25}U_5 + K_{26}U_6 + Q_{f2}$$
$$Q_3 = K_{31}U_1 + K_{32}U_2 + K_{33}U_3 + K_{34}U_4 + K_{35}U_5 + K_{36}U_6 + Q_{f3}$$
$$Q_4 = K_{41}U_1 + K_{42}U_2 + K_{43}U_3 + K_{44}U_4 + K_{45}U_5 + K_{46}U_6 + Q_{f4}$$
$$Q_5 = K_{51}U_1 + K_{52}U_2 + K_{53}U_3 + K_{54}U_4 + K_{55}U_5 + K_{56}U_6 + Q_{f5}$$
$$Q_6 = K_{61}U_1 + K_{62}U_2 + K_{63}U_3 + K_{64}U_4 + K_{65}U_5 + K_{66}U_6 + Q_{f6}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \times \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix}$$

$Q = KU + Qf$;

$Q \rightarrow$ member end force

$U \rightarrow$ member displacement (translation, rotation)

$K \rightarrow$ member stiffness

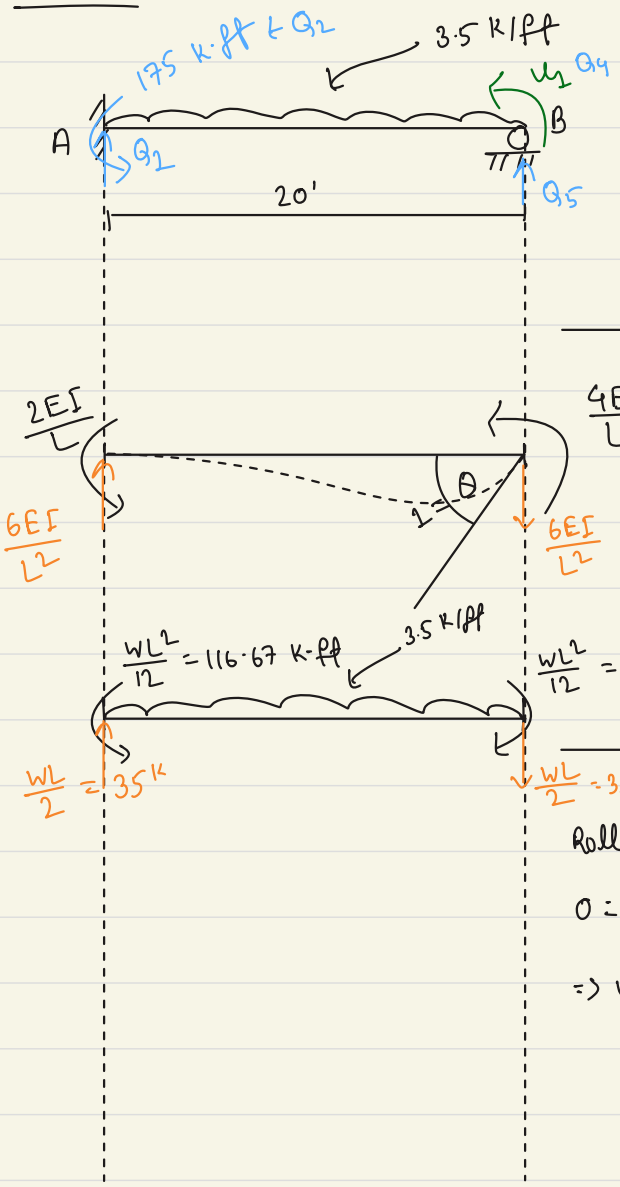
$Qf \rightarrow$ fixed end moment for external loading

Ex. 1

[EI const)

assume if not specified

[Beam inextensible)



DoKI = 1 [only rotation]

Roller (ext), moment = 0

$$0 = \frac{4EI}{L} u_1 - 116.67$$

$$\Rightarrow u_1 = 116.67 \times \frac{L}{4EI}$$

$$= \frac{583.35}{EI}$$

At A, $Q_2 = \frac{2EI}{L} \times u_1 + 116.67$

$$= \frac{2EI}{20} \times \frac{583.35}{EI} + 116.67$$

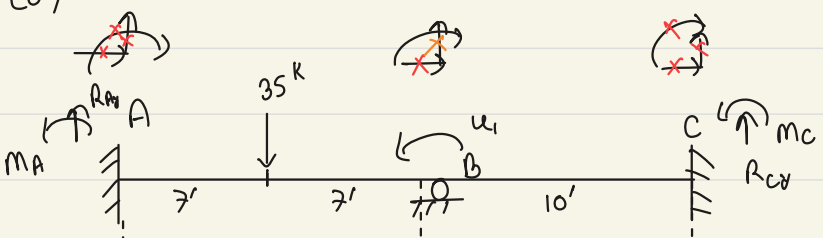
$$= 175 \text{ K-ft}$$

$$Q_1 = \frac{6EI}{L^2} \times u_1 + \frac{WL}{2}$$

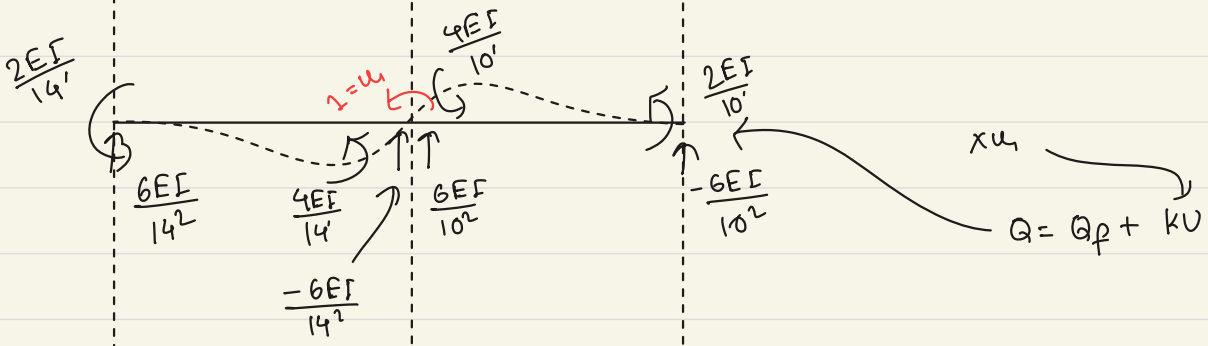
$$= \frac{6 \times 583.35}{20^2} + 35 = 43.75 \text{ K}$$

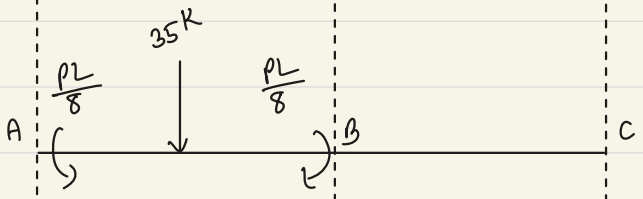
L07

26/07



DOKI = 1



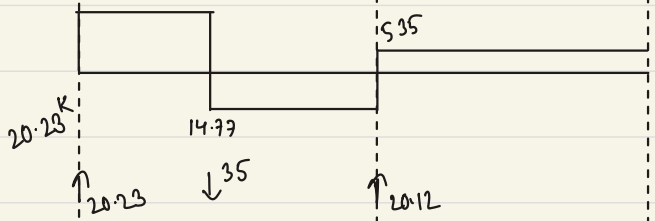


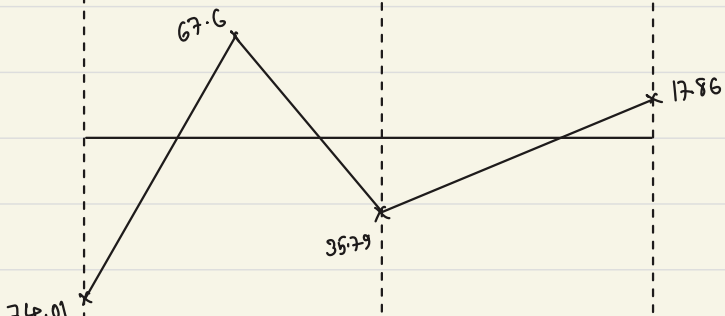
$\Sigma M_{ext} = \Sigma M_{int}$

At B, $0 = -\frac{35 \times 14}{8} + \left[+\frac{4EI}{14} + \frac{4EI}{10}\right] u_1$

$\Rightarrow u_1 = \frac{89.32}{EI}$

Left side ↑ pos
↓ neg
Right ↑ neg
↓ pos





$M_A = \frac{2EI}{14} \times \frac{89.32}{EI} + \frac{35 \times 14}{8}$

$= 74.01$ K-ft

$M_C = \frac{2EI}{10} \times \frac{89.32}{EI} + 0$

$= 17.864$ K-ft

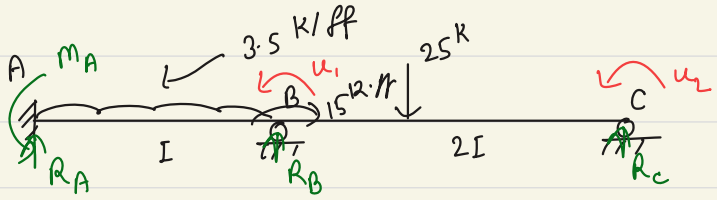
$R_{Ay} = \frac{6}{14^2} \times \frac{89.32}{1} + \frac{35}{2}$

$= 20.234$ K

$R_{Cy} = -\frac{6}{10^2} \times \frac{89.32}{1} = -5.36$ K

$R_{By} = \left(-\frac{6EI}{14^2} + \frac{6EI}{10^2}\right) \times \left(\frac{89.32}{EI}\right) + \frac{P}{2}$

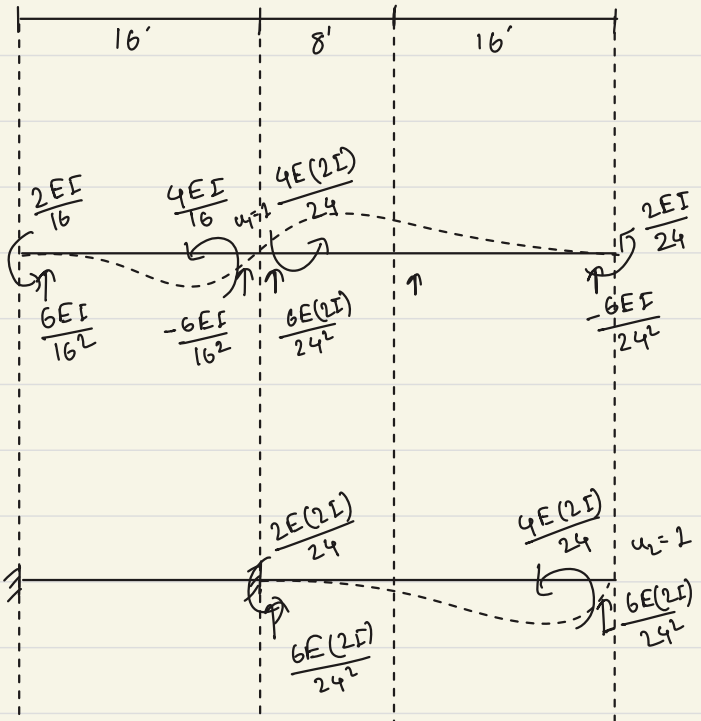
$= 20.12$ K

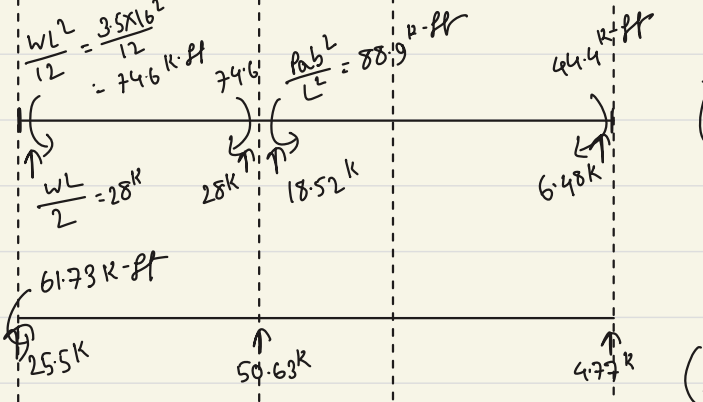


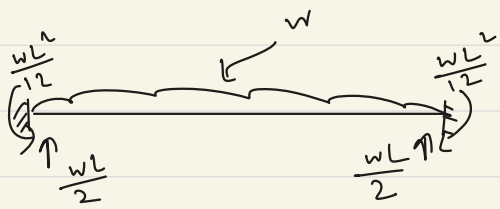
E = const

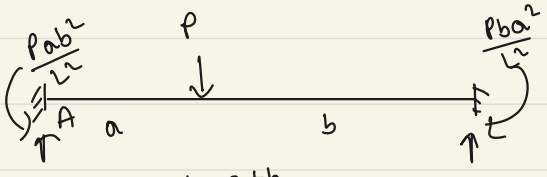
DOKI = 2

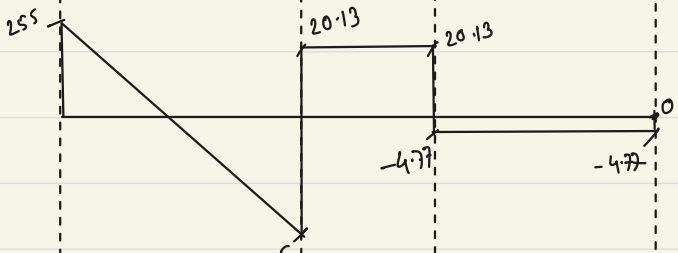
Q = Qf + KU

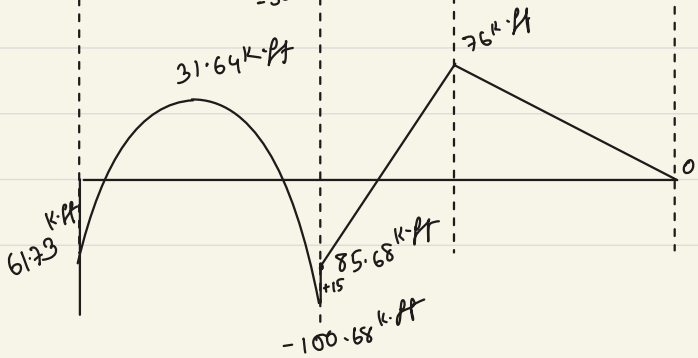












$$m_{(A)} = 0 \Rightarrow \frac{Pab^2}{L^2} - \frac{Pba^2}{L^2} + xL - Pa = 0$$

$$= \frac{Pa + \frac{Pba^2}{L^2} - \frac{Pab^2}{L^2}}{L}$$

$$= \frac{\frac{Pal^2 + Pba^2 - Pab^2}{L^2}}{L}$$

$$= \frac{Pa(L^2 + ba - b^2)}{L^3}$$

$$Q = Q_f + KU$$

$$\Rightarrow -15 = \left(\frac{4EI}{16} + \frac{8EI}{24}\right) u_1 + \frac{4EI}{24} \times u_2 + (-74.6 + 88.9)$$

$$= \frac{7EI}{12} u_1 + \frac{4EI}{24} u_2 + 14.3 \quad \text{—— (I)}$$

$$0 = \frac{4EI}{24} u_1 + \frac{8EI}{24} u_2 - 44.4 \quad \text{—— (II)}$$

(I) $\times 2$ − (ii);

$$u_1 = -\frac{102.84}{EI}$$

$$u_2 = \frac{184.62}{EI}$$

$$\frac{7EI}{12} u_1 - \frac{8EI}{24} u_1 + 14.3 - 88.2 = -15$$

$\Rightarrow u = \cdots$

$$M_A = \frac{2EI}{16} \times u_1 + 0 \times u_2 + 74.6$$

$$= 61.73 \text{ k-ft}$$

$$R_A = \frac{6EI}{16^2} \times u_1 + 0 \times u_2 + 28$$

$$= 25.5^K$$

$$R_B = \frac{-6EI}{16^2} \times u_1 + \frac{12EI}{24^2} \times u_1 + \frac{12EI}{24^2} \times u_2 + (28 + 18.52)$$

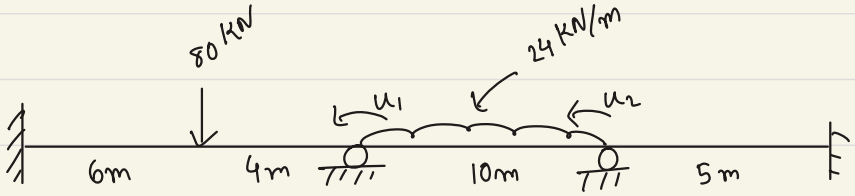
$$= 50.63^K$$

$$R_C = \frac{-12EI}{24^2} \times u_1 + -\frac{12EI}{24^2} \times u_2 + 6.48$$

$$= 4.78^K$$

Mid → upto today's class; all taught; incl. derivation

H.W



EI = const

$u_2 = 192.35 / EI$

$u_4 = ?$

Reaction, SFD, BMD

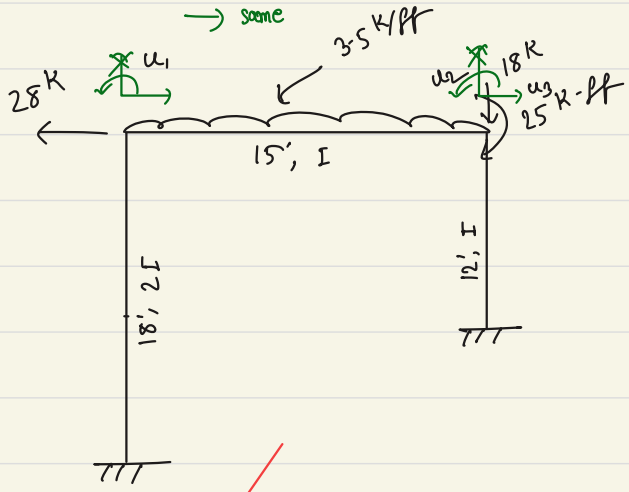
L09 16/08

Mid syllabus: Thursday

- Beam 1°, 2°
- Exclude derivation

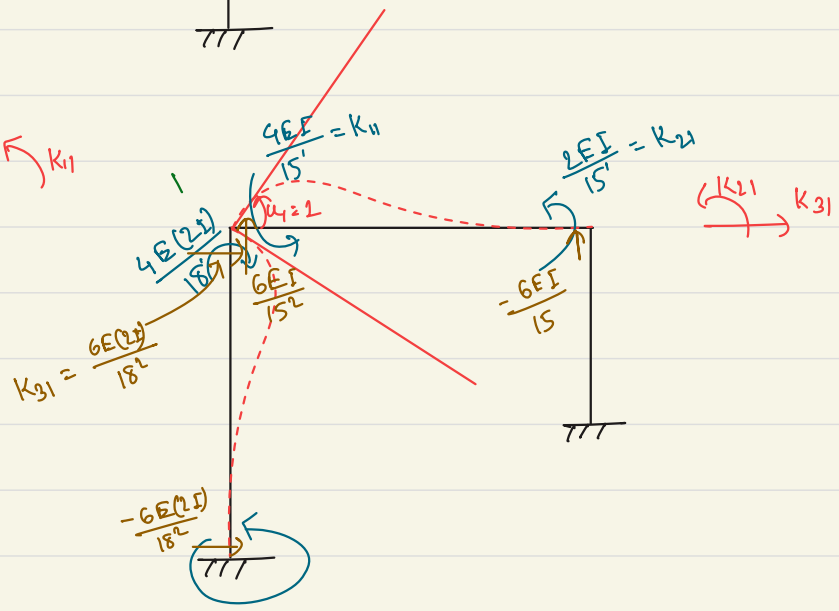
Frame

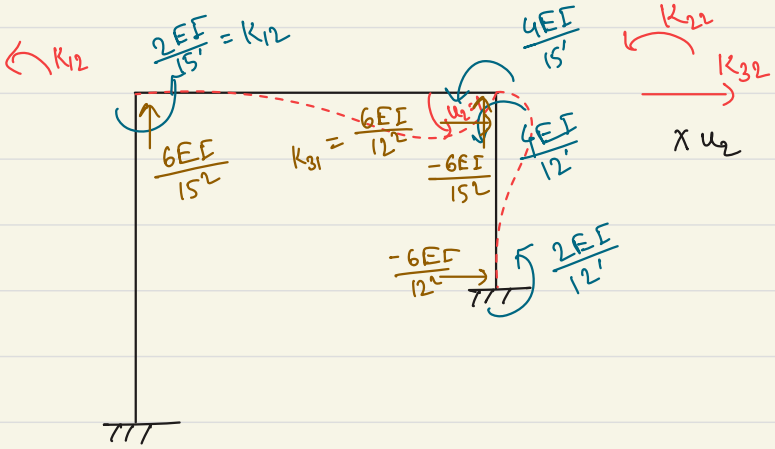
DoKI = 3

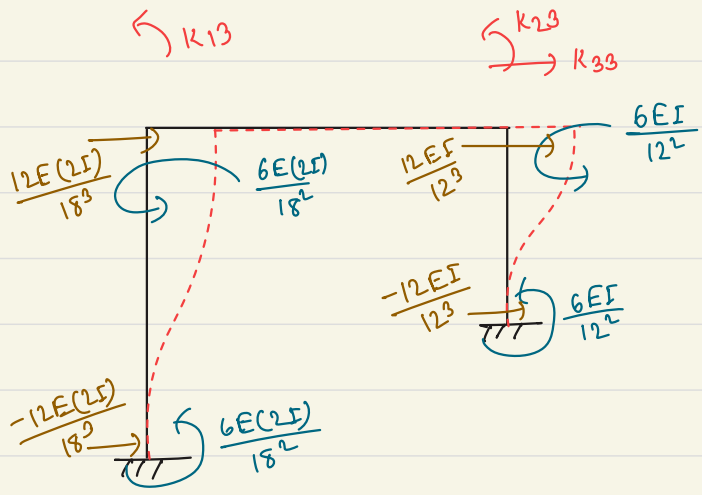


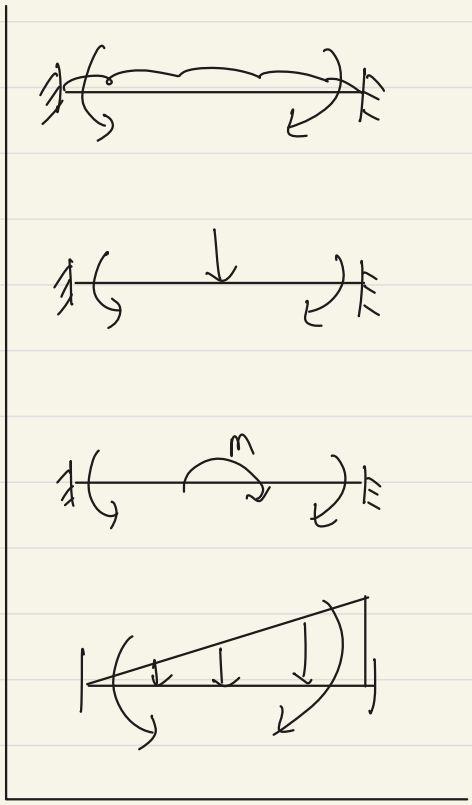
E = const

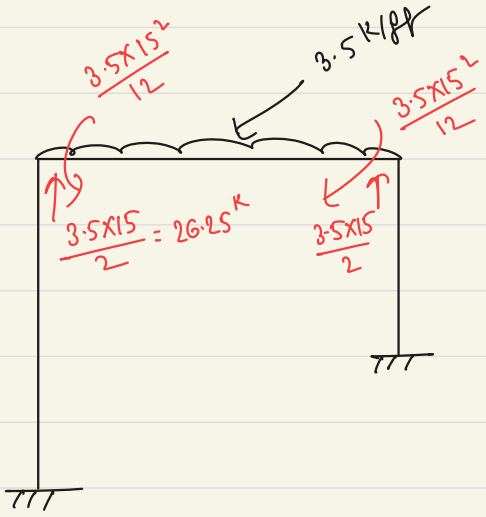
Inextensible











u_1: $\quad Q = Q_f + KU$

$$\Rightarrow 0 = 65.63 + \underbrace{\left(\frac{4EI}{15} + \frac{8EI}{18}\right)}_{K_{11}} u_1 + \underbrace{\frac{2EI}{18}}_{K_{12}} u_2 + \underbrace{\frac{12EI}{18^2}}_{K_{13}} u_3 \quad \text{——— (I)}$$

u_2: $\quad Q = Q_f + KU$

$$\Rightarrow -25 = -65.63 + \frac{2E}{15} u_1 + \left(\frac{4EI}{15} + \frac{4EI}{12}\right) u_2 + \frac{6EI}{12^2} u_3 \quad \text{— (II)}$$

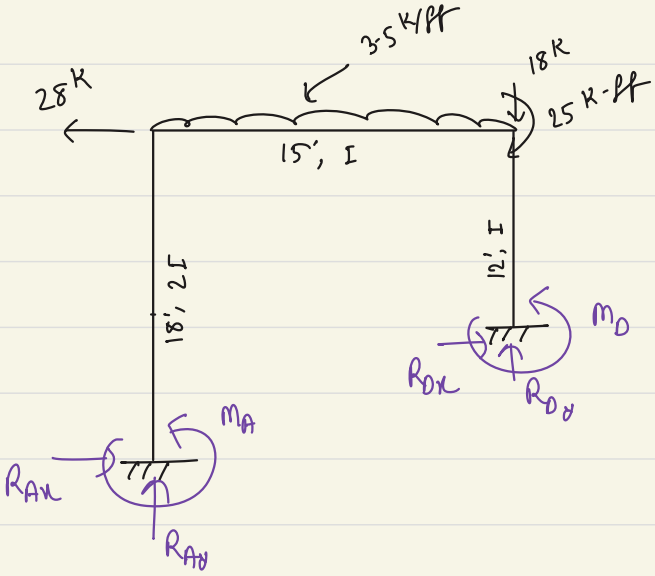
u_3: $\quad Q = Q_f + Ku$

$$\Rightarrow -28 = 0 + \frac{12EI}{18^2} u_1 + \frac{6EI}{12^2}\left(\frac{24EI}{18^3} + \frac{12EI}{12^3}\right) u_3 \quad \text{——— (III)}$$

$$\begin{bmatrix} 0 \\ -25 \\ -28 \end{bmatrix} = \begin{bmatrix} 65.63 \\ -65.63 \\ 0 \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} -65.63 \\ 40.63 \\ 0 \end{bmatrix} = K \times U$$

$$u_1 = \frac{51.42}{EI}; \quad u_2 = \frac{330.54}{EI}; \quad u_3 = -\frac{3949.21}{EI}$$



$$R_{Ay} = 26.25 + \frac{6EI}{15^2} u_1 + \frac{6EI}{15^2} u_2 + 0$$

$$= \text{---}$$

$$= 36.44 \text{ K}$$

$$R_{Ax} = -\frac{6E \times 2I}{18^2} u_1 - \frac{12E \times 2I}{18^3} u_3$$

$$= -\frac{6 \times 2}{18^2} \times 51.42 - \frac{12 \times 2}{18^3} \times (-3949.21)$$

$$= 14.35 \text{ K}$$

$$M_A = \frac{2E \times 2I}{18} u_1 + \frac{6E \times 2I}{18^2}$$

$$= \frac{4}{18} \times 51.42 + \frac{12}{18^2} \times (-3949.21)$$

$$= -134.84 \text{ K-ft}$$

$$R_{Dy} = -18 + 26.25 - \frac{6}{15^2} \times 51.42$$

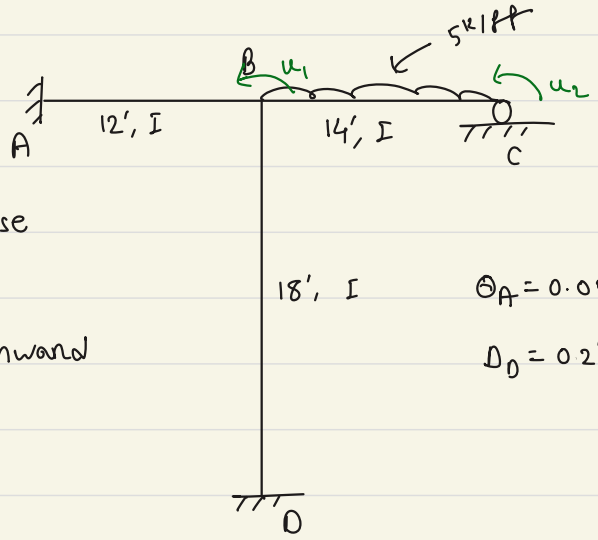
$$- \frac{6}{15^2} \times 330.54 = 34.05 \text{ K}$$

$$R_{Dx} = \text{---} = 13.65 \text{ K}$$

$$M_D = \text{---} = 109.46 \text{ K-ft}$$

Support settlement & rotation

(On exam, may come only one, or both)



Support A rotates clockwise

0.05 rad

$\theta_A = 0.05\,\text{rad}(\curvearrowright)$

Support B settles downward

0.2'

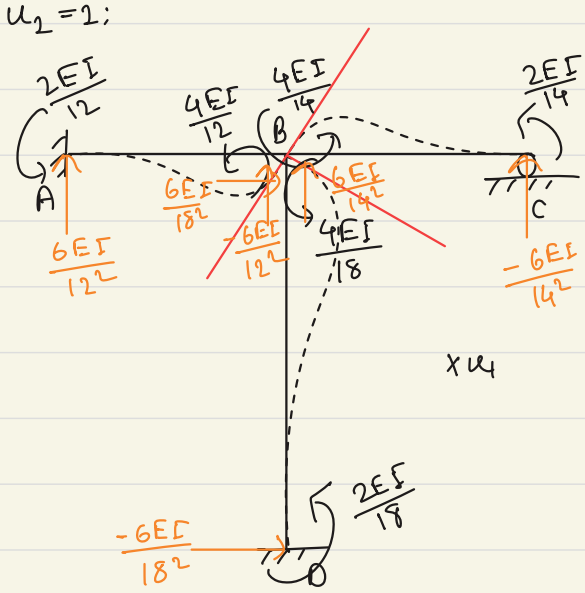
$D_D = 0.2'(\downarrow)$

rotation → Q_{p1}

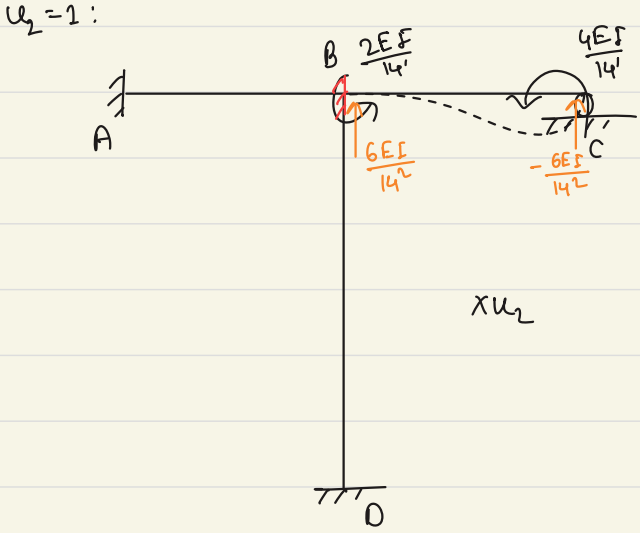
Q_{p2} ← settlement

$Q = Q_p + K\bar{U}$

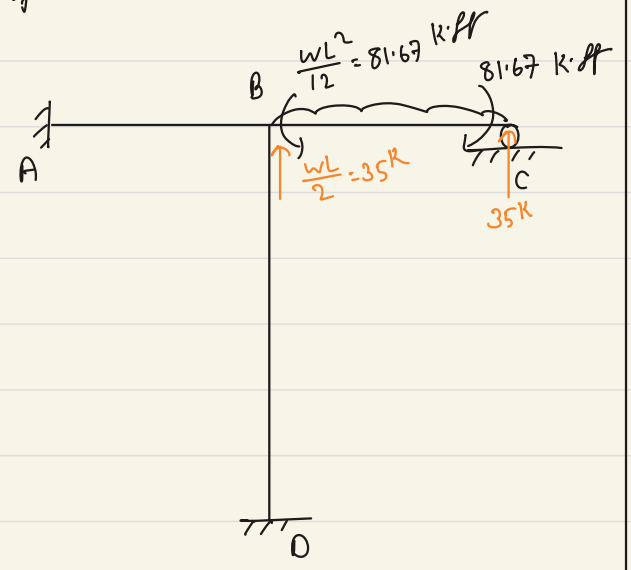
$u_2 = 1$:



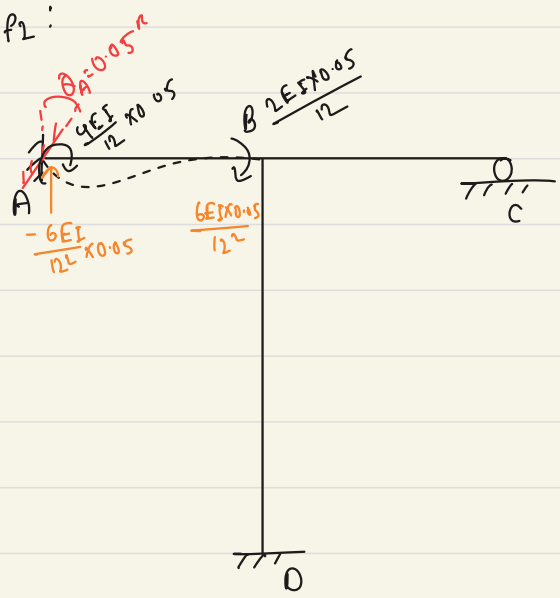
$u_2 = 1$:

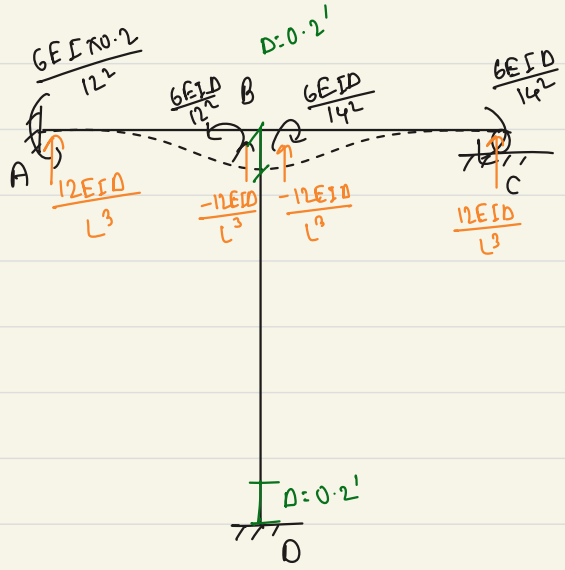


Q_f :



Q_{f2} :





$$0 = Q_f + Q_{f1} + Q_{f2} + Ku_1 + \square u_2$$

u_1: $0 = 81.67 + \frac{-2EI \times 0.05}{12} + \left[\frac{6EI \times 0.2}{12^2} - \frac{6EI \times 0.2}{14^2}\right] + \left[\frac{4EI}{12} + \frac{4EI}{14} + \frac{4EI}{18}\right] \times u_1$

$+ \frac{2EI}{14} \times u_2$

$\Rightarrow 0 = 81.67 + EI\left[\frac{-2 \times 0.05}{12} + \left\{\frac{6 \times 0.2}{12^2} - \frac{6 \times 0.2}{14^2}\right\} + \left\{\frac{4}{12} + \frac{4}{14} + \frac{4}{18}\right\} \times u_1 + \frac{2}{14} u_2\right]$

$\Rightarrow -81.67 = EI\left(-\frac{3}{490} + \frac{53}{63} u_1 + \frac{2}{14} u_2\right)$

$\Rightarrow -81.67 + \frac{3}{490} = \frac{53}{63} u_1 + \frac{2}{14} u_2$

u_2:

$0 = -81.67 - \frac{6EI}{14^2} \times 0.2 + \frac{2EI}{14} \times u_1 + \frac{4EI}{14} \times u_2$

$\Rightarrow 81.67 = EI\left(\frac{3}{490} + \frac{2}{14} u_1 + \frac{4}{14} u_2\right)$

$\Rightarrow 81.67 - \frac{3}{49} = \frac{2}{14} u_1 + \frac{4}{14} u_2$

$\therefore u_1 = -159.08 / EI$

$u_2 = 365.17 / EI$

$u_1 = -\frac{139.25}{EI}$

$u_2 = \frac{462.605}{EI}$

$M_A = 64.875^{k\text{-}ft}$ (↻)

$R_{Ax} = 0^k$

$R_{Ay} = 9.27^k$ (↓)

$R_{Dx} = 2.58^k$ (→)

$M_D = 15.472^{k\text{-}ft}$ (↻)

$R_{Dy} = 49.8^k$ (↑)

$R_{Dy} = 29.47^k$ ↑

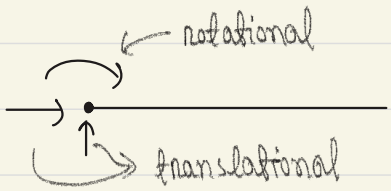
Rough / midterm 29/08

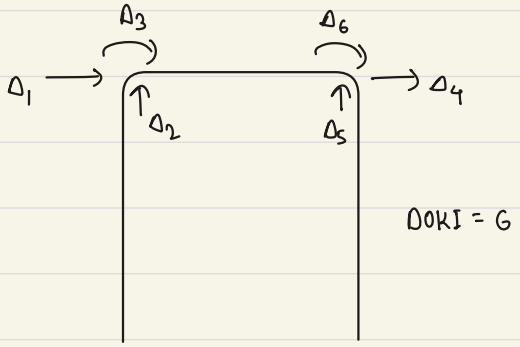
Direct Stiffness Method

DOSI → Static Indeterminacy

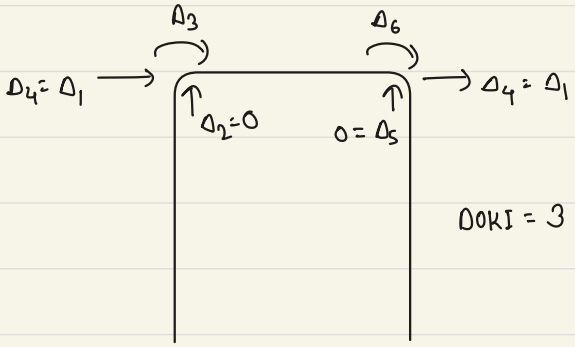
DOKI → Kinetic "

DOKI: Number of independent joint displacement (translational & rotational) to describe the deformed shape.

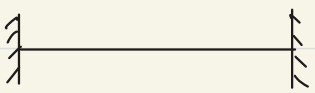




Member is extensible



Member is inextensible

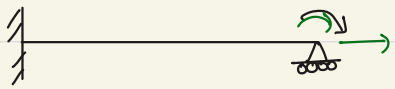


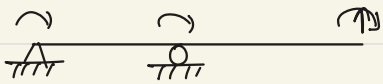
DOSI = 3

DOKI = 0

DOKI = 1 (inextensible)

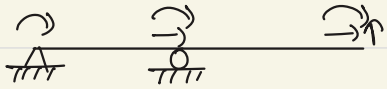
DOKI = 2 (extensible)





(Inextensible)

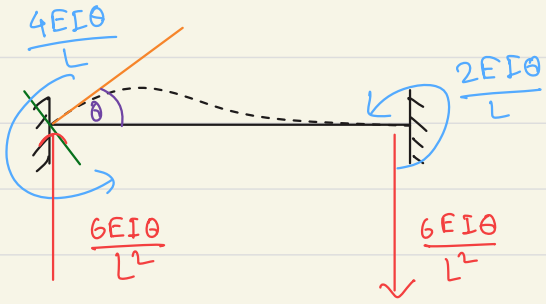
DOKI = 4

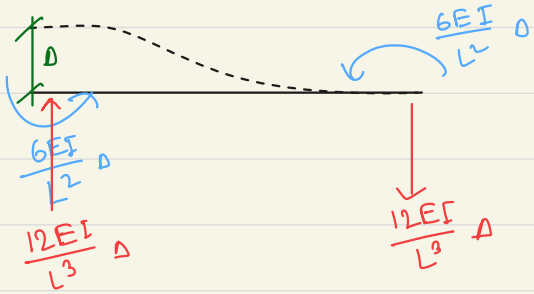


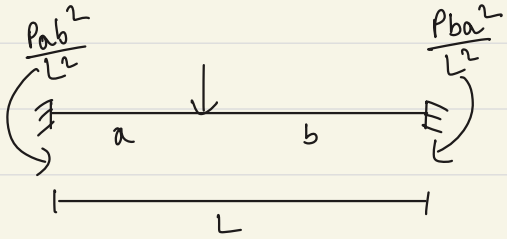
(Extensible)

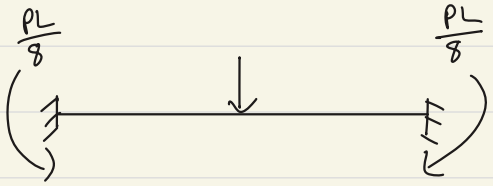
DOKI = 6

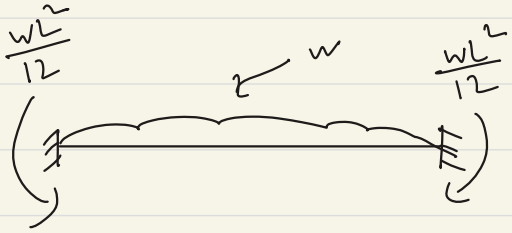
FEm:

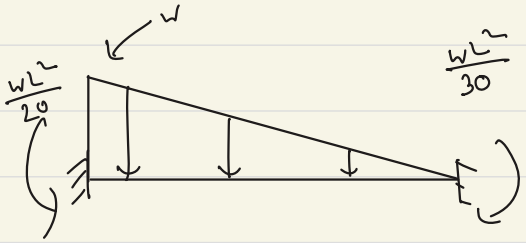




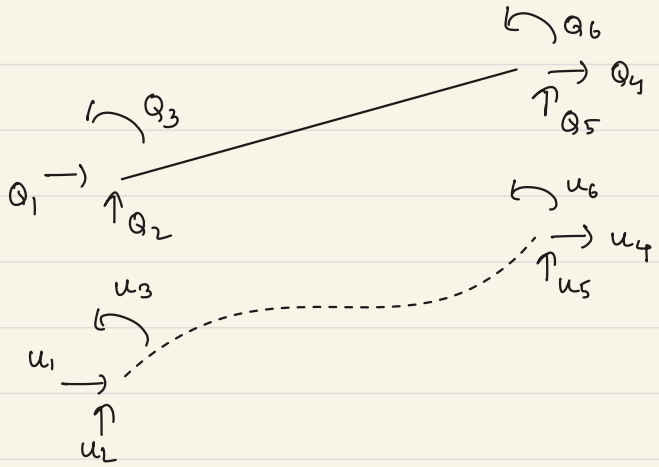




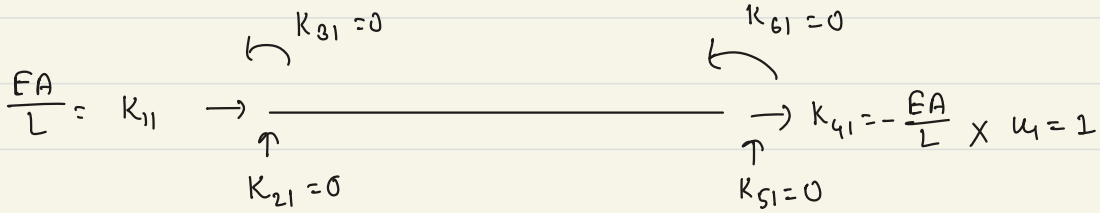




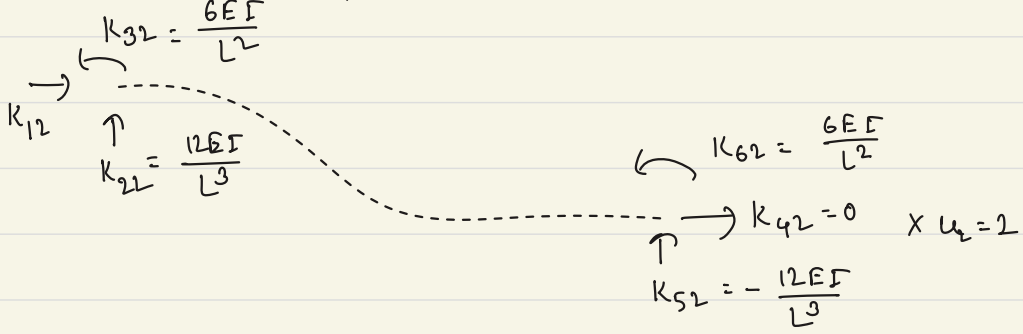
Stiffness Relation: Relationship between member end forces and end displacement in terms of ext. loads applied.



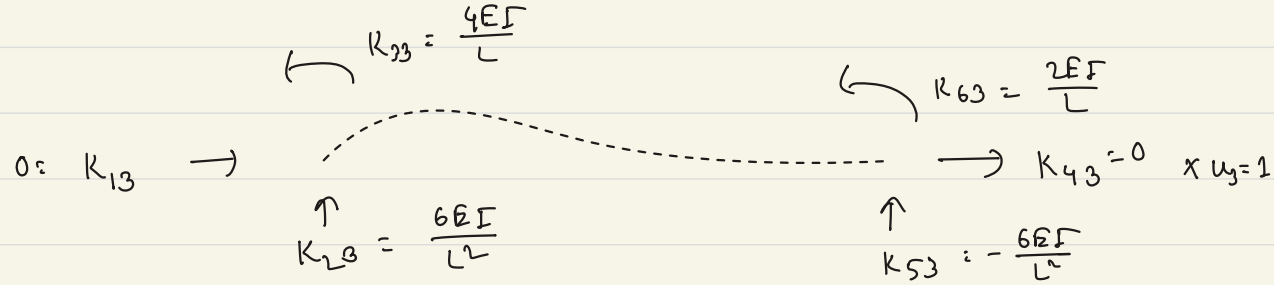
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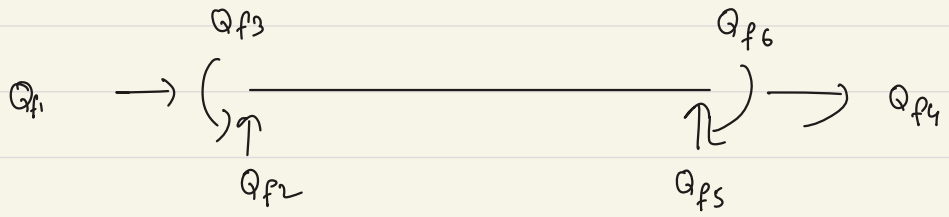
+



+

... u_4, u_5, u_6

+



$$\therefore Q_1 = K_{11}u_1 + K_{12}u_2 + K_{13}u_3 + K_{14}u_4 + K_{15}u_5 + K_{16}u_6 + Q_{f1}$$

$$Q = Ku + Qf$$

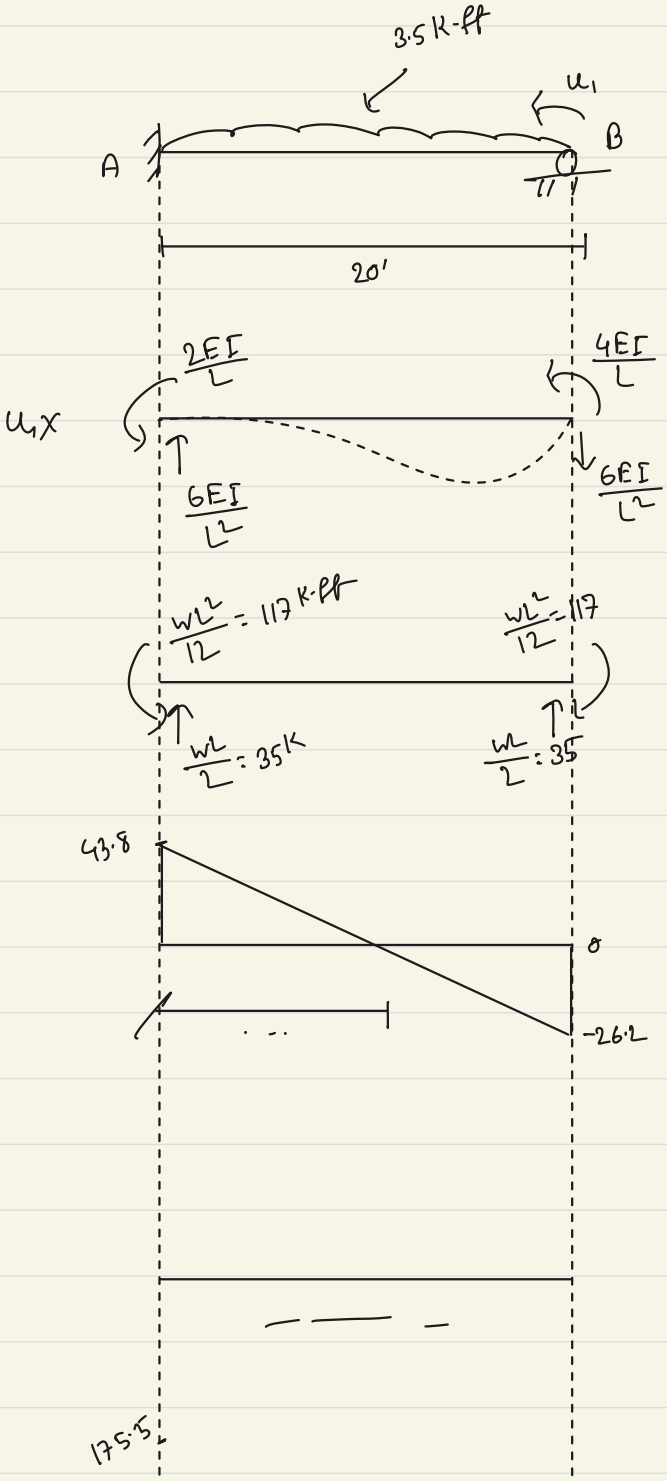
In essence, each displacement (u) corresponds to one member end force (Q); Q is sum of individual displacement load and ext. loading.

↳ Ku ← displacement

↙

Stiffness → moment / force for unit displacement

Ex. 1



DoKI = 1

$Q = ku + Q_f$

At B (roller),

$Q = 0 = \frac{4EI}{20} u_1 - 117$

$\Rightarrow u_1 = \frac{117}{4} \times 20 \times \frac{1}{EI}$

$= \frac{585}{EI}$

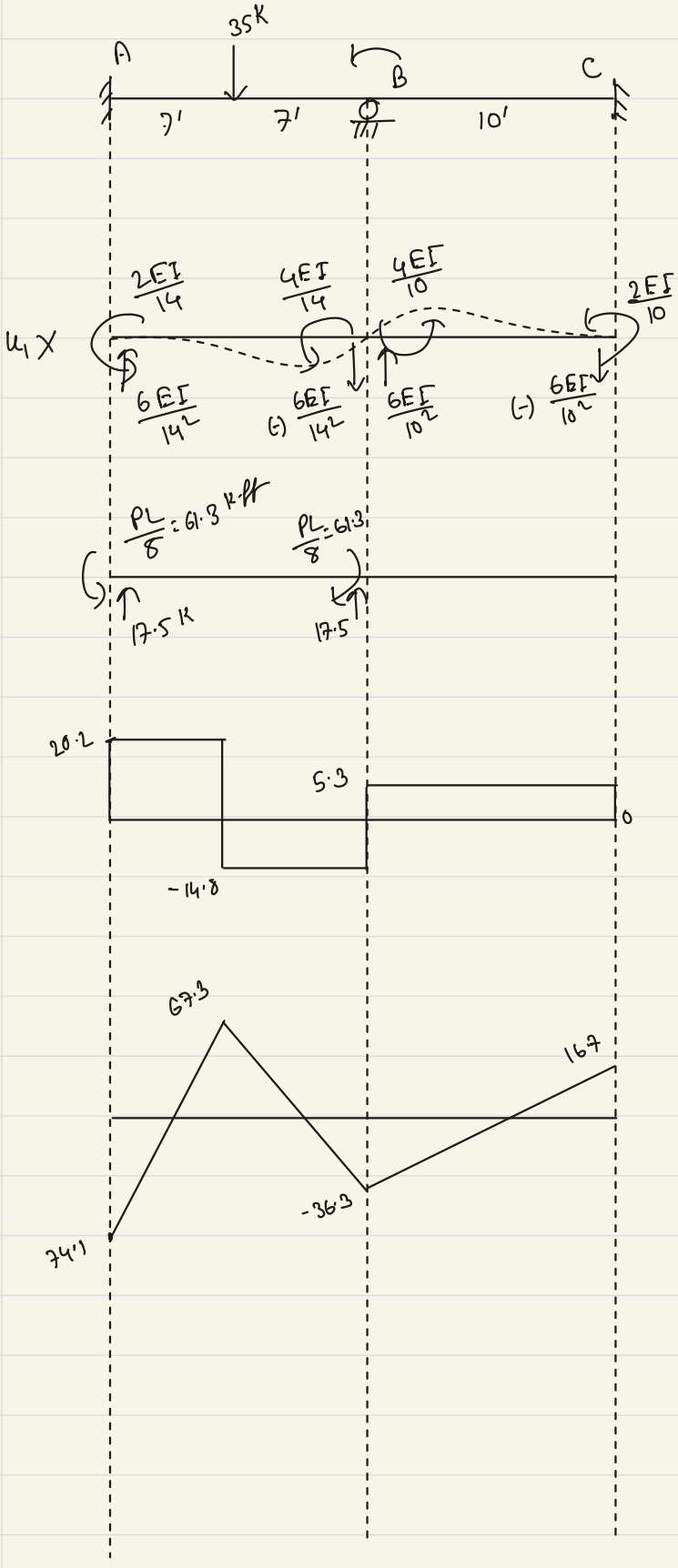
$\therefore Q_A = \frac{2}{20} \times 585 + 117 = 175.5$ K-ft

$Q_B = 0$

$R_{AV} = \frac{6}{20^2} \times 585 + 35 = 43.8$ K

$R_{BV} = -\frac{6}{20} \times 585 + 35 = 26.2$ K

Ex. 2



DOF = 1

Q = Ku + Qf

At B (roller):

$$Q = 0 = \left[\frac{4EI}{14} + \frac{4EI}{10}\right] \times u_1 - 61.3$$

$$\Rightarrow u_1 = \frac{61.3 \times 35}{24EI} = \frac{89.4}{EI}$$

$$M_A = \frac{2}{14} \times 89.4 + 61.3 = 74.1 \text{ K-ft}$$

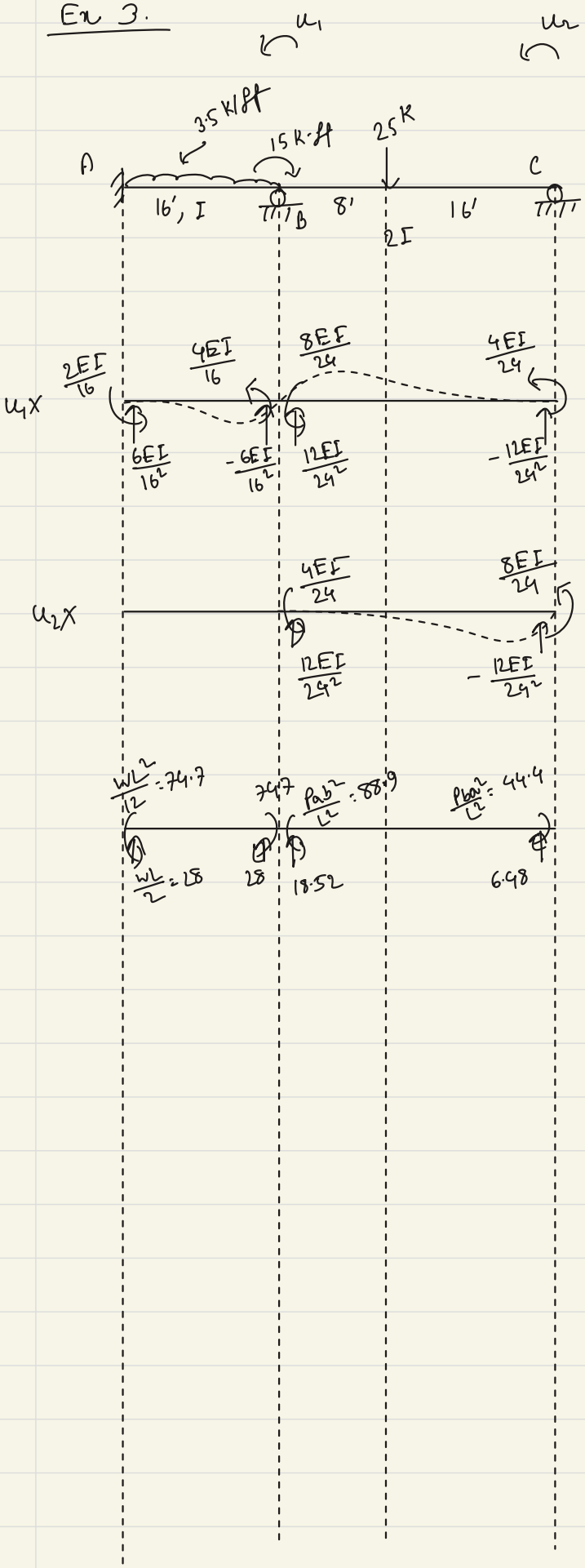
$$M_C = \frac{2}{10} \times 89.4 + 0 = 17.9 \text{ K-ft}$$

$$R_{Ay} = \frac{6}{14^2} \times 89.4 + 17.5 = 20.2 \text{ K}$$

$$R_{Cy} = -\frac{6}{10^2} \times 89.4 = -5.4 \text{ K}$$

$$R_{By} = \left(-\frac{6}{14^2} + \frac{6}{10^2}\right) \times 89.4 + 17.5 = 20.1 \text{ K}$$

Ex 3.



DoKI = 2

$Q = Ku + Q_f$

At B,

$$\Sigma M_B = 0 \Rightarrow +15 + \left[\frac{4EI}{16} + \frac{8EI}{24}\right] u_1 + \frac{4EI}{24} u_2$$

$$-74.7 + 88.9$$

$$\Rightarrow \frac{7}{12} EI \times u_1 + \frac{4}{24} EI \times u_2 + 14.2 = -15$$

At C, $0 = \frac{4EI}{24} u_1 + \frac{8EI}{24} u_2 - 44.4 = 0$

$u_1 = -102.8/EI$

$u_2 = 184.6/EI$

$M_A = \frac{2}{16} \times (-102.8) + 74.7 = 61.85$ k·ft

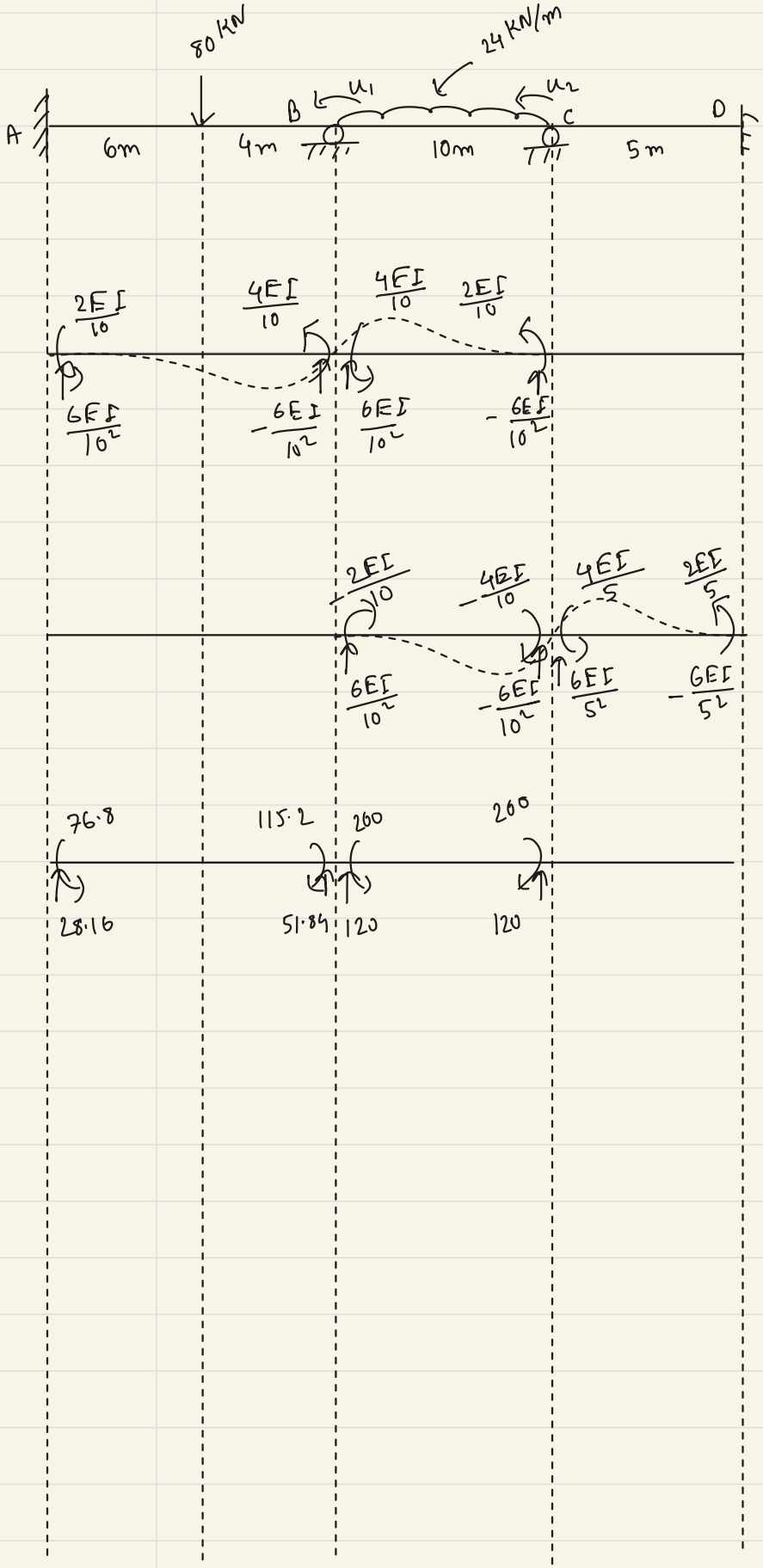
$M_C = \frac{4}{24} \times (-102.8) + \frac{8}{24}(184.6) - 44.4 = 0$

$R_{Ay} = \frac{6}{16^2} \times (-102.8) + 28 = 25.6$ K

$R_{By} = \left(-\frac{6}{16^2} + \frac{12}{24^2}\right) \times (-102.8) + \frac{12}{24^2} \times 184.6 + 28$

$+ 18.52 = 50.63$ K

$R_{Cy} = 4.78$ K



DoKI = 2

$Q = Ku + Qf$

At B,

$$0 = \left(\frac{4EI}{10} + \frac{4EI}{10}\right) u_1 + \left(\frac{-2EI}{10}\right) u_2$$

$$-115.2 + 200 = 0$$

$$\therefore \frac{4}{5} EI\, u_1 - \frac{2EI}{10} u_2 = -84.8$$

At C,

$$0 = \frac{2EI}{10} u_1 \left(-\frac{4EI}{10} + \frac{4EI}{5}\right) u_2$$

$$-200 = 0$$

$$\therefore \frac{2EI}{10} u_1 - \frac{2EI}{5} u_2 = 200$$

$u_1 = -269 \quad -632$

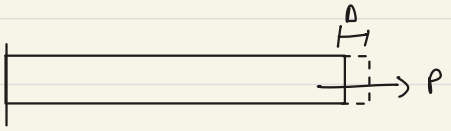
Plane Grid



$\theta = \frac{TL}{GJ}$ → torsional rigidity

G = Modulus of torsion / shear modulus / modulus of rigidity

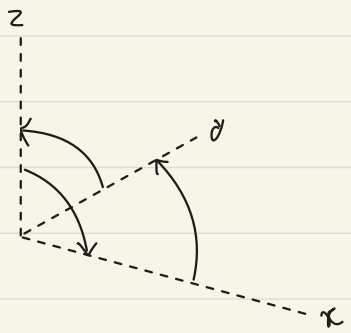
J = Torsional constant



$\frac{PL}{AE} \Rightarrow \frac{P}{D} = \frac{AE}{L}$

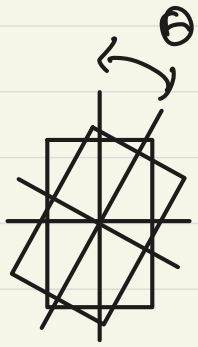
$T/\theta = \frac{GJ}{L}$

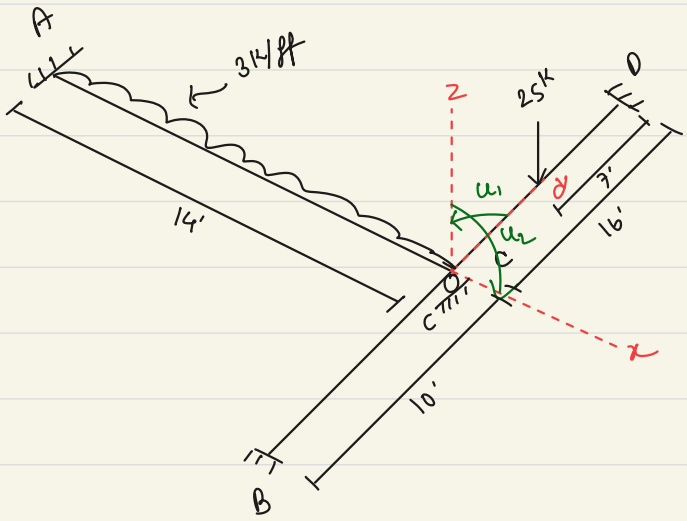
$\frac{M}{\theta} = \frac{4EI}{L}$



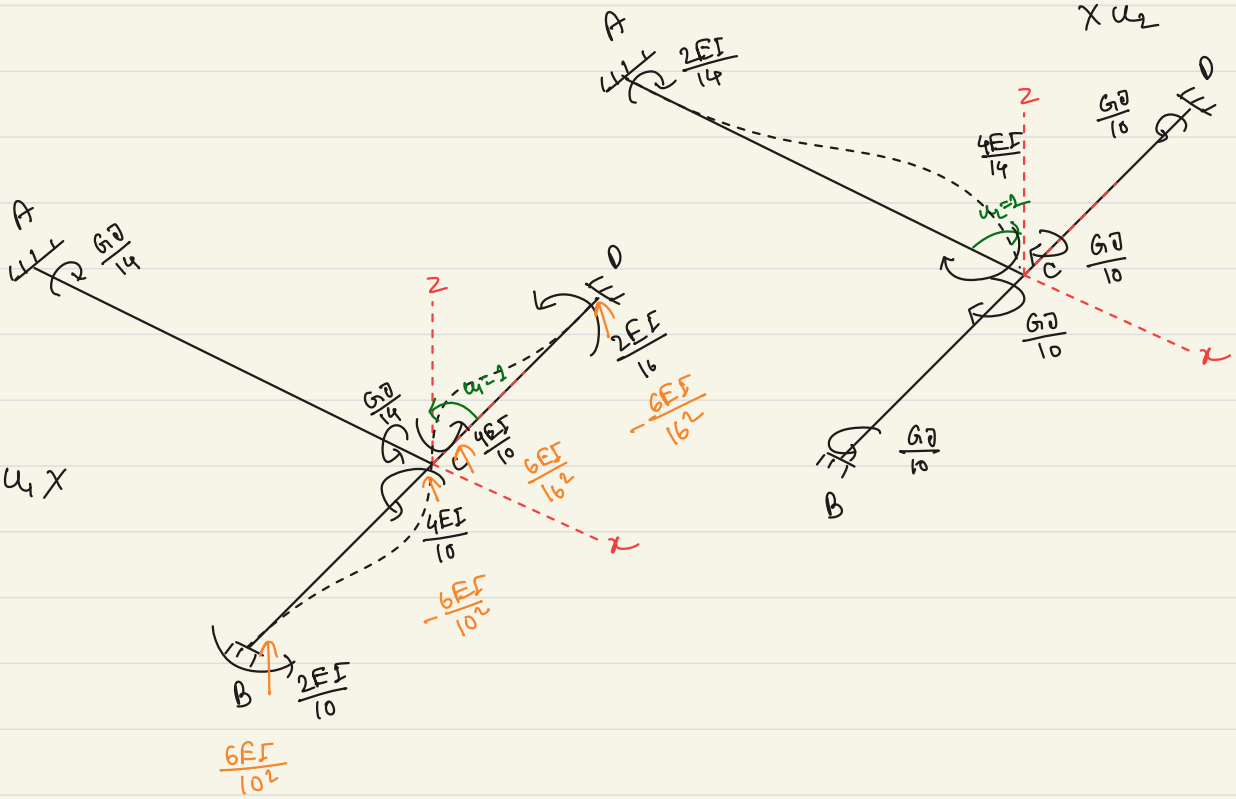
Sign conv.

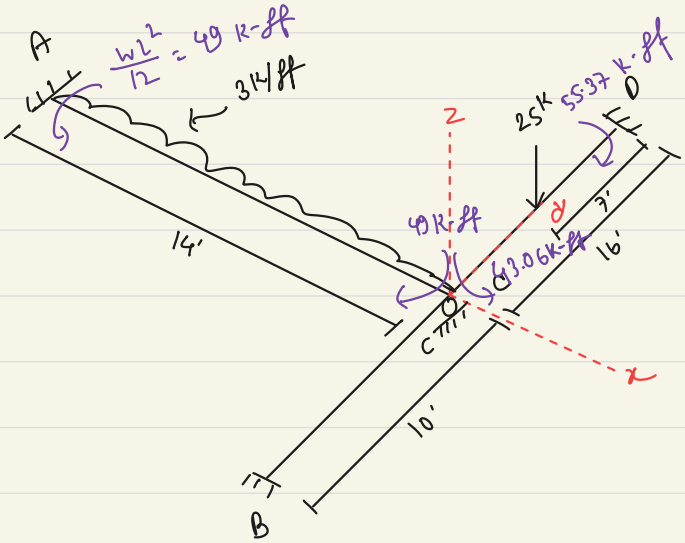
Pos (+) x to y, y to z, z to




A
31k/ft
14'
C
25k
D
3'
16'
10'
B
x
z
u_1
u_2

DoKI = 2


A
$\frac{2EI}{14}$
D
$\frac{GJ}{10}$
$\frac{4EI}{14}$
$u_4=1$
C
$\frac{GJ}{10}$
$\frac{GJ}{10}$
B
$\frac{GJ}{10}$
x
z
X u_2
$\frac{GJ}{14}$
u_4 X
$\frac{2EI}{16}$
$-\frac{6EI}{16^2}$
$\frac{4EI}{10}$
$\frac{6EI}{16^2}$
$-\frac{6EI}{10^2}$
$\frac{2EI}{10}$
$\frac{6EI}{10^2}$


A
$\frac{wL^2}{12}$ = 49 K-ft
31k/ft
14'
C
25k
55.97 K-ft
D
49K-ft
43.06K-ft
3'
16'
10'
B
x
z

u_4, on its axis all mmnt

$$0 = \left(\frac{4EI}{16} + \frac{4EI}{10} + \frac{GJ}{14}\right) \times u_4 + 0 \times u_2 + 436$$

$$0 = 0 \times u_1 + \left(\frac{4EI}{14} + \frac{GJ}{16} + \frac{GJ}{10}\right) \times u_2 + 49$$

$GJ = \frac{EI}{3}$

$$\therefore u_4 = \frac{-63.92}{EI}; \quad u_2 = -\frac{144.17}{EI}$$

$R_C = 29.094^{K}$

$M_D = -63.36 \text{ K-ft}$

$T_D = -3 \text{ k-ft}$

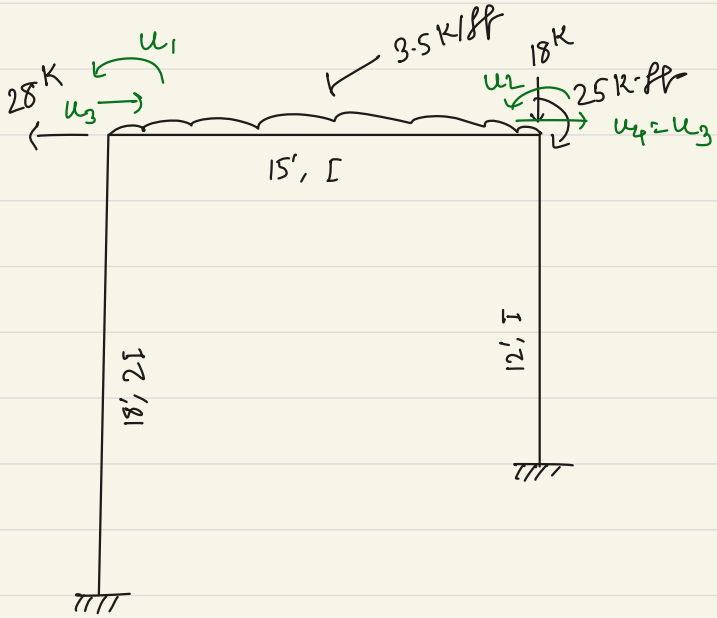
CT - 2

Next cls: Truss

Frame

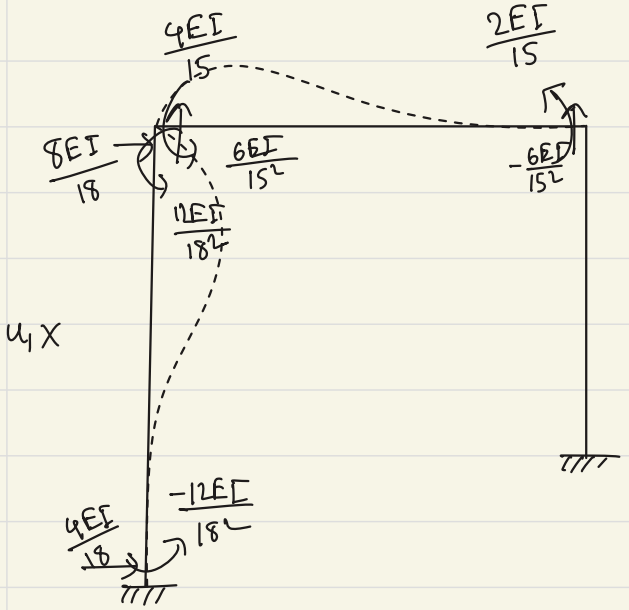
Rough / CT

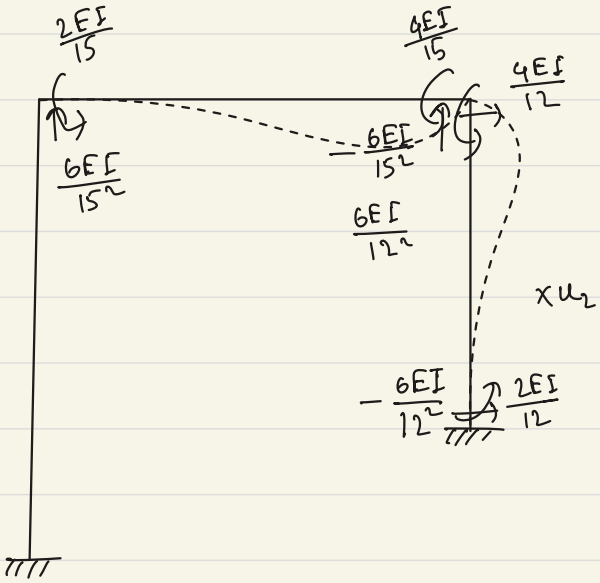
02/09

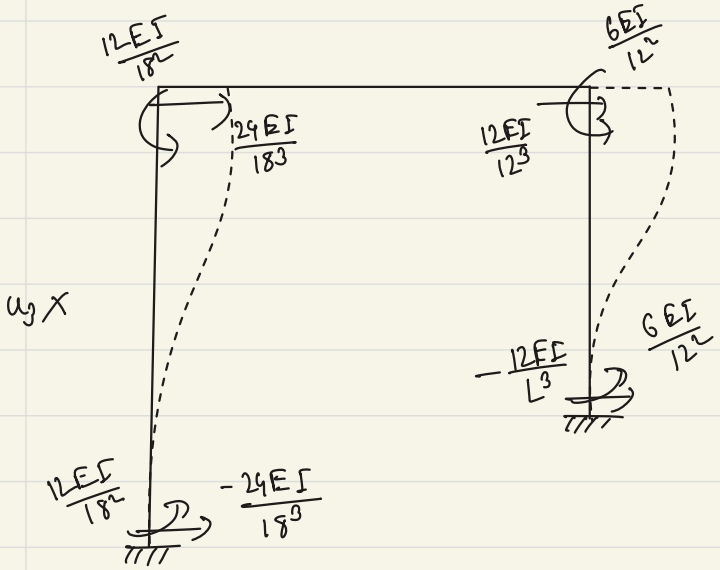


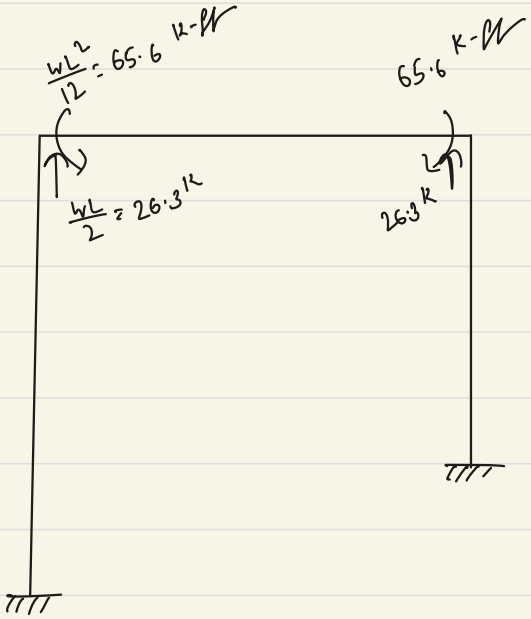
DoKI = 3

Q = Ku + Qf









u_1: $Q = Ku + Q_f$

$$\Rightarrow 0 = \left(\frac{4EI}{15} + \frac{8EI}{18}\right)u_1 + \frac{2EI}{15}u_2 + \frac{12EI}{18^2}u_3 + 65.6$$

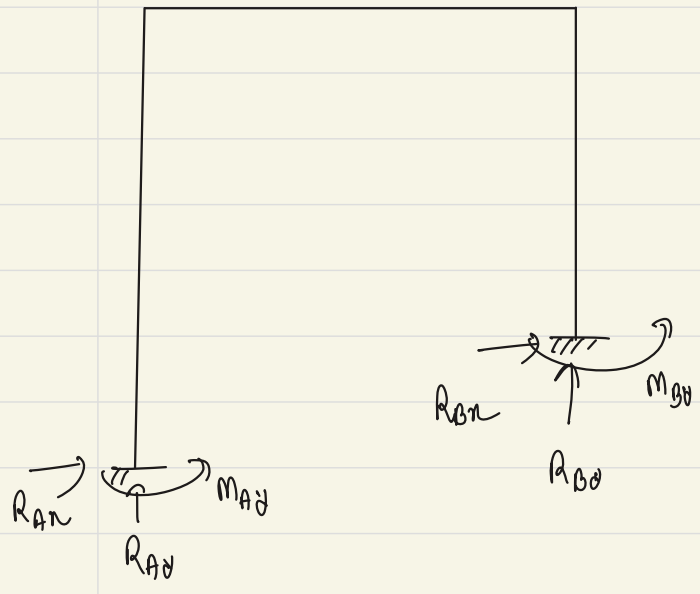
u_2: $Q = Ku + Q_f$

$$\Rightarrow -25 = \frac{2EI}{15}\times u_1 + \left(\frac{4EI}{15} + \frac{4EI}{12}\right)u_2 + \frac{6EI}{12^2}u_3 - 65.6$$

u_3: $Q = Ku + Q_f$

$$\Rightarrow -28 = \frac{12EI}{18^2}u_1 + \frac{6EI}{12^2}u_2 + \left(\frac{24EI}{18^3} + \frac{12EI}{12^3}\right)u_3$$

$$\therefore u_1 = \frac{51.5}{EI}; \quad u_2 = \frac{330.5}{EI}; \quad u_3 = \frac{-3949.1}{EI}$$



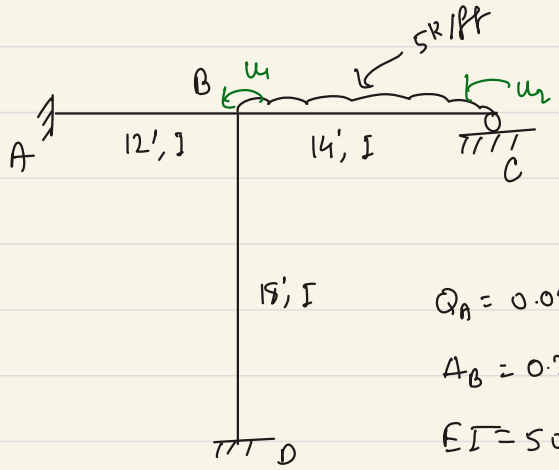
$$M_{AY} = \frac{4}{18}51.4 + \frac{12}{18^2}(-3949.1) = -134.8\ K\text{-}ft$$

$$M_{BY} = \frac{2}{12}\times 330.5 + \frac{6}{12^2}(-3949.1) = -109.4\ K\text{-}ft$$

$$R_{AY} = \frac{6}{15^2}\times 51.4 + \frac{6}{15^2}\times 330.5 + 26.3 = 36.5\ K$$

$$R_{AX} = \frac{-12}{18^2}\times 51.4 + \frac{-24}{18^3}(-3949.1) = 14.3\ K$$

Support settlement & rotation

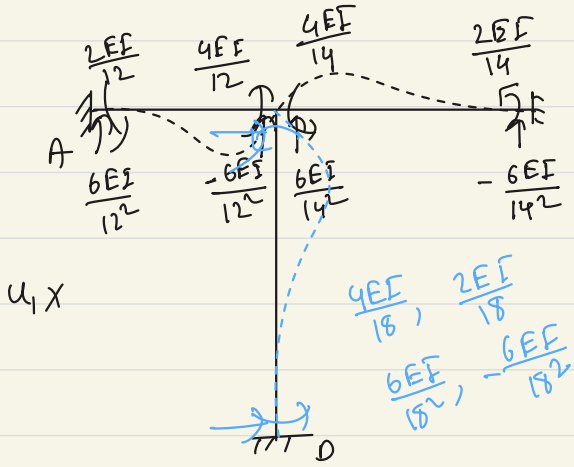


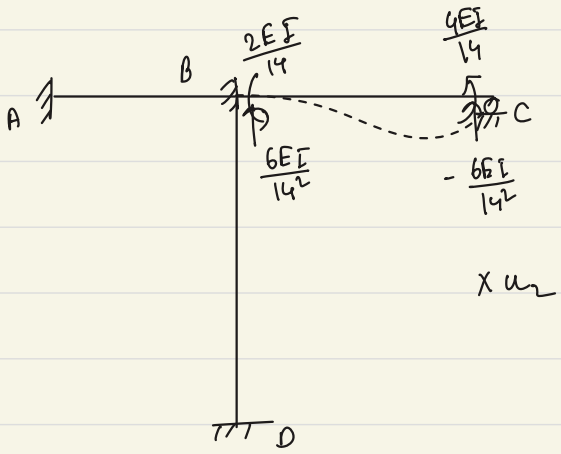
DoKI = 2

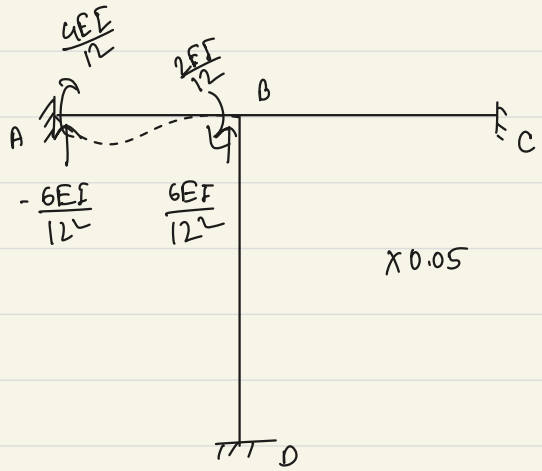
$Q_A = 0.05^r$ (↻)

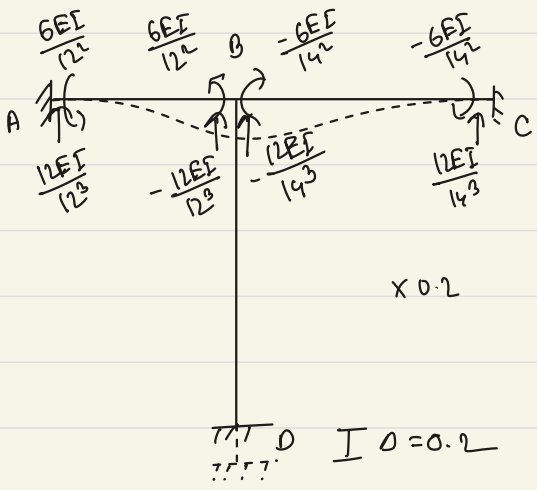
$\Delta_B = 0.2'$ (↓)

$EI = 5000\ k\text{-}ft^4$

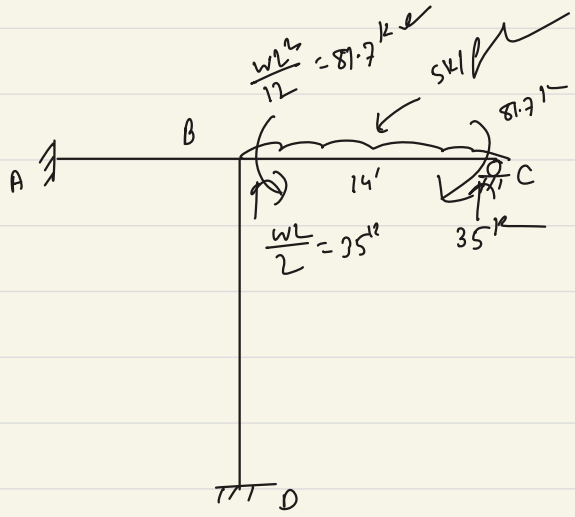








I D = 0.2



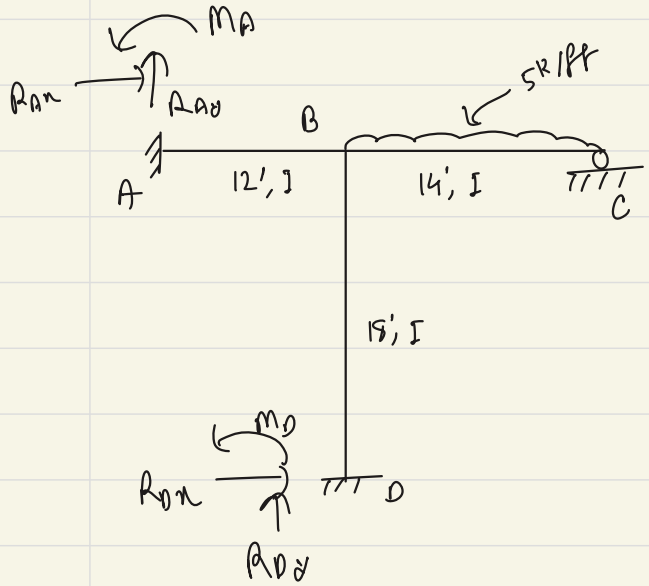
u_1: $Q = Ku + Qf + Q_i$

$$0 = \left(\tfrac{4}{12} + \tfrac{4}{14} + \tfrac{4}{18}\right)u_1 + \tfrac{2}{14}u_2 - \frac{2EI}{12} \times 0.05 + EI\left(\frac{6}{12^2} - \frac{6}{14^2}\right) \times 0.2 + 81.7$$

u_2: $Q = Ku + Qf + Q_i$

$$0 = \tfrac{2}{14}u_1 + \tfrac{4}{14}u_2 + EI\frac{-6}{14^2} \times 0.2 - 81.7$$

$$\therefore u_1 = \frac{-1393}{EI} \; ; \quad u_2 = \frac{462.7}{EI}$$



$R_{Ax} = 0$

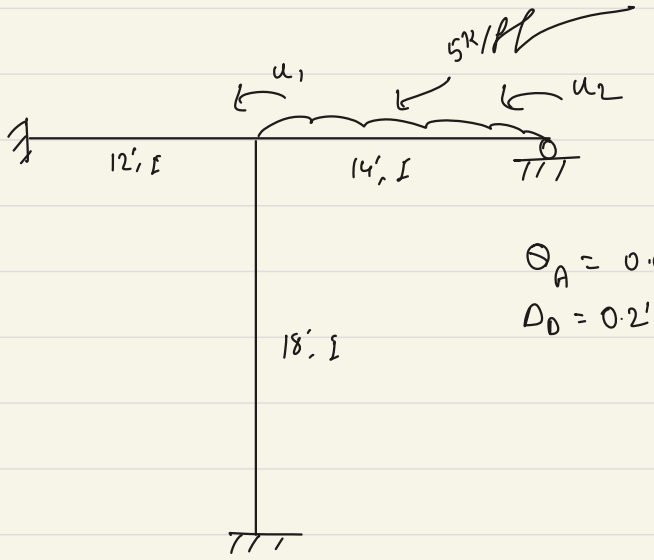
$R_{Ay} = \frac{6}{12^2}u_1 - \frac{6EI}{12^2} \times 0.05 + \frac{12EI}{12^3} \times 0.2$

$= -9.3$ K

$M_D = \frac{2}{18} \times u_4 = -15.5$ K-ft

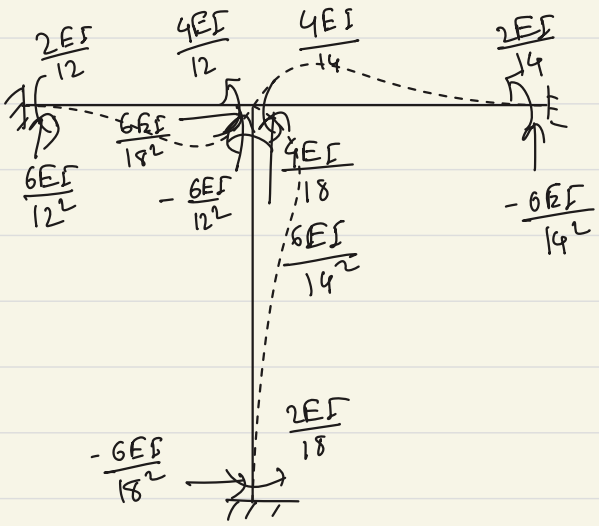
$M_A = \frac{2}{12}u_4 + \frac{-4EI}{12} \times 0.05 + \frac{6EI}{12^2} \times 0.2$

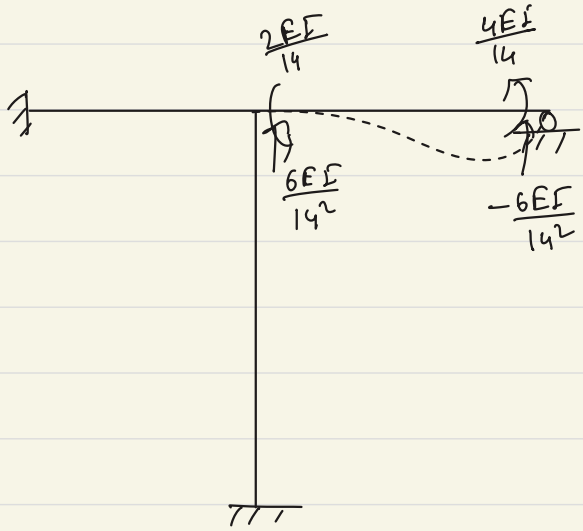
$= -64.9$ K

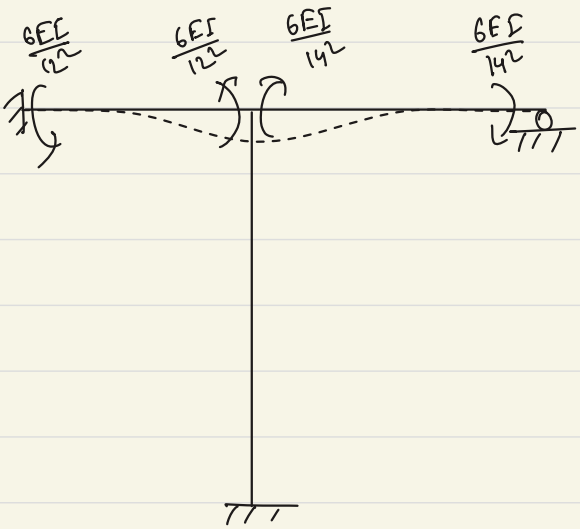


$\theta_A = 0.05^R$ (↻)

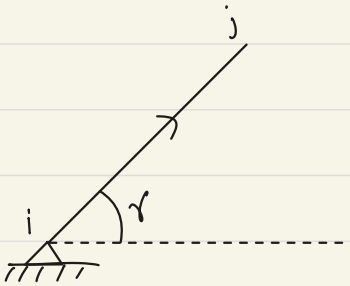
$\Delta_D = 0.2'$ (↓)

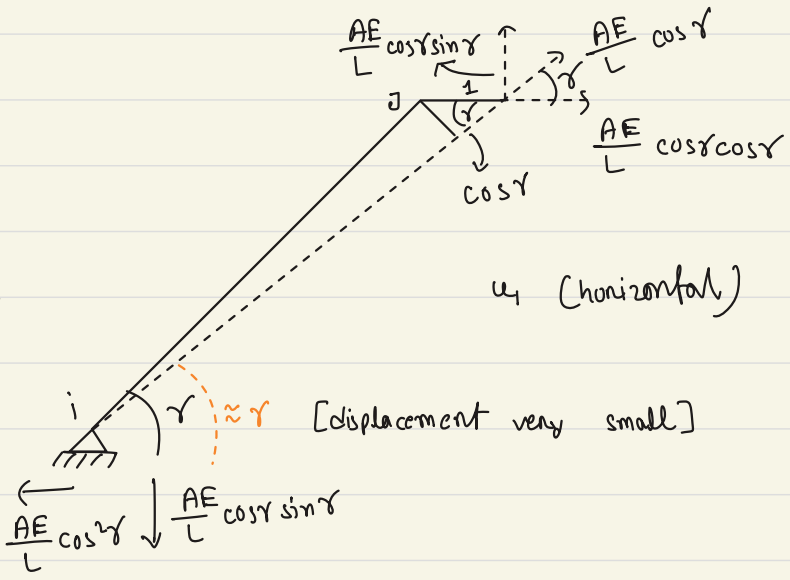






Truss



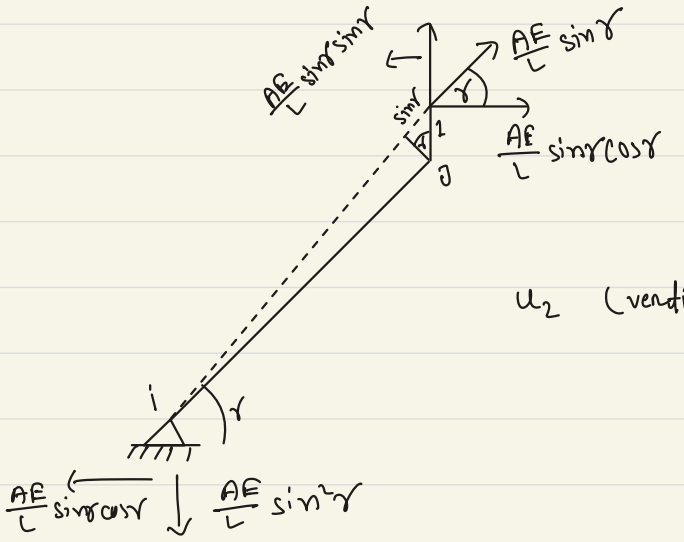


u_1 (horizontal)

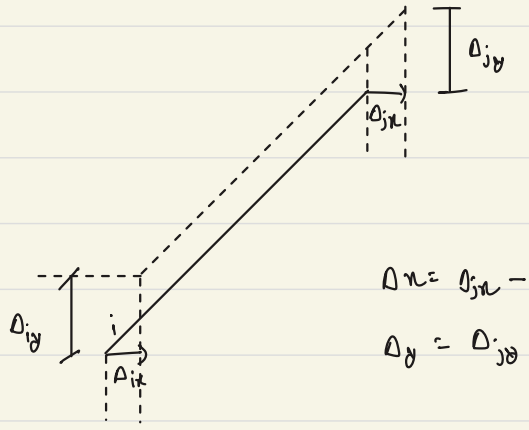
[displacement very small]

$\delta = \frac{FL}{AE}$

$\Rightarrow F = \frac{AE\delta}{L}$

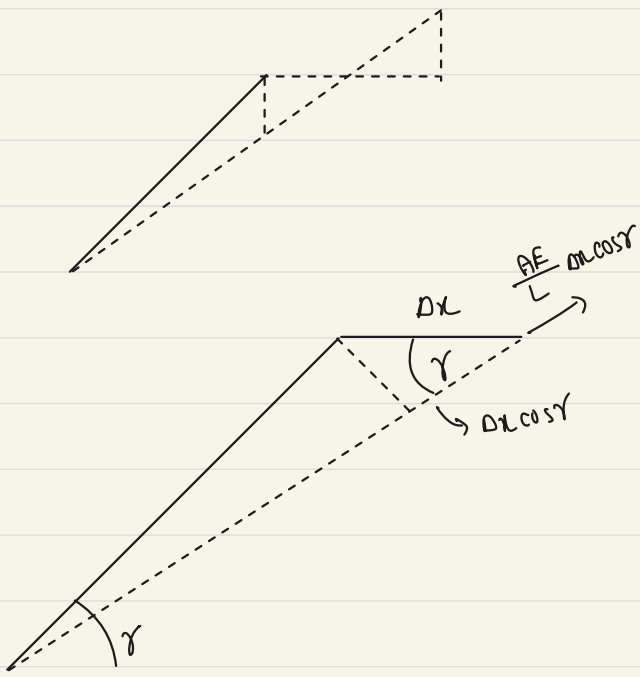


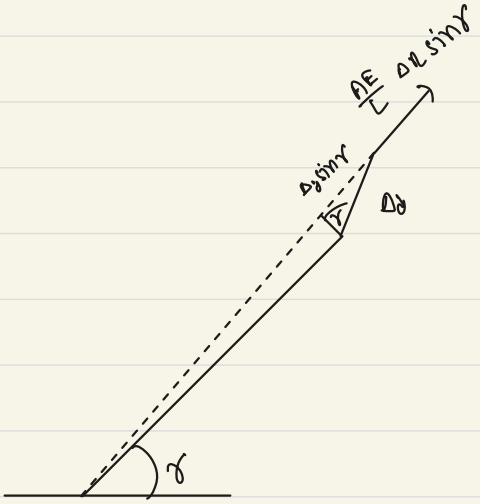
u_2 (vertical)



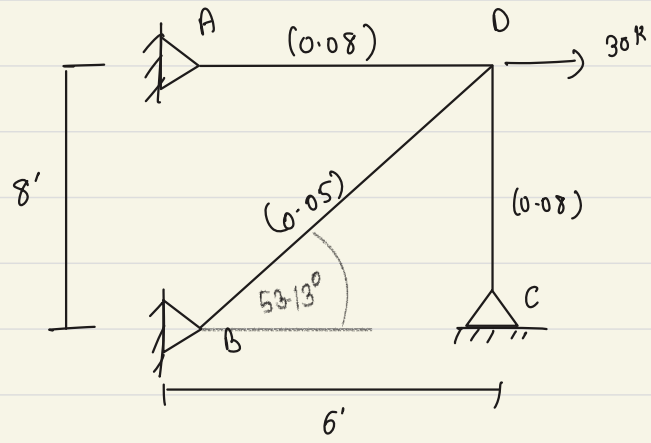
$\Delta x = \delta_{jx} - \delta_{ix}$

$\Delta y = \delta_{jy} - \delta_{iy}$



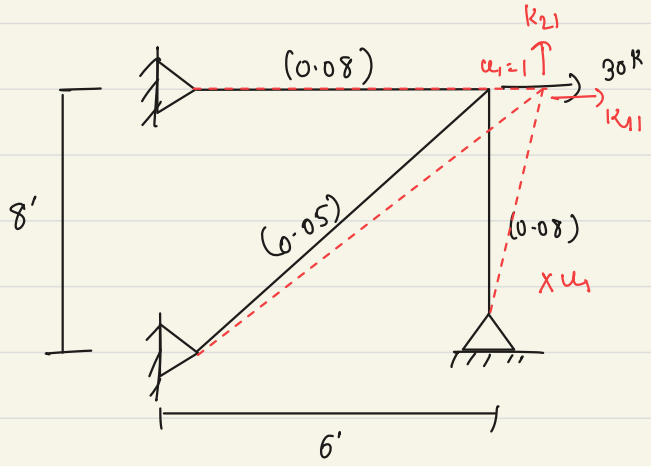


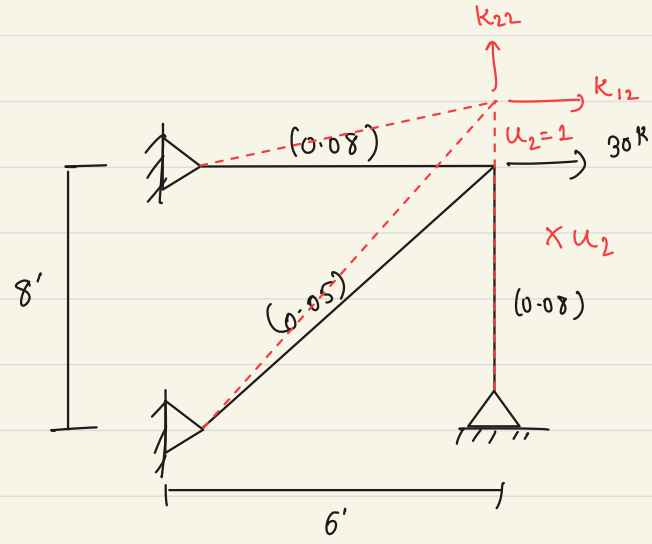
$$\therefore F = \frac{AE}{L}\left(\Delta_i \cos\gamma + \Delta_j \sin\gamma\right)$$



$E = 1000\ K/ft^2$

DoKI = 2





$Q = Ku \qquad [no\ Q_f]$

$$K_{11} = \sum_{AO,BO,CO} \frac{AE}{L}\cos^2\gamma$$

$$= \left(\frac{AE}{L}\cos^2\gamma\right)_{AO} + \left(\frac{AE}{L}\cos^2\gamma\right)_{BO} + \left(\frac{AE}{L}\cos^2\gamma\right)_{CO}$$

$$= E\cos^2\gamma\,(\cdots)$$

$$= 15.13$$

$$K_{21} = \sum_{AD,BD,CD} \left(\frac{AE}{L} \cos\gamma \sin\gamma \right)$$

$$= 2.4$$

$$K_{12} = \sum_{AD,BD,CD} \left(\frac{AE}{L} \cos\gamma \sin\gamma \right)$$

$$= 2.4$$

$$K_{22} = \sum_{AD,BD,CD} \left(\frac{AE}{L} \sin^2\gamma \right)$$

$$= 13.2$$

$$O_j = KU$$

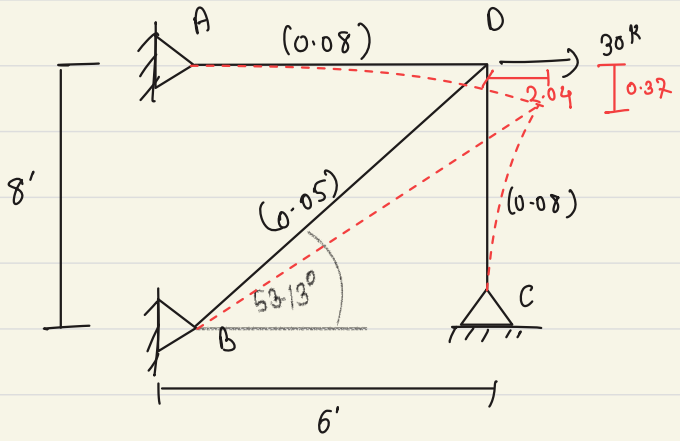
$$30 = K_{11} u_1 + K_{12} u_2$$

$$0 = K_{21} u_1 + K_{22} u_2$$

$$\begin{bmatrix} 30 \\ 0 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u_1 = 2.04$$

$$u_2 = -0.37$$

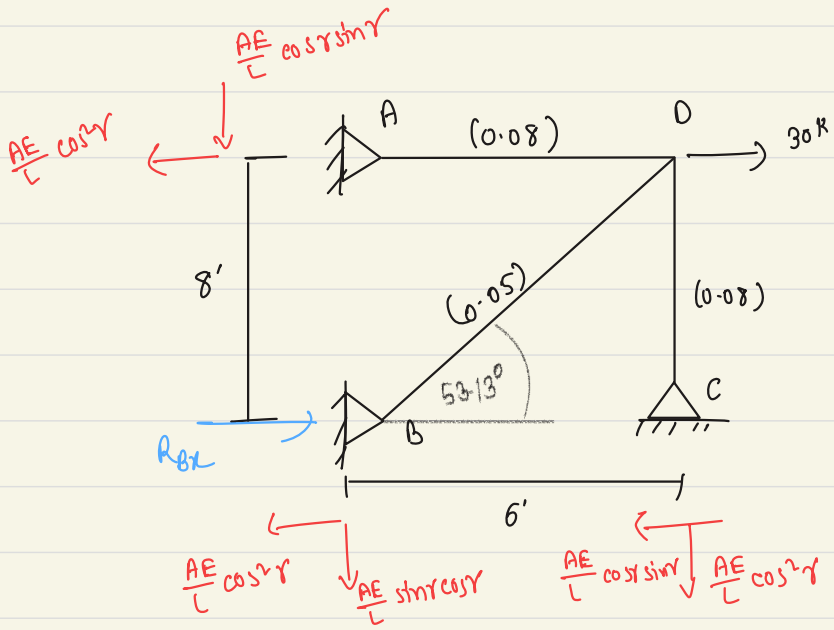


$$F_{AD} = \frac{AE}{L}\left(D_x \cos\gamma + D_y \sin\gamma\right)$$

$$= 27.23^{K} \quad (T)$$

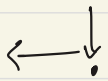
$F_{BD} = 4.64 \quad (T)$

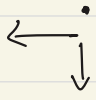
$F_{CD} = -3.77 \quad (C)$

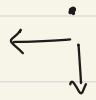


For $u = 1 \longrightarrow$

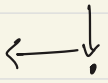
For u_1 :







For u_2 :





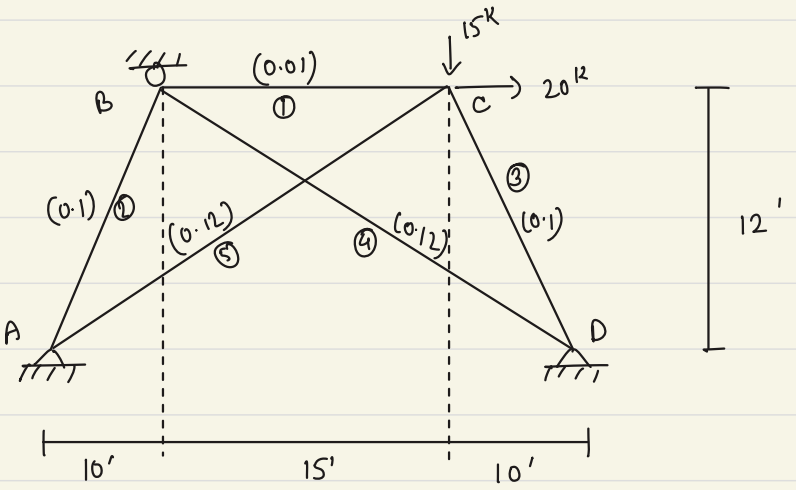


$Q = Ku$

$R_{Bx} = \left(-\frac{AE}{L}\cos^2\gamma\right)u_1 + \left(-\frac{AE}{L}\sin\gamma\cos\gamma\right) \times u_2$

$= -2.785^{K}\ (\leftarrow)$

$R_{By} = -37.1^{K}\ (\downarrow)$



(−) → area

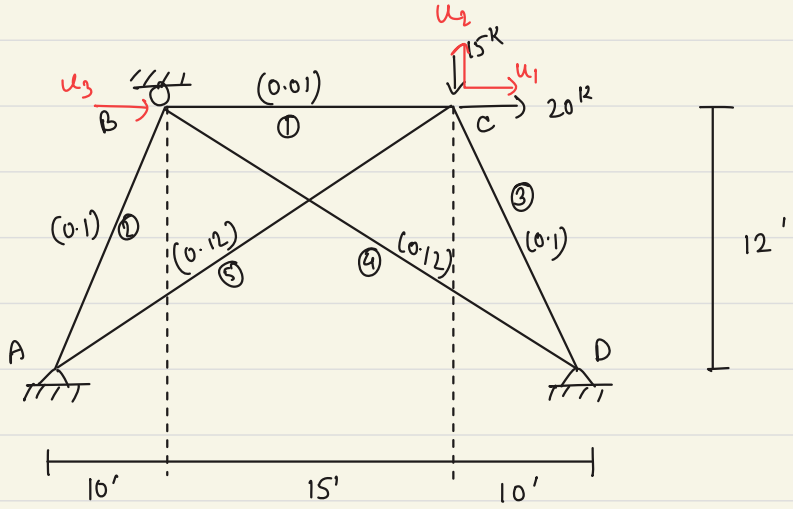
⊙ → member no.

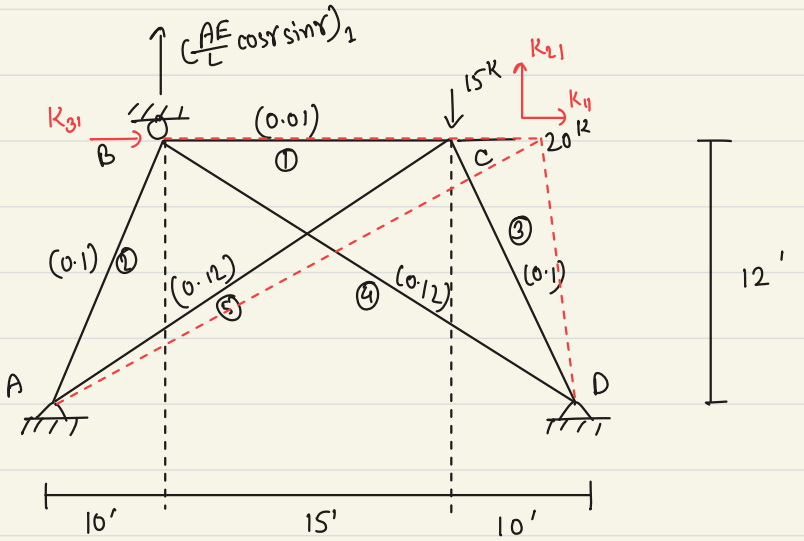
E = 5000 k/ft²

DOKI = 3

Instead of fixed joint be pin @ the joints

Truss always extensible

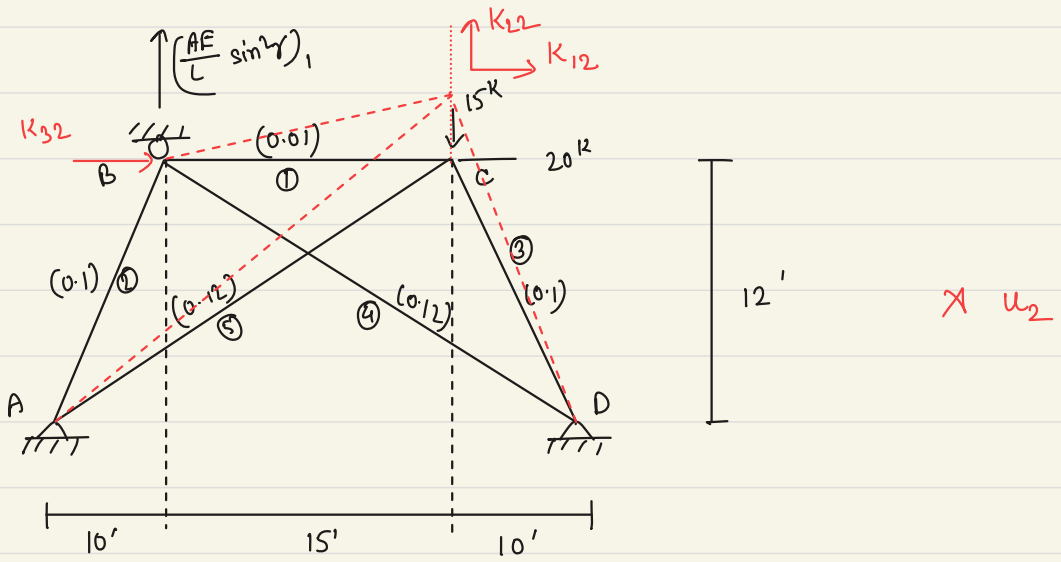




✗ u_1

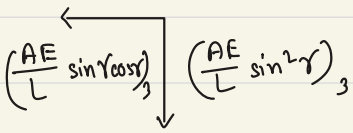
$\left(\frac{AE}{L}\cos^2\gamma\right)_5$ $\left(\frac{AE}{L}\cos\gamma\sin\gamma\right)_5$

$\left(\frac{AE}{L}\cos^2\gamma\right)_3$ $\left(\frac{AE}{L}\sin\gamma\cos\gamma\right)_3$

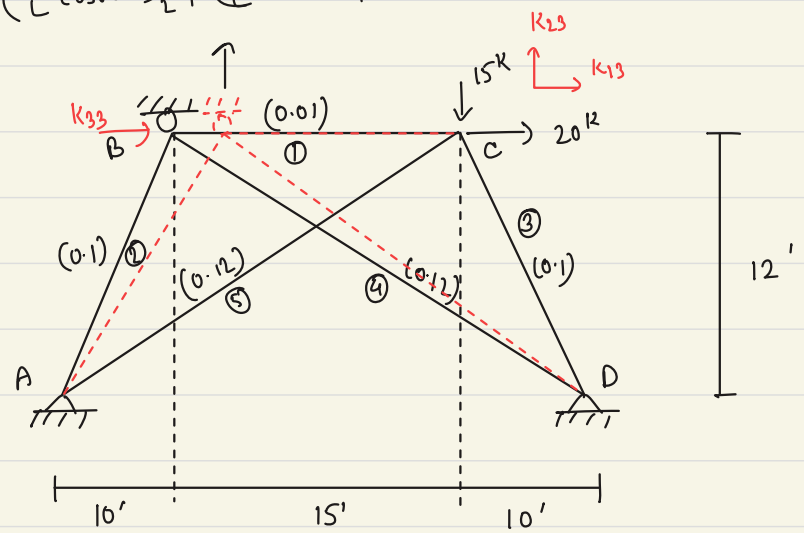


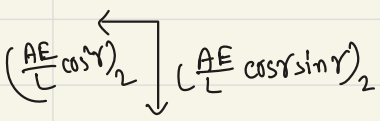
$\times u_2$

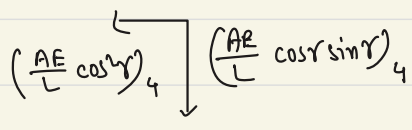




$\left(\frac{AE}{L}\cos\gamma\sin\gamma\right)_2 + \left(\frac{AE}{L}\cos\gamma\sin\gamma\right)_4 + \left(\frac{AE}{L}\cos\gamma\sin\gamma\right)_1$







Member Name	Member Endpoint	i	j	L	A	γ	$\cos\gamma$	$\sin\gamma$	$\cos\gamma\sin\gamma$	
1	BC	B	C	15	0.1	0	1	0	0	
2	AB	A	B	15.62	0.1	50.19	0.41	0.59	0.49	
3	DC	D	C	15.62	0.1	129.81	0.41	0.59	-0.49	
4	DB	D	B	27.73	0.12	154.36	0.81	0.19	-0.39	
5	AC	A	C	27.73	0.12	25.64	0.81	0.19	0.39	

$$K_{11} = \sum_{1,3,5} \frac{AE}{L} \cos^2\gamma$$

$$= E\left(\frac{0.1}{15} \times 1 + \frac{0.1}{15.62} \times 0.91 + \frac{0.12}{27.73} \times 0.81\right)$$

$$= 63.98$$

$R_{Ax} = 3.78^K \ (\leftarrow)$

$R_{Dx} = 16.22^K \ (\leftarrow)$

$R_{Ay} = 3.50^K \ (\downarrow)$

$R_{By} = 1.30^K \ (\uparrow)$

$R_{Dy} = 17.2^K \ (\uparrow)$

$K_{12} = K_{21} = -7.283$

$K_{31} = K_{13} = -33.33$

$K_{22} = 22.937$

$K_{32} = K_{23} = 0$

$K_{33} = 64.04$

not dependent on reaction forces

$u_1, u_2 \ldots$ new

$F_{AB} = 3.67^K \ (T)$

$F_{AC} = 1.59^K \ (T)$

$F_{CD} = 20.43^K \ (C)$

$F_{BC} = 5.42^K \ (T)$

$F_{BD} = 3.49^K \ (C)$

$$\begin{bmatrix} 20 \\ 15 \\ 0 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ \text{——} & \text{——} & \text{——} \\ \text{——} & \text{——} & \text{——} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

---

if D point only asked, perform on D only

Recap / Matrix next cls

On computer

← must ques

Direct stiffness method — Analysis of Indeterminate Structure

Background:

DOSI : Degree of static Indeterminacy

- → < 0 → unstable, determinate
  = 0 → stable, determinate
  > 0 → stable, indeterminate

→ Internal static indeterminacy (I_{si}) → $3m - 3j + 3 - R_n$

External static indeterminacy (I_{se}) → $R - 3$

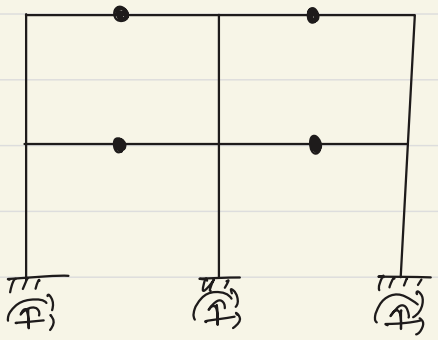
$I_s = I_{si} + I_{se}$

→ $(R + 3m) - (3j + R_n)$

(R + 3m): total unknown; (3j + R_n): total reactions

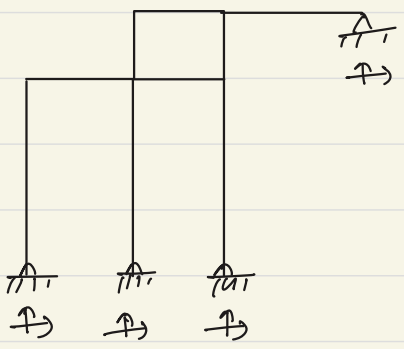
$R = 9$
$m = 14$
$j = 13$
$R_n = 4$

} 8°



$R = 8$
$m = 9$
$j = 9$
$R_n = 0$

} 8° ??



DOKI : Degree of Kinematic Indeterminacy

Number of independent joint displacement required to describe deformed shape.

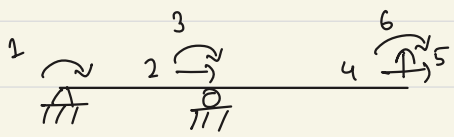
↳ translational

rotational

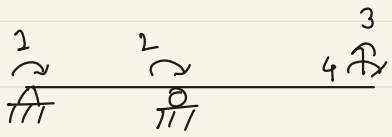
$I_K = 3j - k$ [K → const. displacement given by support]

$2j - k$ [truss]

$6j - k$ [3D]



extensible → 6



inextensible → 4

(numbered from 1st node to last

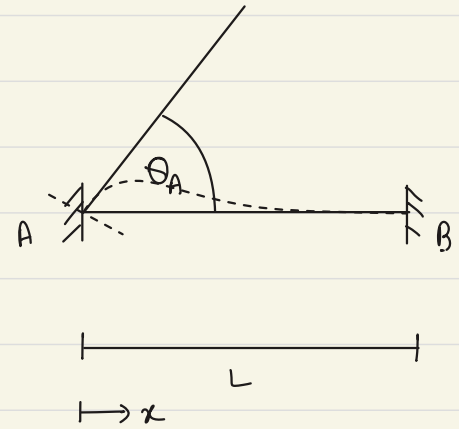
x → y → rotate)

Rotational stiffness :

Moment required to make unit rotation

$\rightarrow \frac{m}{\theta}$

Fixed End Moment :



$v(x)$ = vert. defl.

$EI\, v''''(x) = P$ [Euler - Bernoulli Beam Theory]

on, for external loading $(P) = 0$,

$EI\, v'''(x) = C_1 \longrightarrow$ Shear (i)

$EI\, v''(x) = C_1 x + C_2 \longrightarrow$ moment (ii)

$EI\, v'(x) = C_1 \frac{x^2}{2} + C_2 x + C_3 \longrightarrow$ slope (iii)

$EI\, v(x) = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4 \rightarrow$ Deflection (iv)

Boundary conditions : when, $x=0,\ v=0,\ \theta = \theta_A$

$x=L,\ v=0,\ \theta = 0$

Putting $x=0,\ v=0,\ \theta=\theta_A$ in (iv) & (iii) :

$0 = C_4$

$EI\,\theta_A = C_3$

on, $EI\,\theta_x = C_1 \frac{x^2}{2} + C_2 x + EI\,\theta_A$ — (v)

$EI\, v_x = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + EI\,\theta_A x$ — (vi)

Putting $x = L$, $v = 0$, $\theta = 0$ in (v) & (vi):

$$0 = C_1 \frac{L^3}{6} + C_2 \frac{L^2}{2} + EI\theta_A L \quad \text{—(vii)}$$

$$0 = C_1 \frac{L^2}{2} + C_2 L + EI\theta_A \quad \text{—(viii)}$$

$$\frac{L}{3} \times (viii) - (vii) \Rightarrow C_2\left(\frac{L^2}{3} - \frac{L^2}{2}\right) + EI\theta_A\left(\frac{1}{3}L - L\right) = 0$$

$$\Rightarrow C_2\left(-\frac{1}{6}L^2\right) = \left(\frac{2}{3}L\right) EI\theta_A$$

$$\Rightarrow C_2 = -EI\theta_A \times 4L^{-1}$$

$$= -\frac{4EI\theta_A}{L}$$

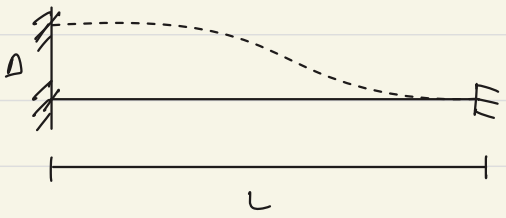
$$(viii) \Rightarrow 0 = C_1 \frac{L^2}{2} - \frac{4EI\theta_A}{L} \times L + EI\theta_A$$

$$\Rightarrow -C_1 \frac{L^2}{2} = -3EI\theta_A$$

$$\Rightarrow C_1 = \frac{6EI\theta_A}{L^2}$$

Translation:

$EI v''''(x) = p$ Load



$p=0$,

$EI v'''(x) = c_1$ Shear (i) $\longmapsto x$

$EI v''(x) = c_1 x + c_2$ Moment (ii)

$EI v'(x) = c_1 \frac{x^2}{2} + c_2 x + c_3$ Slope (iii)

$EI v(x) = c_1 \frac{x^3}{6} + c_2 \frac{x^2}{2} + c_3 x + c_4$ Deflection (iv)

Boundary conditions:

when $x=0,\ v=\Delta,\ \theta=0$

$x=L,\ v=0,\ \theta=0$

Putting $x=0,\ v=\Delta,\ \theta=0$ on (iii) $\Rightarrow 0 = c_3$

(iv) $\Rightarrow EI\Delta = c_4$

$\therefore EI v'(x) = c_1 \frac{x^2}{2} + c_2 x$ — (v)

$EI v(x) = c_1 \frac{x^3}{6} + c_2 \frac{x^2}{2} + 0 + EI\Delta$ — (vi)

Putting $x=L,\ v=0,\ \theta=0$ on (v) $\Rightarrow 0 = c_1 \frac{L^2}{2} + c_2 L$ — (vii)

(vi) $\Rightarrow 0 = c_1 \frac{L^3}{6} + c_2 \frac{L^2}{2} + 0 + EI\Delta$ — (viii)

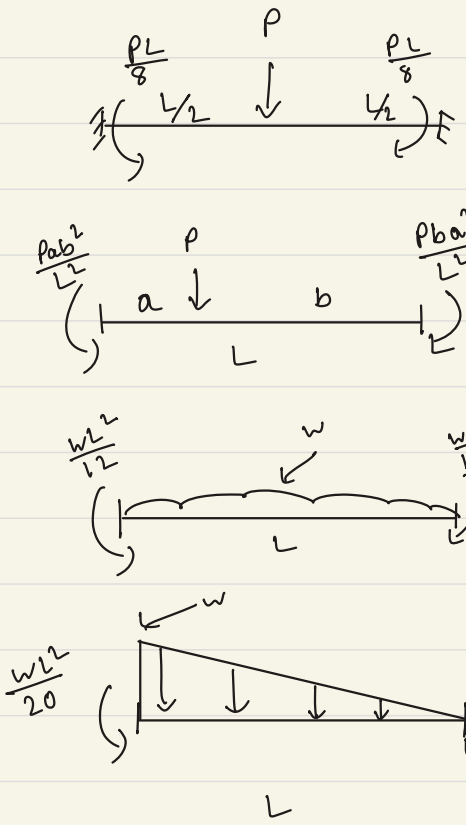
(vii) $\times \frac{L}{3}$ − (viii) $\Rightarrow c_1 \times 0 + c_2\left(\frac{L^2}{3} - \frac{L^2}{2}\right) - EI\Delta = 0$

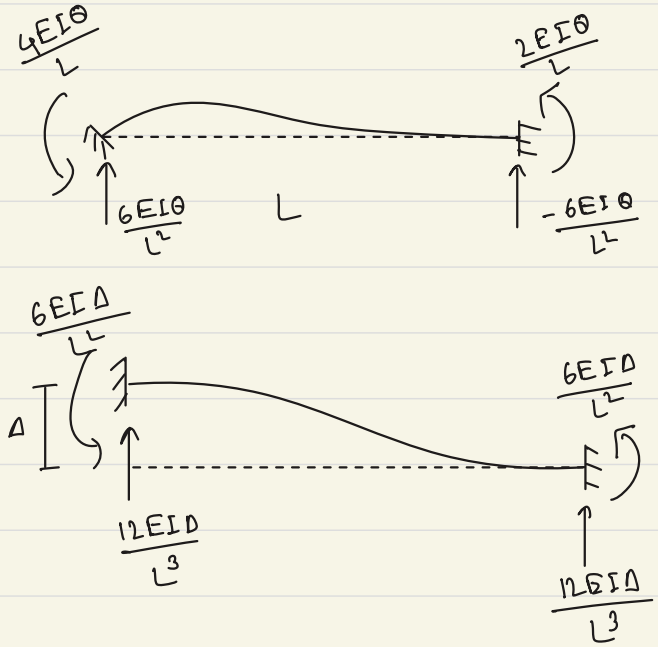
$\Rightarrow c_2 = -\frac{6EI}{L^2}\Delta$

(vii) $\Rightarrow 0 = c_1 \frac{L^2}{2} - \frac{6EI}{L}$

$\Rightarrow c_1 = \frac{12EI}{L^3}\Delta$

Fixed End moments :





matrix algebra:

Addition, substraction → connesponding el.

multiplication → scalar → multiply each el.

matrices → $\underbrace{\text{num of col}}_{1^{st}\ m} = \underbrace{\text{num of row}}_{2^{nd}\ m}$

$$\underset{m \times n}{A} \times \underset{n \times s}{B} = \underset{m \times s}{C}$$

$$C_{ij} = \sum A_{iK} B_{Kj}$$

row x col [connesponding]

$$A_{11}x_1 + A_{12}x_2 + A_{13}x_3 = P_1$$
$$A_{21}x_1 + A_{22}x_2 + A_{23}x_3 = P_2$$
$$A_{31}x_1 + A_{32}x_2 + A_{33}x_3 = P_3$$

↓

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

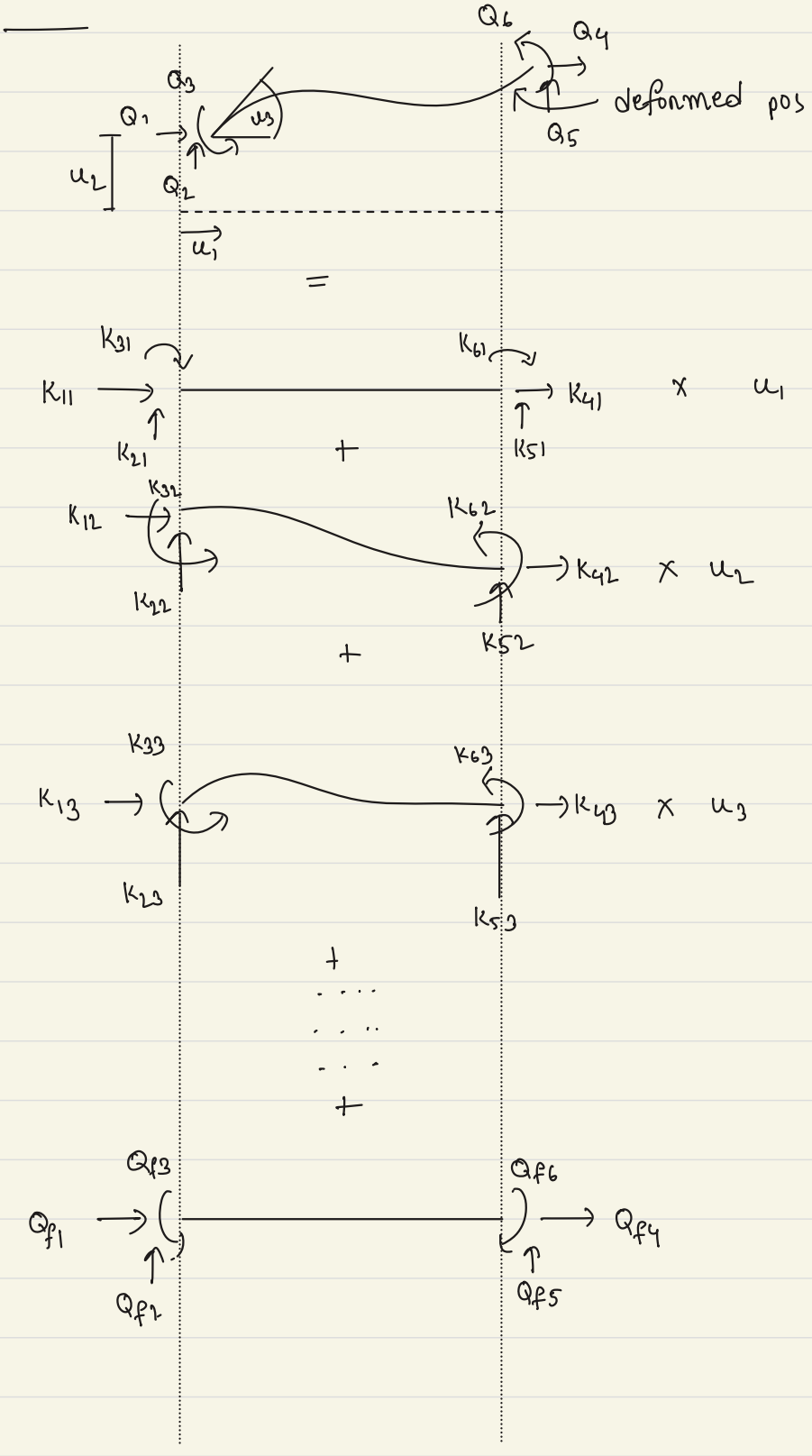
↓

$$Ax = P$$
$$\Rightarrow A^{-1}Ax = A^{-1}P$$
$$\Rightarrow x = A^{-1}P$$

Inverse of symmetric matrix is the " "

Stiffness Relation: member end forces and member end displacement in terms of external loading.

$$Q = Ku + Q_f$$



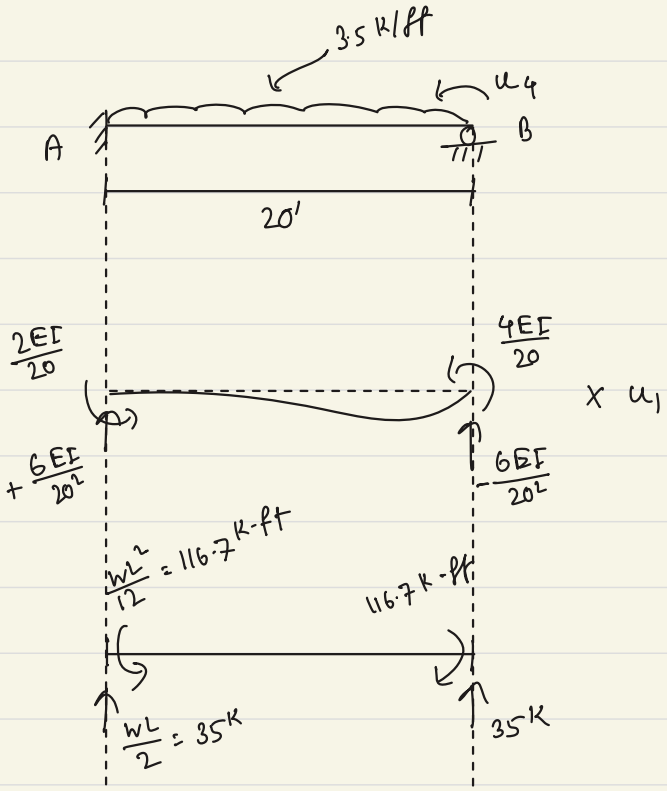
$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix}$$

$$Q = ku + Q_f$$

Beam:

Ex. 1 DoK I = 2

$Q = Ku + Q_f$



At B:

$$0 = u_4\left[\frac{4EI}{20}\right] + (-116.7)$$

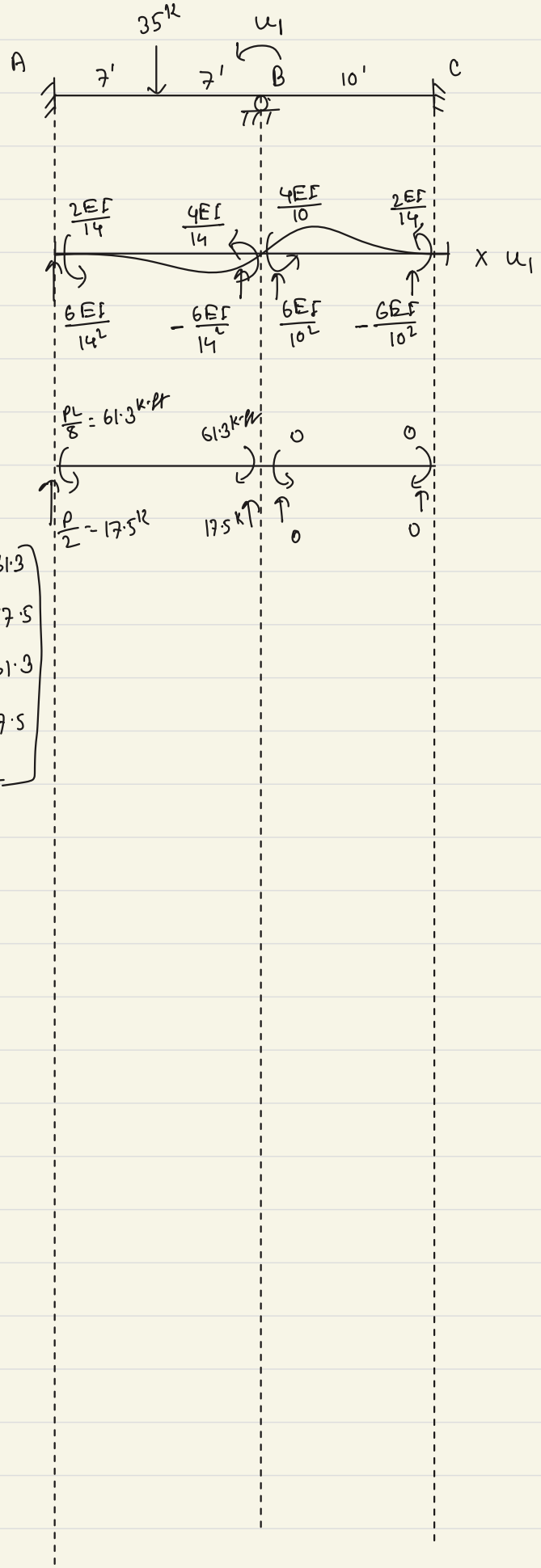
$$\Rightarrow u_4 = 583.5 \ \frac{1}{EI}$$

$$[Q_4] = [K_{44}]\,[u_4] + [Q_{f_4}]$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 6/20^2 \\ 0 & 0 & 0 & 2/20 \\ 0 & 0 & 0 & -6/20^2 \\ 0 & 0 & 0 & 4/20 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 583.5 \end{bmatrix} + \begin{bmatrix} 35 \\ 116.7 \\ 35 \\ -116.7 \end{bmatrix}$$

$$= \begin{bmatrix} 43.75 \\ 175.05 \\ 26.25 \\ 0 \end{bmatrix} \begin{matrix} \rightarrow R_{AY} \\ \rightarrow M_A \\ \rightarrow R_{BY} \\ \rightarrow M_B \end{matrix}$$

Ex. 2



Dof = 1

$Q = ku + Q_f$

At B, $0 = u_1\left[\frac{4EI}{14} + \frac{4EI}{10}\right] - 61.3$

$\Rightarrow u_1 = \frac{89.4}{EI}$

M_A, M_B, R_{AY}, B_{BY} :

$$\begin{bmatrix} M_A \\ R_{AY} \\ M_B \\ R_{BY} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2/14 & 0 \\ 0 & 0 & 6/14^2 & 0 \\ 0 & 0 & \frac{4}{14}+\frac{4}{10} & 0 \\ 0 & 0 & \frac{-6}{14^2}+\frac{6}{10^2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 89.4 \\ 0 \end{bmatrix} + \begin{bmatrix} 61.3 \\ 17.5 \\ -61.3 \\ 17.5 \end{bmatrix}$$

$$= \begin{bmatrix} 74.1 \\ 20.2 \\ 0 \\ 20.1 \end{bmatrix}$$

Ex. 3

$$\begin{bmatrix} -15 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{16}+\frac{8}{24} & \frac{4}{24} \\ \frac{4}{24} & \frac{8}{24} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} -74.7+88.9 \\ -44.4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -103.4 \\ +184.9 \end{bmatrix}$$

$$\therefore u_1 = \frac{-103.4}{EI}; \quad u_2 = \frac{184.9}{EI}$$

$$M_A = -103.4\left(\tfrac{2}{16}\right) + 74.7 = 61.8 \text{ K-ft}$$

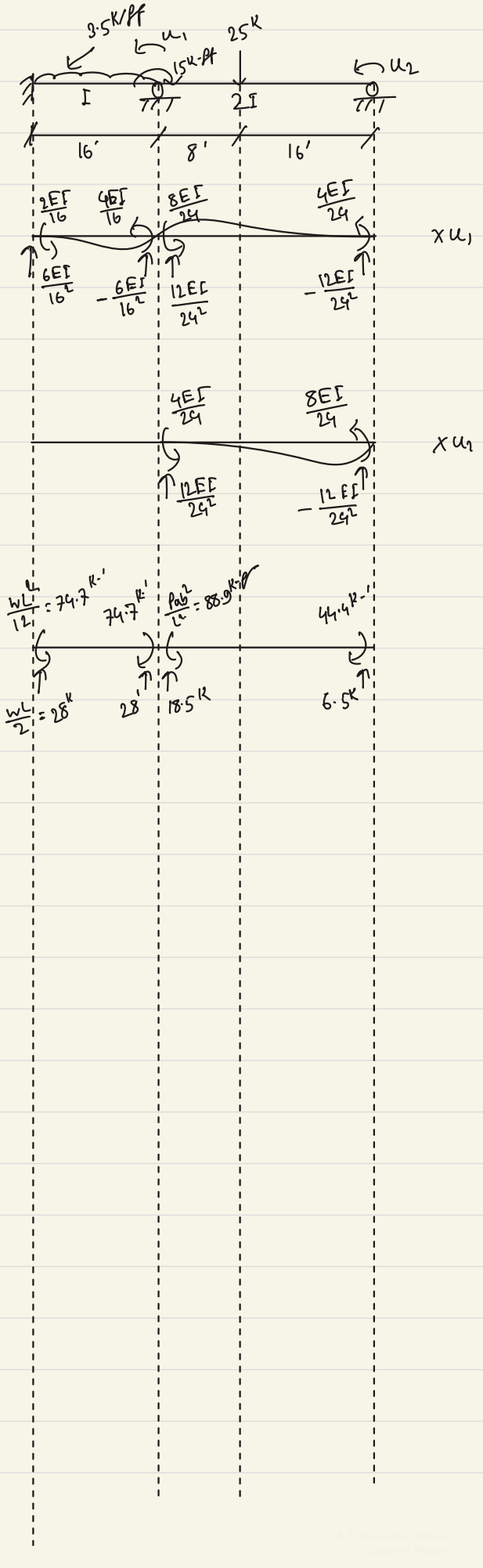
$$R_{Ay} = -103.4\left(6/16^2\right) + 28 = 25.6 \text{ K}$$

$$R_{By} = -103.4\left(-\frac{6}{16^2}+\frac{12}{24^2}\right) + 184.9\left(\frac{12}{24^2}\right) + (28+18.5)$$

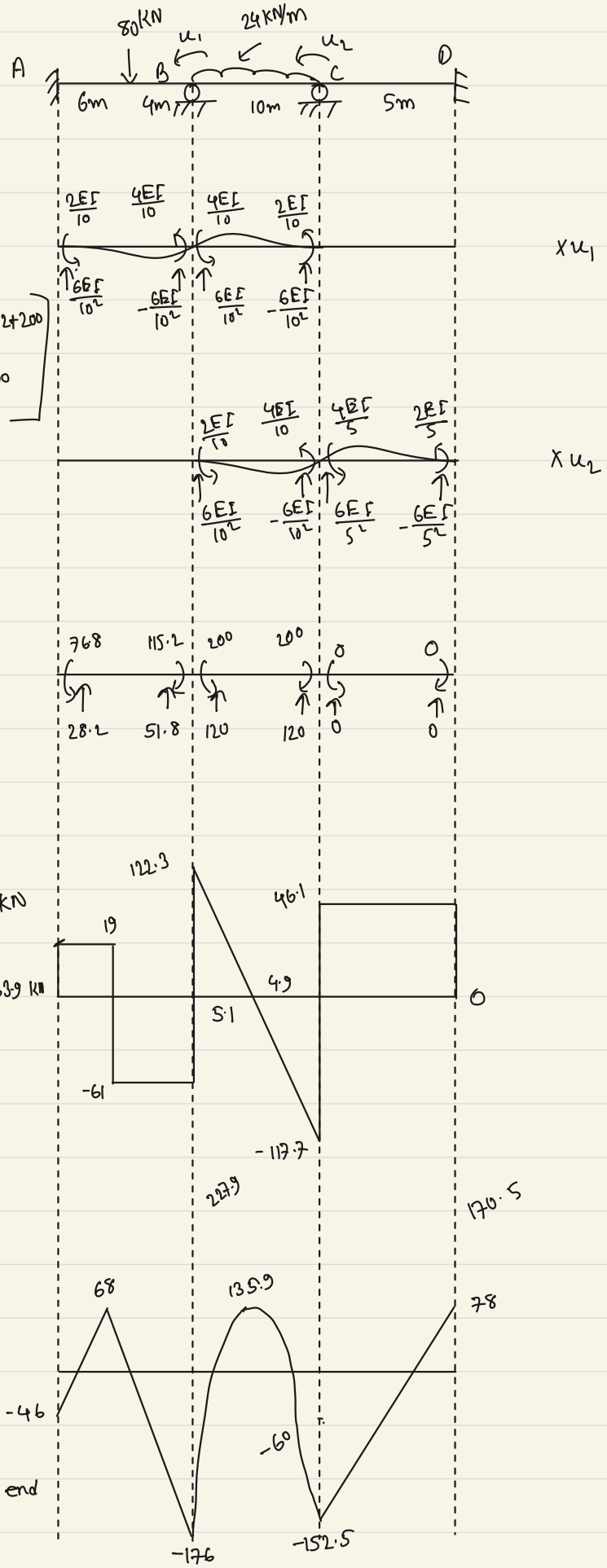
$$= 50.6 \text{ K}$$

$$R_{Cy} = -103.4\left(-\frac{12}{24^2}\right) + 184.9\left(-\frac{12}{24^2}\right) + 6.5$$

$$= 4.8 \text{ K}$$



H.W 1/



OokI = 2

$Q = Ku + Qf$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{10}+\frac{4}{10} & \frac{2}{10} \\ \frac{2}{10} & \frac{4}{10}+\frac{4}{5} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} -115.2+200 \\ -200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -154 \\ 192.3 \end{bmatrix}$$

$M_A = -154\left[\frac{2}{10}\right] + 76.8 = 46 \text{ kN.m}$

$R_{Ay} = -154\left[6/10^2\right] + 28.2 = 19.0 \text{ KN}$

$R_{By} = 192.3\left[\frac{6}{10^2}\right] + 51.8 + 120 = 183.3 \text{ KN}$

$R_{Cy} = -154\left[-\frac{6}{10^2}\right] + 192.3\left[-\frac{6}{10^2} + \frac{6}{5^2}\right] + 120 = 163.9 \text{ KN}$

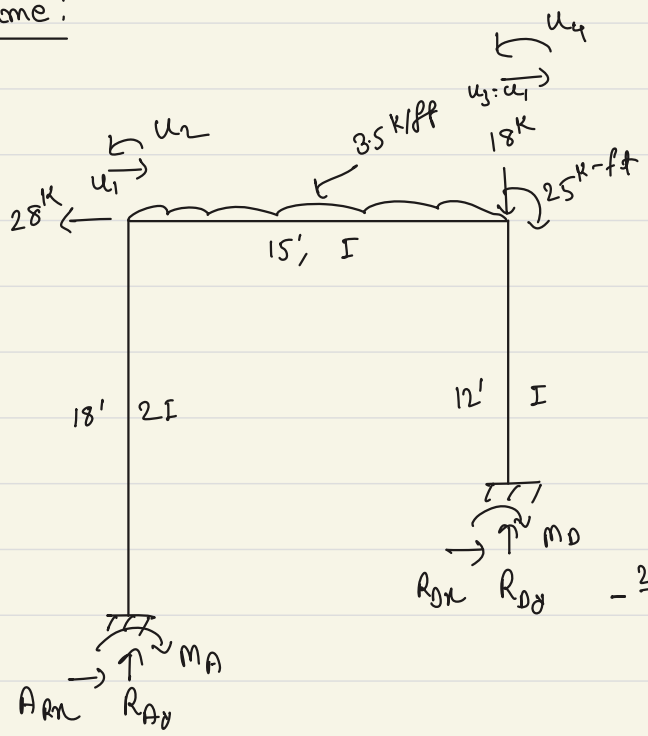
$R_{Dy} = 192.3\left[-\frac{6}{5^2}\right] + 0 = -46.1 \text{ KN}$

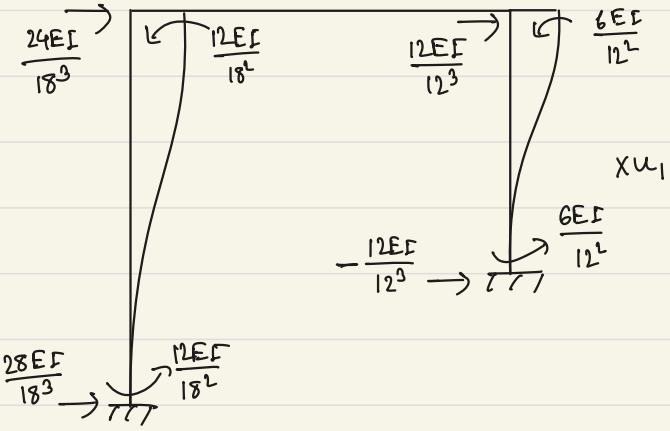
Stiffness method → ↻ → +

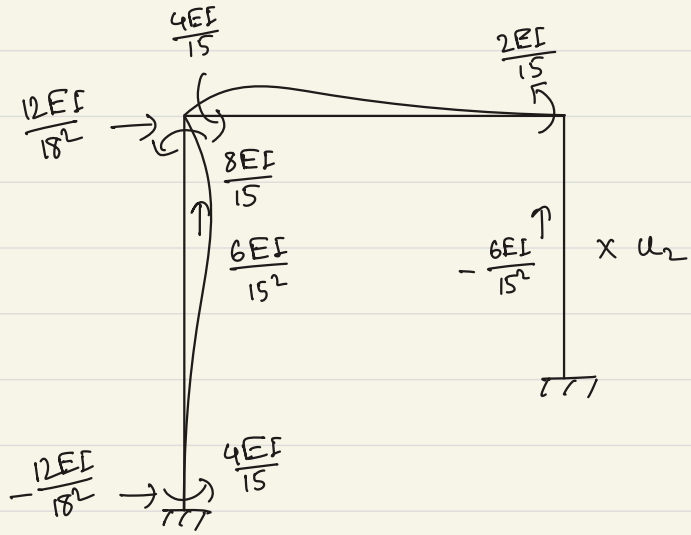
convention → ↷ → +

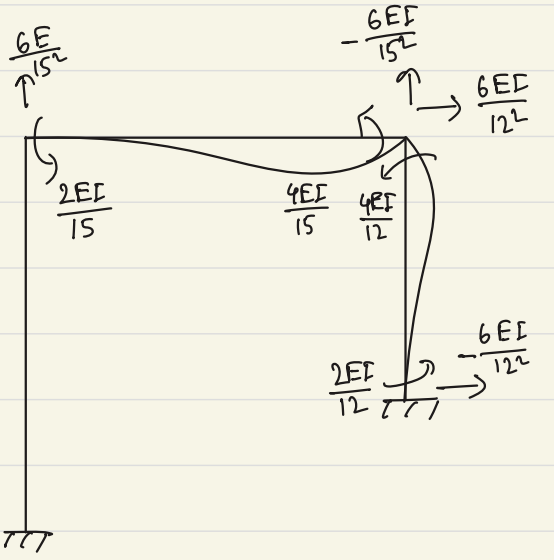
sign changes from other end

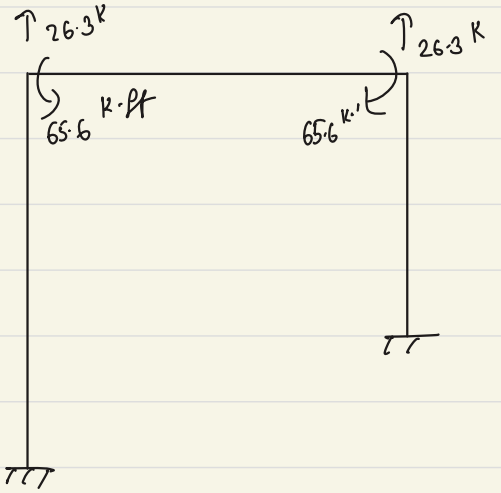
Frame:


u4
u3 = u4
u2
3.5 K/ft
18K
u1
28K
25 K-ft
15', I
18' 2I
12' I
MD
RDx RDy
MA
ARx RAy


24EI/18^3
12EI/18^2
12EI/12^3
6EI/12^2
X u1
-12EI/12^3
6EI/12^2
-28EI/18^3
12EI/18^2


4EI/15
2EI/15
12EI/18^2
8EI/15
6EI/15^2
-6EI/15^2
X u2
-12EI/18^2
4EI/15


6E/15^2
-6EI/15^2
6EI/12^2
2EI/15
4EI/15
4EI/12
2EI/12
-6EI/12^2


26.3K
26.3 K
65.6 K·ft
65.6 K·ft

$$Q = Ku + Qf$$

$$\Rightarrow \begin{bmatrix} -28 \\ 0 \\ -25 \end{bmatrix} = \begin{bmatrix} 24/18^3 + \frac{12}{12^3} & 12/18^2 & 6/12^2 \\ 12/18^2 & 8/15 + 4/15 & 2/15 \\ 6/12^2 & 2/15 & 4/15 + 4/12 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 65.6 \\ -65.6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -39261 \\ 44.7 \\ 330.37 \end{bmatrix}$$

$$M_A = -133.5 \text{ K}^{-1}$$

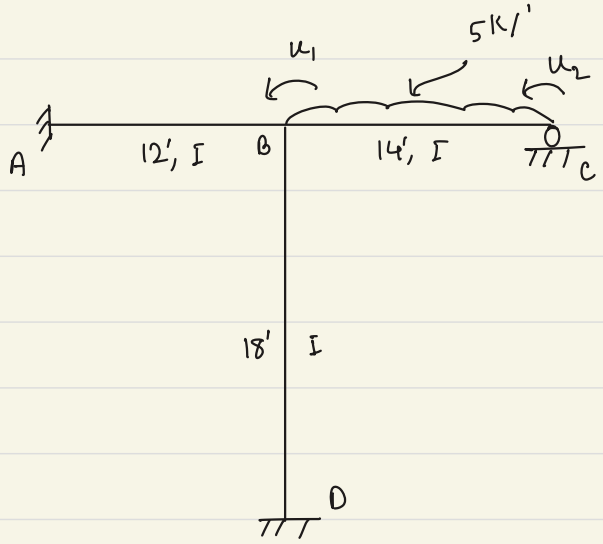
$$R_{Dy} = \dots$$

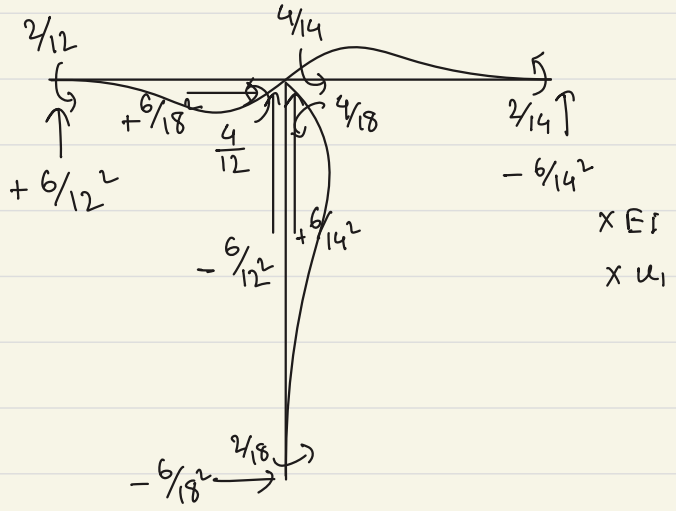
Support settlement & Rotation:

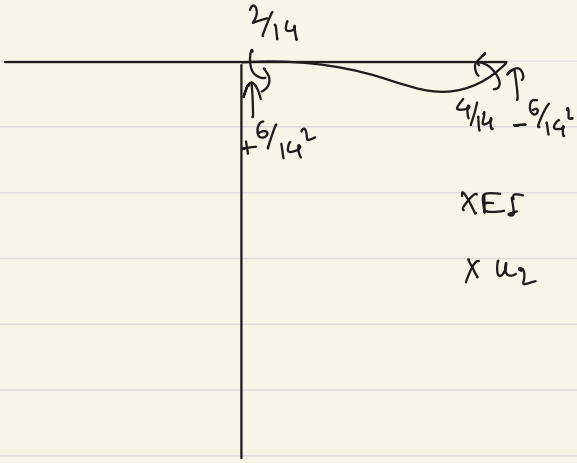
$\theta_A = 0.05^R$ (↻)

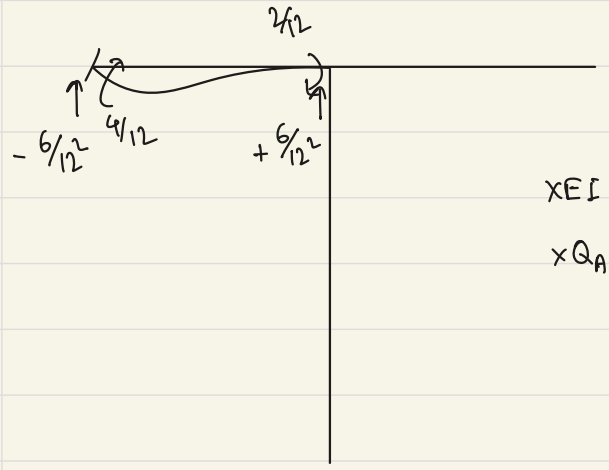
$D_B = 0.2'$ (↓)

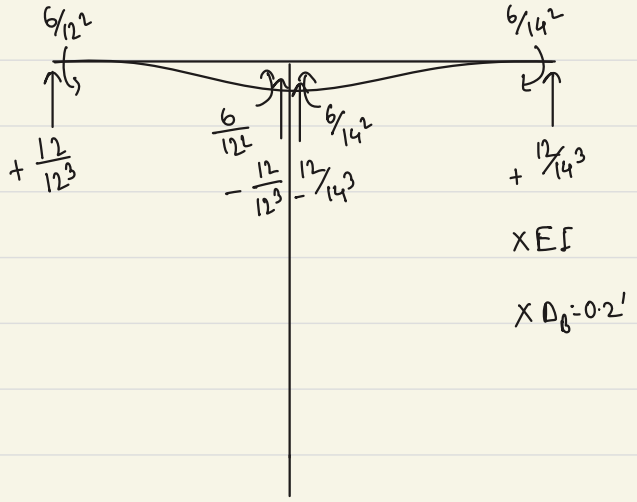
$EI = 5000\ k\text{-}ft^4$

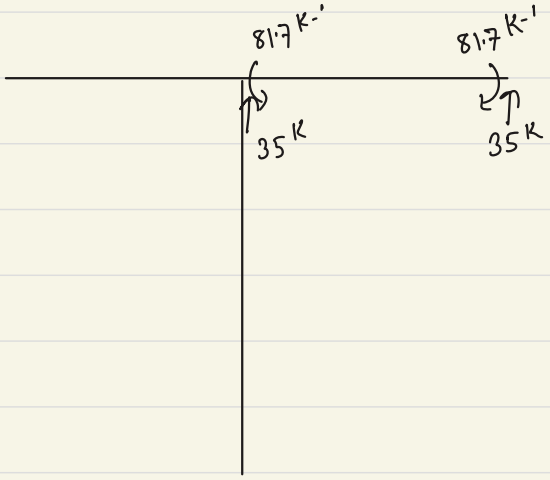












$$Q = Ku + Qf$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{12}+\frac{4}{14}+\frac{4}{18} & \frac{2}{14} \\ \frac{2}{14} & \frac{4}{14} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} -\frac{2}{12} \\ 0 \end{bmatrix} 0.05 \times 5000 + 5000 \times 0.2 \begin{bmatrix} \frac{6}{12^2}-\frac{6}{14^2} \\ -\frac{6}{14^2} \end{bmatrix} + \begin{bmatrix} 81.7 \\ -81.7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -139.3 \\ 462.7 \end{bmatrix}$$

$$M_A = -139.3 \times \left[\frac{2}{12}\right] - \frac{4}{12} \times 5000 \times 0.05 + \frac{6}{12^2} \times 5000 \times 0.2 = 64.9 \text{ K-ft} \quad (2)$$

Plane Grid:

$$\frac{M}{\theta} = \frac{4EI}{L}$$

$$\frac{T}{\theta} = \frac{G\vartheta}{L}$$

$$\theta = \frac{TL}{G\vartheta}$$

T : Torsion

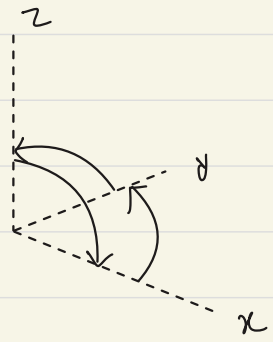
G : modulus of rigidity

ϑ : Torsional const

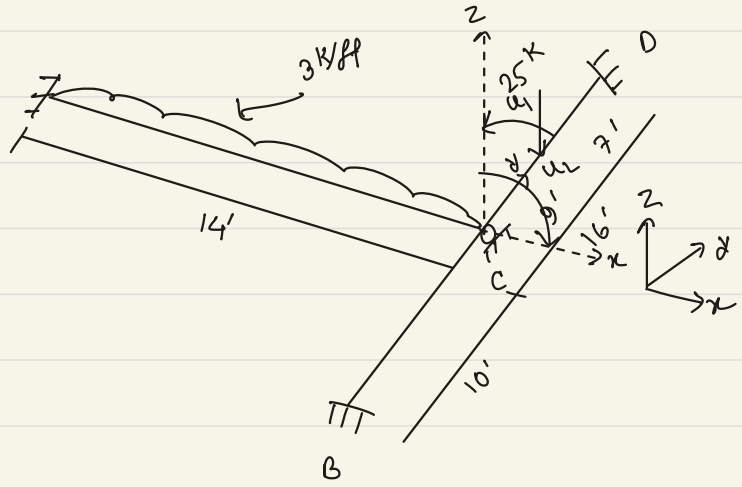
Sign conv. :

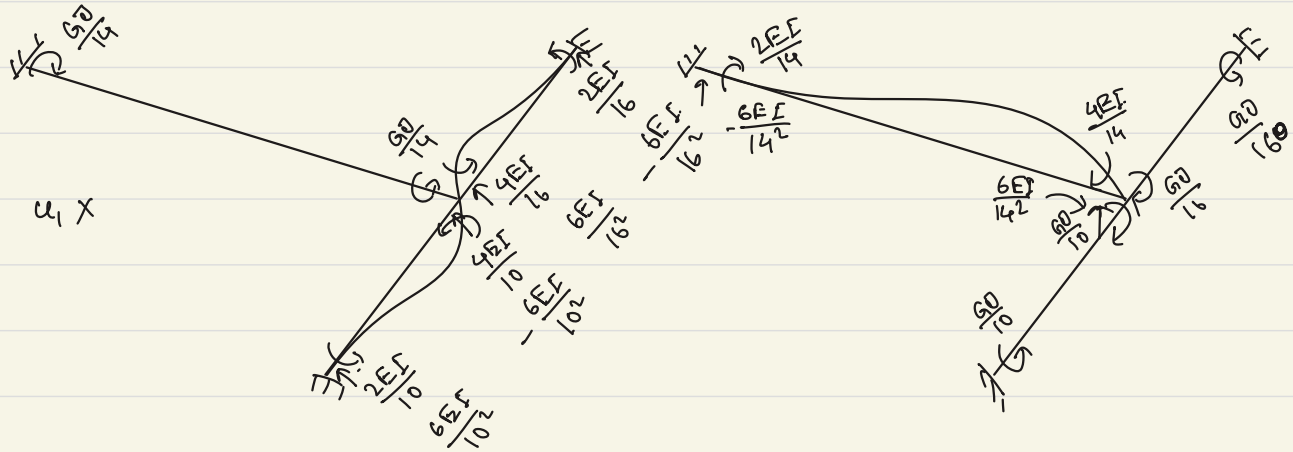
$x \to y,$
$y \to z,$
$y \to z,$
} (pos)

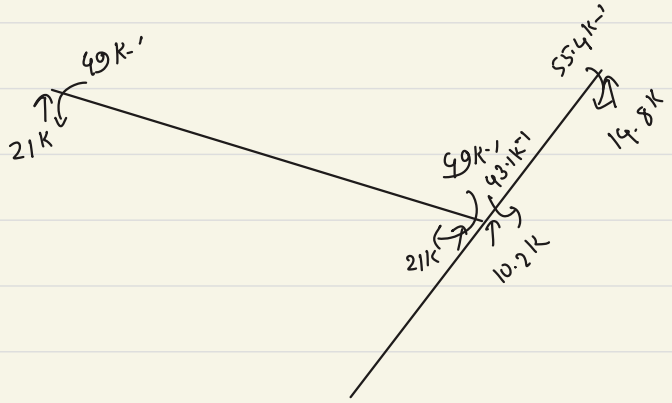
follow for mmt sign



Ex.







$$[GD = \frac{EI}{3}]$$

$$G = \frac{E}{2(1+\nu)};\ \nu < 0.5 \to 0.5$$

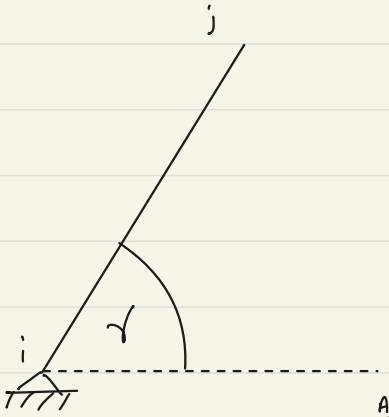
$$= \frac{E}{3}$$

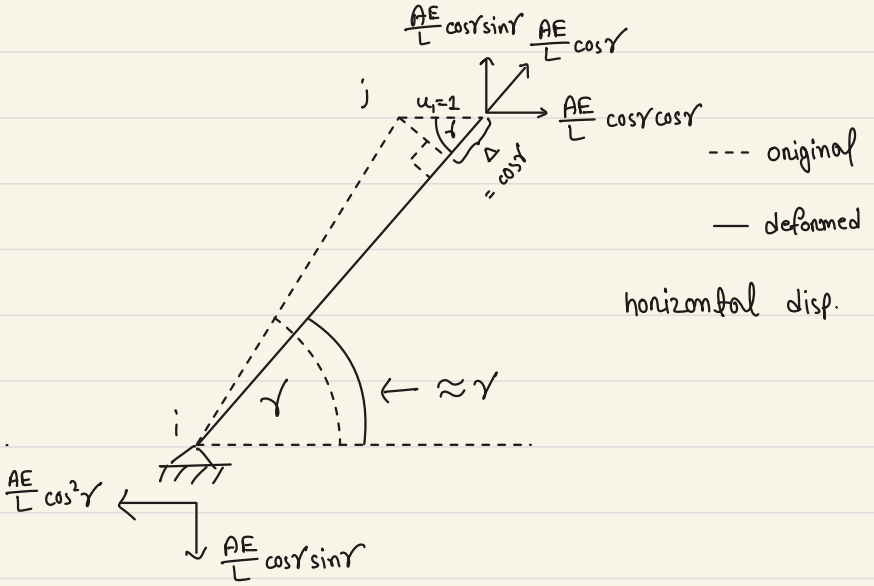
$$Q = Ku + Q_f$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{16} + \frac{4}{10} + \frac{1}{3\times14} & 0 \\ 0 & \frac{4}{14} + \frac{1}{3\times10} + \frac{1}{3\times16} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 43.1 \\ 49 \end{bmatrix} \Rightarrow \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -64.0 \\ -144.2 \end{bmatrix}$$

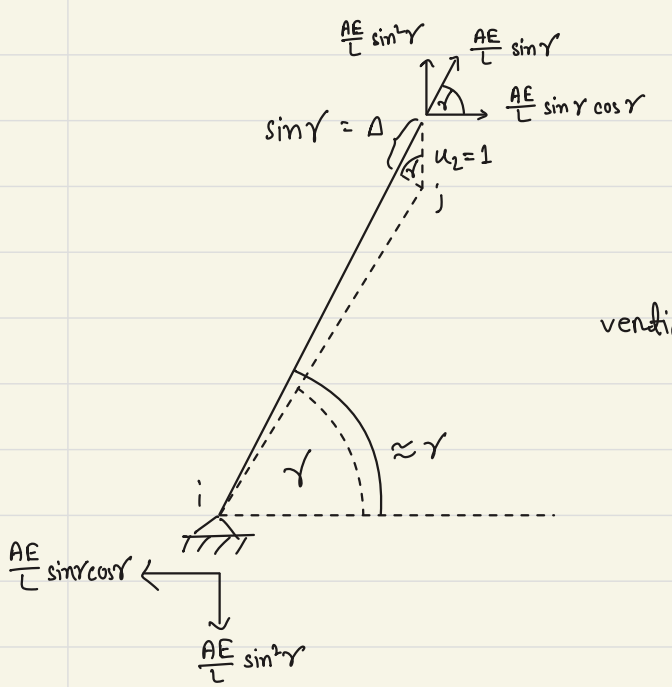
$$T_D = \frac{1}{3\times16} \times (-144.2) = -3\ \text{K-ft}$$

Truss :





$$\Delta = \frac{PL}{AE} \Rightarrow P = \frac{AE}{L}\Delta$$



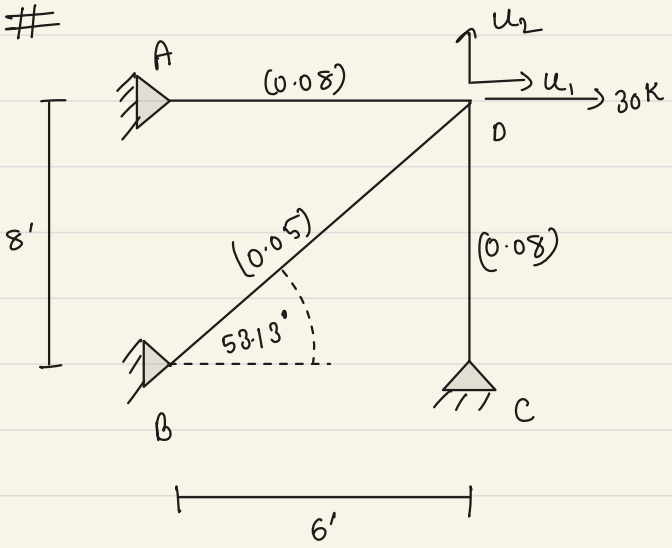
$$F = \frac{AE}{L}(\Delta x\cos\gamma + \Delta y \sin\gamma)$$

$$\Delta x = \Delta_{jx} - \Delta_{ix}$$

$$\Delta y = \Delta_{jy} - \Delta_{iy}$$

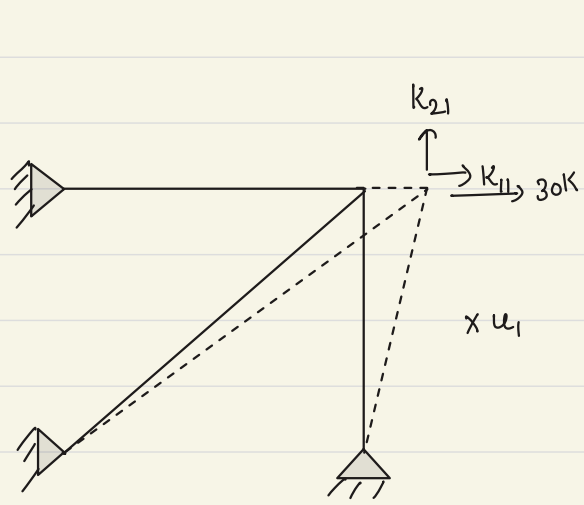
Member	L	A	γ	$\cos^2\gamma$	$\sin^2\gamma$	$\cos\gamma\sin\gamma$

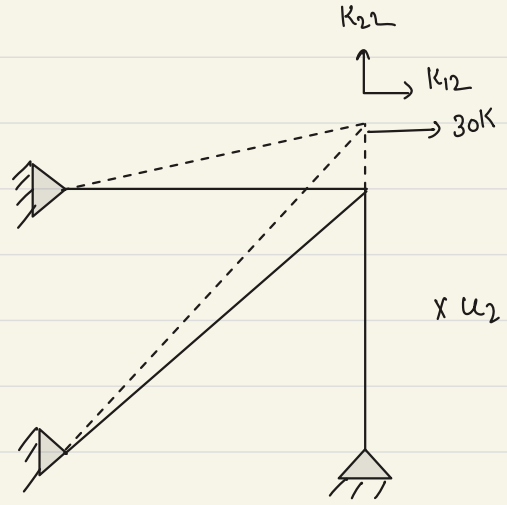
#



$E = 1000 K/ft^2$

DOKI = 2





Member	L	A	$\frac{AE}{L}$	γ	$\cos^2\gamma$	$\sin^2\gamma$	$\sin\gamma\cos\gamma$
AD	6	0.08	13.33	0	1	0	0
BD	10	0.05	5	53.13	0.36	0.64	0.48
CD	8	0.08	10	90	0	1	0

$$K_{11} = \sum_{AD,BD,CD} \frac{AE}{L}\cos^2\gamma = 13.33 \times 1 + 5 \times 0.36 + 10 \times 0 = 15.13$$

$$K_{21} = \sum_{AD,BD,CD} \frac{AE}{L}\cos\gamma\sin\gamma = 13.33 \times 0 + 5 \times 0.48 + 10 \times 0 = 2.4$$

$$K_{12} = 2.4; \quad K_{22} = \sum_{AD,BD,CD} \frac{AE}{L}\sin^2\gamma = 13.33 \times 0 + 5 \times 0.64 + 10 \times 1 = 13.2$$

$$Q = Ku$$

$$\begin{bmatrix} 30 \\ 0 \end{bmatrix} = \begin{bmatrix} 15.13 & 2.4 \\ 2.4 & 13.2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 2.04 \\ -0.37 \end{bmatrix}$$

Member: $AD = \frac{AE}{L}[\Delta x \cos\gamma + \Delta y \sin\gamma]$

$= 13.33[2.04 \times 1 - 0.37 \times 0] = 27.19^K$ (T)

$BD = 5[2.04 \times \sqrt{0.36} - 0.37 \times \sqrt{0.64}] = 4.64^K$ (T)

$CD = 10[2.04 \times 0 - 0.37 \times 1] = 3.7^K$ (C)

Reaction:

$$R_{Ax} = \frac{AE}{L}\cos^2\gamma \times u_1 + \frac{AE}{L}\sin\gamma\cos\gamma \times u_2$$

$= 13.33[1 \times 2.04] = 27.19^K$ ($\leftarrow$)

$$R_{Bx} = \frac{AE}{L}\cos^2\gamma \times u_1 + \frac{AE}{L}\sin\gamma\cos\gamma \times u_2$$

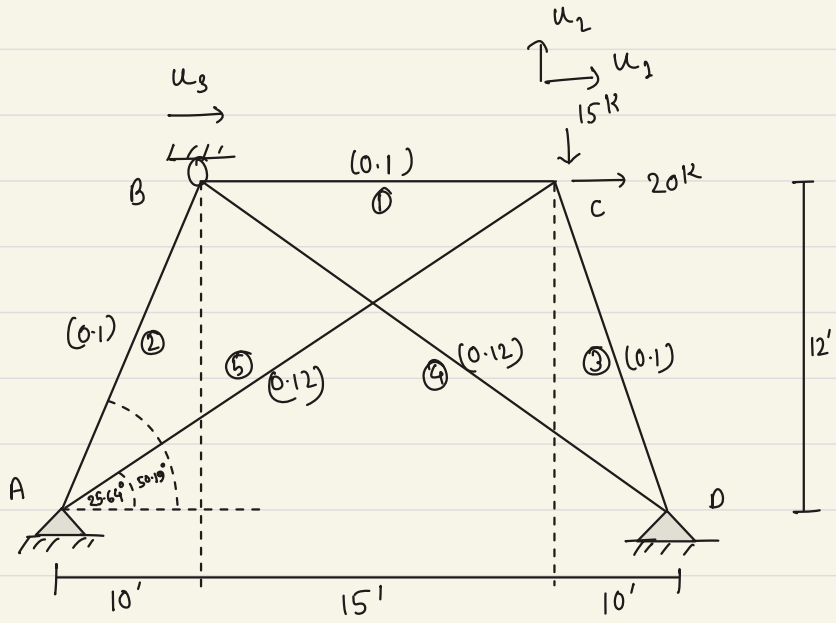
$= 5[0.36 \times 2.04 + 0.48 \times (-0.37)] = 2.78^K$ ($\leftarrow$)

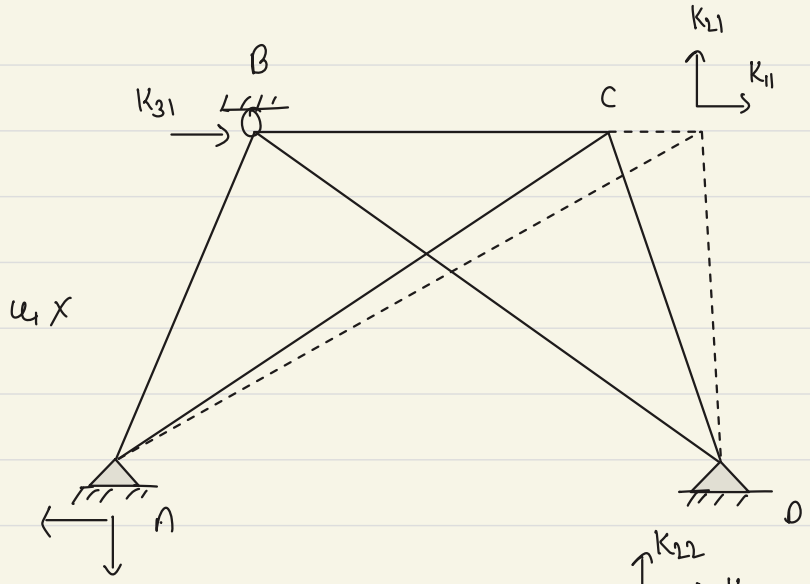
$$R_{By} = \frac{AE}{L}\cos\gamma\sin\gamma \times u_1 + \frac{AE}{L}\sin^2\gamma \times u_2]$$

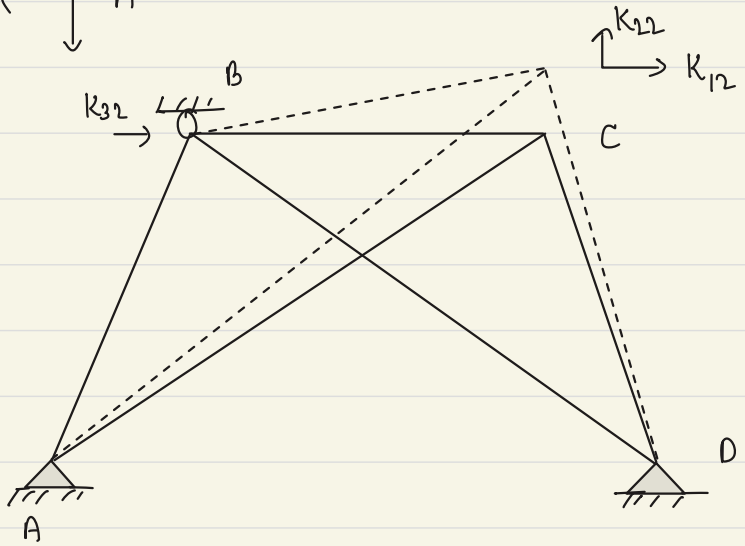
$= 5[0.48 \times 2.04 + 0.64(-0.37)] = 3.7^K$ ($\downarrow$)

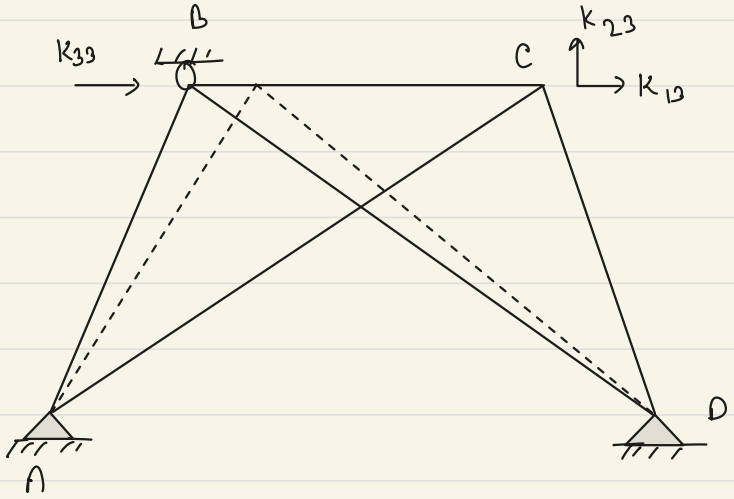
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$E = 5{,}000\ k\text{-}ft^2$









Member	L_H	γ	A	L	$\frac{AE}{L}$	$\cos^2\gamma$	$\sin^2\gamma$	$\sin\gamma\cos\gamma$
BC	15	0	0.1	15	33.33	1	0	0
AB	10	50.19	0.1	15.62	32.01	0.41	0.59	0.49
CD	10	129.81	0.1	15.62	32.01	0.41	0.59	−0.49
OB	25	154.36	0.12	27.73	21.64	0.81	0.19	-0.39
AC	25	25.64	0.12	27.73	21.64	0.81	0.19	0.39

$$K_{11} = \sum_{AC,BC,DC} \frac{AE}{L}\cos^2\gamma = [21.64\times0.81 + 33.33\times1 + 32.01\times0.41] = 63.98$$

$$K_{21} = K_{12} = \sum_{AC,BC,DC} \frac{AE}{L}\sin\gamma\cos\gamma = [21.64\times0.39 + 33.33\times0 + 32.01\times(-0.49)] = -7.26$$

$$K_{31} = K_{13} = \sum_{-BC} \frac{AE}{L}\cos^2\gamma = [-33.33\times1] = -33.33$$

$$K_{22} = \sum_{AC,BC,DC} \frac{AE}{L}\sin^2\gamma = [21.64\times0.19 + 33.33\times0 + 32.01\times0.59] = 23.0$$

$$K_{33} = \sum_{AB,DB,BC} \frac{AE}{L}\cos^2\gamma = [32.01\times0.41 + 21.04\times0.81 + 33.33\times1] = 68.5$$

$$K_{32} = K_{23} = 0$$

$$Q = Ku$$

$$\begin{bmatrix} 20 \\ -15 \\ 0 \end{bmatrix} = \begin{bmatrix} 6898 & -7.26 & -33.33 \\ -7.26 & 23.0 & 0 \\ -33.33 & 0 & 63.5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$u_1 = 0.35$

$u_2 = -0.54$

$u_3 = 0.18$

$R_{Ay} = 3.56^{K} \ (\downarrow)$

$$R_{Ay} = u_1 \left[\frac{AE}{L} \cos\gamma \sin\gamma\right]_{AC} + u_2 \left[\frac{AE}{L} \sin^2\gamma\right]_{AC}$$

$$u_3 \left[\frac{AE}{L} \cos\gamma \sin\gamma\right]_{AB}$$

$$R_{Bx} = u_1 \left[\frac{AE}{L} \cos^2\gamma\right]_{BC} + u_2 \left[\frac{AE}{L} \cos\gamma \sin\gamma\right]_{BC}$$

$$+ u_3 \left[\frac{AE}{L} \cos^2\gamma\right]_{AB,\,OB}$$

$= 17.18^{K} \ (\leftarrow)$

$$R_{By} = u_1 \left[\frac{AE}{L} \sin\gamma\cos\gamma\right]_{BC} + u_2 \left[\frac{AE}{L} \sin^2\gamma\right]_{BC} + u_3 \left[\frac{AE}{L} \sin\gamma \cos\gamma\right]_{AB,OB}$$

$= 1.3^{K} \ (\uparrow)$

$$F_{CD} = \frac{AE}{L}\left[\Delta x \cos\gamma + \Delta y \sin\gamma\right]$$

$$= 32.01\left[0.35 \times \sqrt{0.41} - 0.54\sqrt{0.59}\right] = 6.1^{K}\ (C) \quad \times$$

$$F_{AB} = \frac{AE}{L}\left[\Delta x \cos\gamma + \Delta y \sin\gamma\right]$$

$$= 32.01\left[0.18 \times \sqrt{0.41}\right] = 3.69^{K}\ (T) \quad \checkmark$$

Hand calculation / Two span beam:

$$Q = Ku + Q_f$$

$$\begin{bmatrix} 0 \\ -28 \end{bmatrix} = \begin{bmatrix} 4/16 & 2/16 \\ 2/16 & 4/16 + 8/20 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 91.4 \\ -54.8 + 100 \end{bmatrix}$$

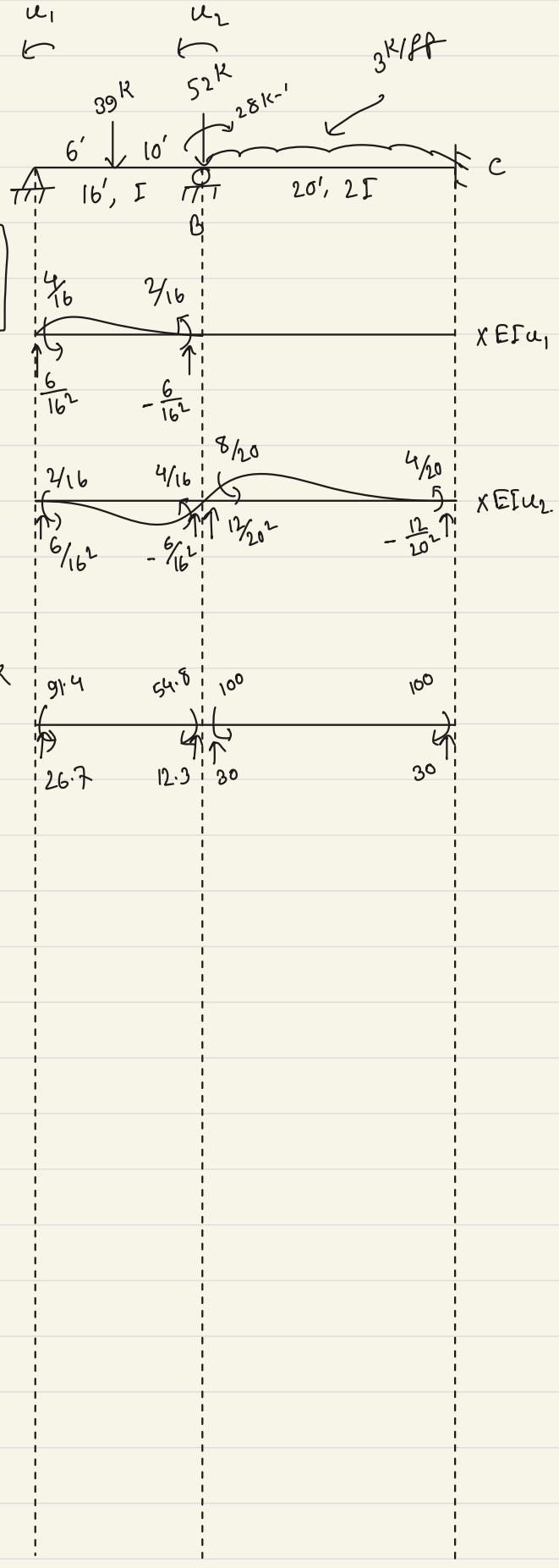
$$\Rightarrow u_1 = -342.2$$

$$u_2 = -46.8$$

$$M_C = -46.8\left(\frac{4}{20}\right) - 100 = -109.4 \text{ k-ft}$$

$$R_{Ay} = -342.2\left(\frac{6}{16^2}\right) - 46.8\left(\frac{6}{16^2}\right) + 26.7 = 17.6 \text{ K}$$

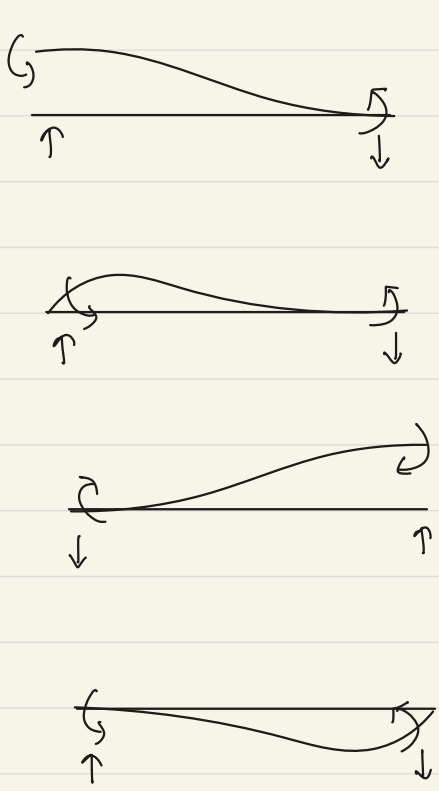
. . .



Formalization:

Stiffness of a beam element:

$$K = EI \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix}$$



Memo:

1. One of: $\frac{12}{L^3}, \frac{6}{L^2}, \frac{4}{L}, \frac{2}{L}$
2. Sign: Index 3

$$\begin{array}{c} \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} & & 3 & \end{array} \\ \begin{bmatrix} + & + & - & + \\ + & + & - & + \\ - & - & \boxed{+}_{3,3} & - \\ + & + & - & \end{bmatrix} \end{array}$$

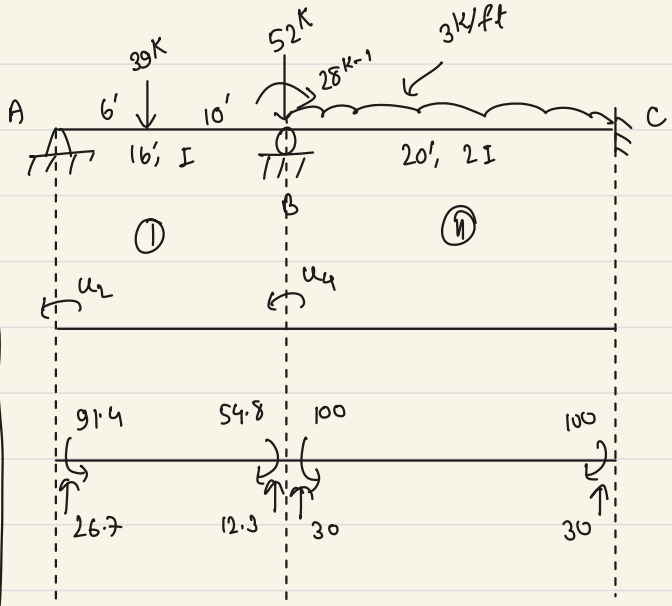
3. Diagonal → $\frac{12}{L^3}, \frac{4}{L}, \frac{12}{L^3}, \frac{4}{L}$
4. 2, 4 & 4, 2 → $\frac{2}{L}$
5. 1, 3 & 3, 1 → $\frac{12}{L^3}$
6. Rest $6/L^2$

Element stiffness matrix:

$$\begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix}$$

#/

$$P_j = KU + P_m$$



$$K = \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix}$$

$$P_j = KU + P_m$$

$$\begin{bmatrix} R_{AY} \\ M_A \\ R_{BY} - 52 \\ M_B - 28 \\ R_{CY} \\ M_C \end{bmatrix} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \frac{12}{16^3} & \frac{6}{16^2} & -\frac{12}{16^3} & \frac{6}{16^2} & 0 & 0 \\ \frac{6}{16^2} & \frac{4}{16} & -\frac{6}{16^2} & \frac{2}{16} & & \\ -\frac{12}{16^3} & -\frac{6}{16^2} & \frac{12}{16^3}+\frac{12\times2}{20^3} & -\frac{6}{16^2}+\frac{6\times2}{20^2} & -\frac{12\times2}{20^3} & \frac{6\times2}{20^2} \\ \frac{6}{16^2} & \frac{2}{16} & -\frac{6}{16^2}+\frac{6\times2}{20^2} & \frac{4}{16}+\frac{4\times2}{20} & -\frac{6\times2}{20^2} & \frac{2\times2}{20} \\ 0 & 0 & -\frac{12\times2}{20^3} & -\frac{6\times2}{20^2} & \frac{12\times2}{20^3} & -\frac{6\times2}{20^2} \\ 0 & 0 & \frac{6\times2}{20^2} & \frac{2\times2}{20} & -\frac{6\times2}{20^2} & \frac{4\times2}{20} \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ u_2 \\ 0 \\ u_4 \\ 0 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 26.7 \\ 91.4 \\ 12.3+30 \\ -54.8+100 \\ 30 \\ -100 \end{bmatrix}$$

Extract unconstraint DOFs: 2 & 4 → u_1, u_2 → $K_{22}, K_{24}, K_{42}, K_{44}$

$$\begin{bmatrix} 0 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{4}{16} & \frac{2}{16} \\ \frac{2}{16} & \frac{4}{16}+\frac{8}{20} \end{bmatrix} \begin{bmatrix} u_2 \\ u_4 \end{bmatrix} + \begin{bmatrix} 91.4 \\ 100-54.8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_2 \\ u_4 \end{bmatrix} = \begin{bmatrix} -342.2 \\ -46.8 \end{bmatrix}$$

Extract restraint coordinates: 1, 3, 5, 6

$$\begin{bmatrix} R_{Ay} \\ R_{By} \\ R_{Cy} \\ m_C \end{bmatrix} + \begin{bmatrix} 0 \\ -52 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{6}{16^2} & \frac{6}{16^2} \\ -\frac{6}{16^2} & -\frac{6}{16^2}+\frac{12}{20^2} \\ 0 & -\frac{12}{20^2} \\ 6 & \frac{4}{20} \end{bmatrix} \begin{bmatrix} -342.2 \\ -46.8 \end{bmatrix} + \begin{bmatrix} 26.7 \\ 12.3+30 \\ 30 \\ -100 \end{bmatrix}$$

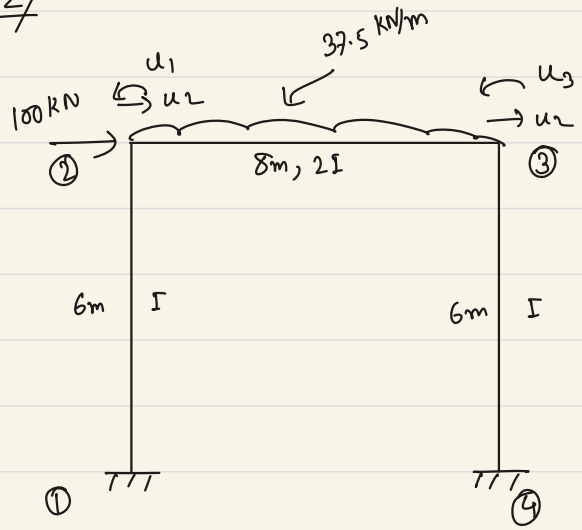
$R_{Ay} \rightarrow 17.6$

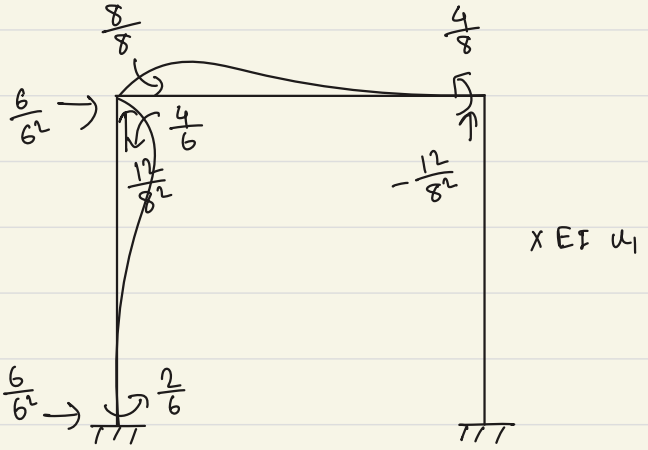
$R_{By} \rightarrow 102.0$

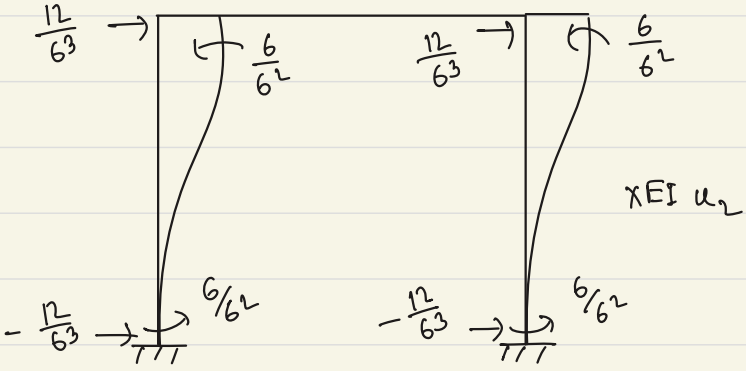
$R_{Cy} \rightarrow 31.4$

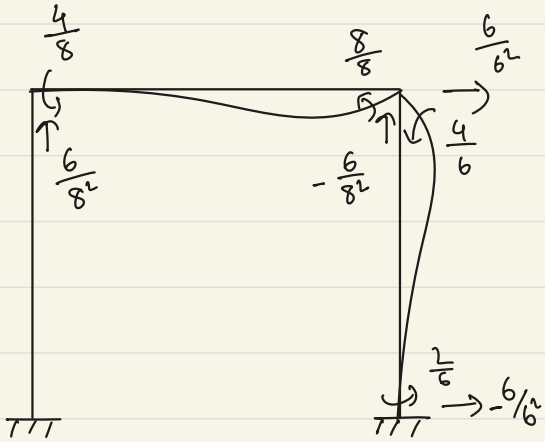
$m_C \rightarrow 109.3$

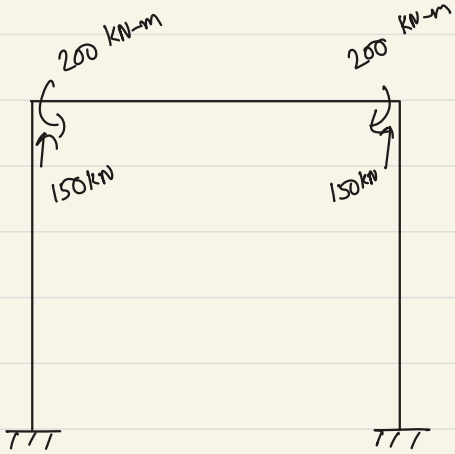
CT 2/











$$Q = ku + Qf$$

$$\Rightarrow \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{8}{8}+\frac{4}{6} & \frac{6}{6^2} & \frac{4}{8} \\ \frac{6}{6^2} & \frac{12}{6^3}+\frac{12}{6^3} & \frac{6}{6^2} \\ \frac{4}{8} & \frac{6}{6^2} & \frac{8}{8}+\frac{4}{6} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 200 \\ 0 \\ -200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -261.4 \\ 1170 \\ 81.4 \end{bmatrix}$$

$$R_{AY} = \frac{12}{8^2} \times (-261.4) + \frac{6}{8^2} \times 81.4 + 150 = 108.6^{K}$$

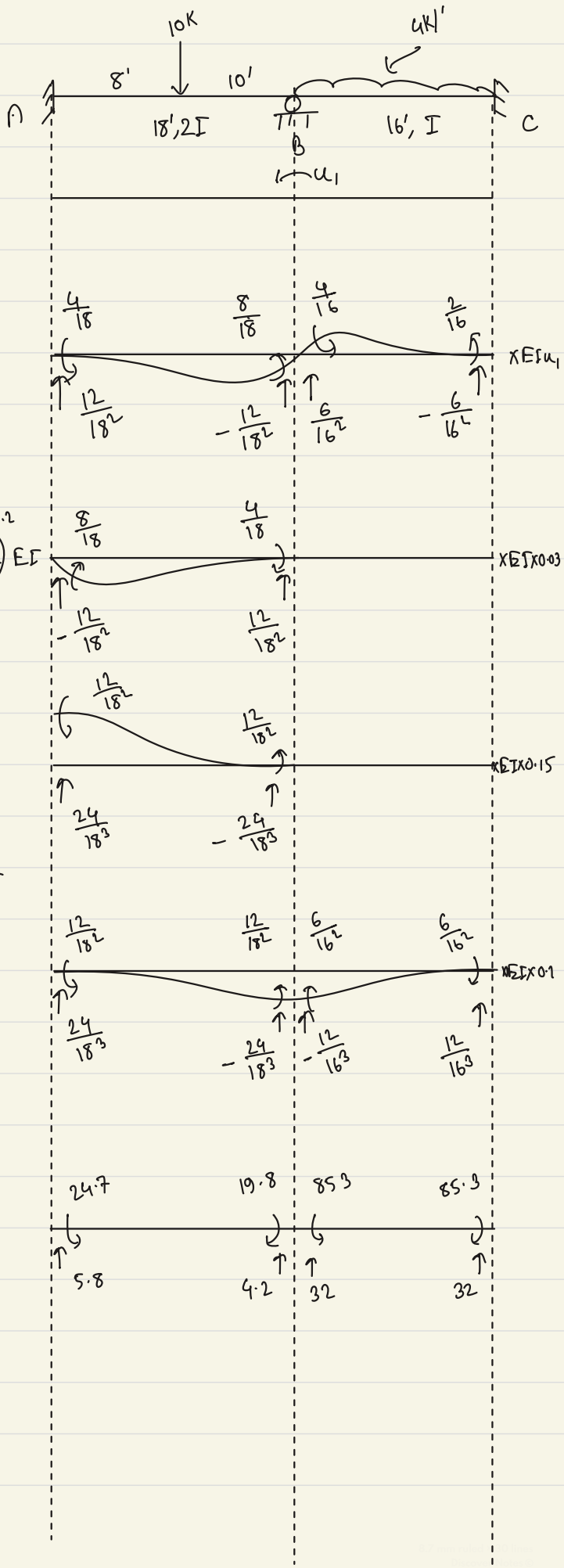
SB/LI/Ex.2

$EI = 4500\ k\text{-}ft^2$

$\theta_A = 0.03^R$ ↻

$\Delta_A = 0.15'$ ↑

$\Delta_B = 0.2'$ ↓



$Q = Ku + Q_f$

At B:

$$0 = \left(\frac{8}{18} + \frac{4}{16}\right)u_1 + \left(\frac{4}{18} + \frac{12}{18^2} + \frac{12}{18^2} - \frac{6}{16^2}\right)EI$$

(0.03, 0.15, 0.2)

$$-19.8 + 85\ 3$$

$$\Rightarrow u_1 = \frac{-72.73}{8/18 + 4/16} = -104.7$$

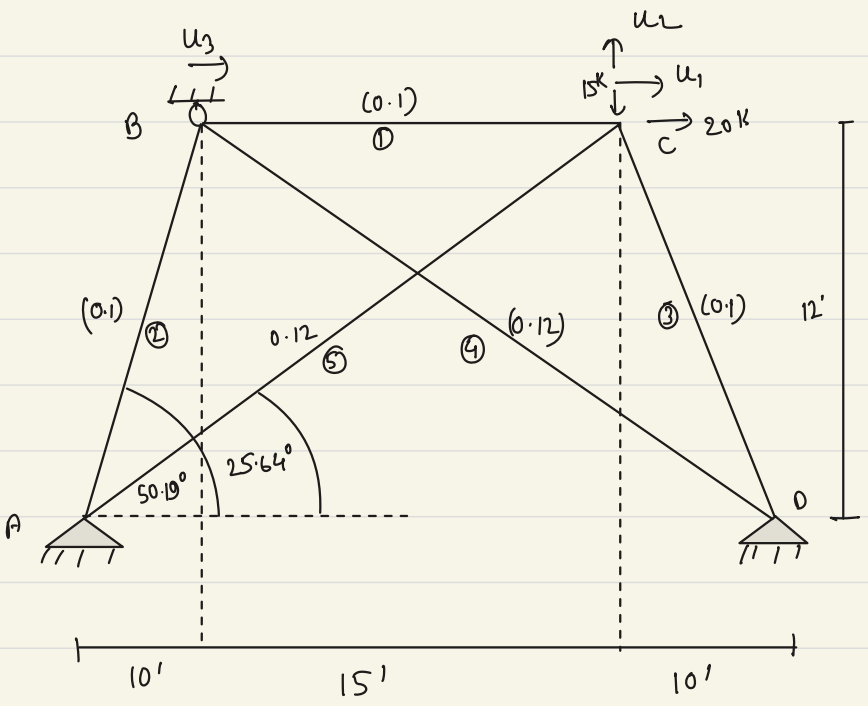
$$\therefore M_A = -104.7\left[\frac{4}{18}\right] + EI\left[-\frac{8}{18}\times 0.03 + \frac{12}{18^2}\times 0.15 + \frac{12}{18^2}\times 0.2\right] + 24.7$$

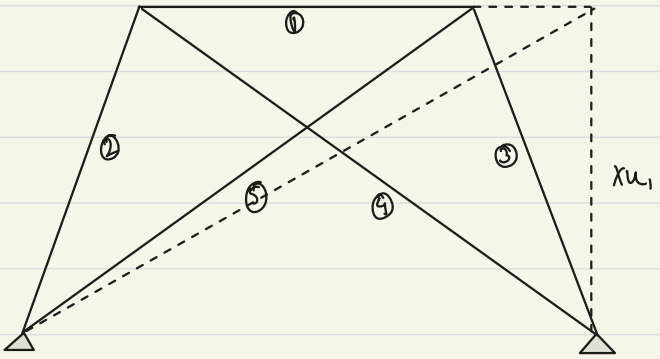
$$= -0.23^{K-'}$$

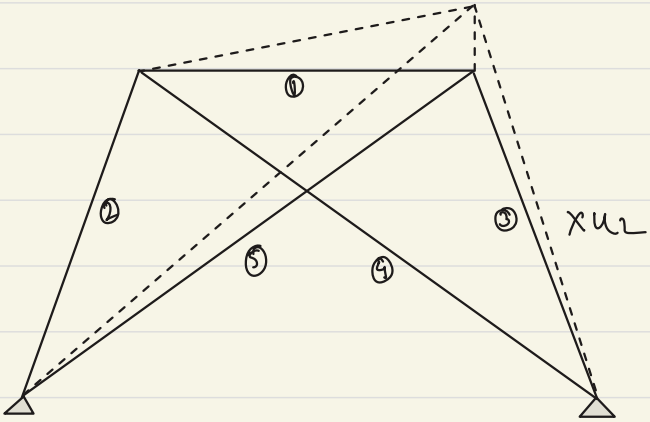
Recap / Final

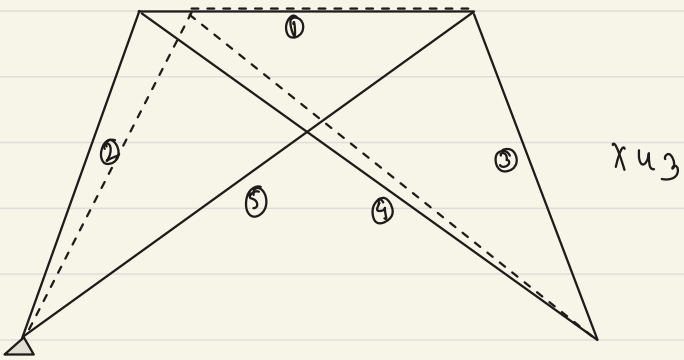
14/10

$EI = 5000\ k\text{-}ft^2$









Member	LH	γ	A	L	$\frac{AE}{L}$	$\cos^2\gamma$	$\sin^2\gamma$	$\sin\gamma\cos\gamma$
1	15	0	0.1	15	33.33	1	0	0
2	10	50.19	0.1	15.62	32.01	0.41	0.59	0.49
3	10	129.81	0.1	15.62	32.01	0.41	0.59	-0.49
4	25	154.36	0.12	27.73	21.64	0.81	0.19	-0.39
5	25	25.64	0.12	27.73	21.64	0.81	0.19	0.39

$$K_{11} = \sum_{1,3,5} \frac{AE}{L}\cos^2\gamma = [33.33\times 1 + 32.01\times 0.41 + 21.64\times 0.81] = 63.98$$

$$K_{12} = K_{21} = \sum_{1,3,5} \frac{AE}{L}\cos\gamma\sin\gamma = [33.33\times 0 + 32.01\times -0.49 + 21.64\times 0.39] = -7.25$$

$$K_{13} = K_{31} = \sum_{-1} \frac{AE}{L}\cos^2\gamma = [-33.33\times 1) = -33.33$$

$$K_{22} = \sum_{1,3,5} \frac{AE}{L}\sin^2\gamma = [33.33\times 0 + 32.01\times 0.59 + 21.64\times 0.19] = 23.0$$

$$K_{23} = K_{32} = \sum_{-1} \frac{AE}{L}\cos\gamma\sin\gamma = 0$$

$$K_{33} = \sum_{1,2,4} \frac{AE}{L}\cos^2\gamma = [33.33\times 1 + 32.01\times 0.41 + 21.64\times 0.81] = 63.98$$

$$Q = Ku$$

$$\begin{bmatrix} 20 \\ -15 \\ 0 \end{bmatrix} = \begin{bmatrix} 63.98 & -7.25 & -33.33 \\ -7.25 & 23.0 & 0 \\ -33.33 & 0 & 63.98 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\Rightarrow \quad u_1 = 0.344$$
$$u_2 = -0.543$$
$$u_3 = 0.179$$

$$R_{Ay} = \left[\frac{AE}{L}\sin\gamma\cos\gamma\right]u_1 + \left[\frac{AE}{L}\sin^2\gamma\right]u_2 + \left[\frac{AE}{L}\sin\gamma\cos\gamma\right]u_3$$

$$= 3.48^{K} \ (\downarrow)$$

$$F_{CD} = \frac{AE}{L}\left[D_x\cos\gamma + D_y\sin\gamma\right]$$

$$= 20.4^{K} \ (C) \qquad [\cos\gamma \rightarrow (-)]$$