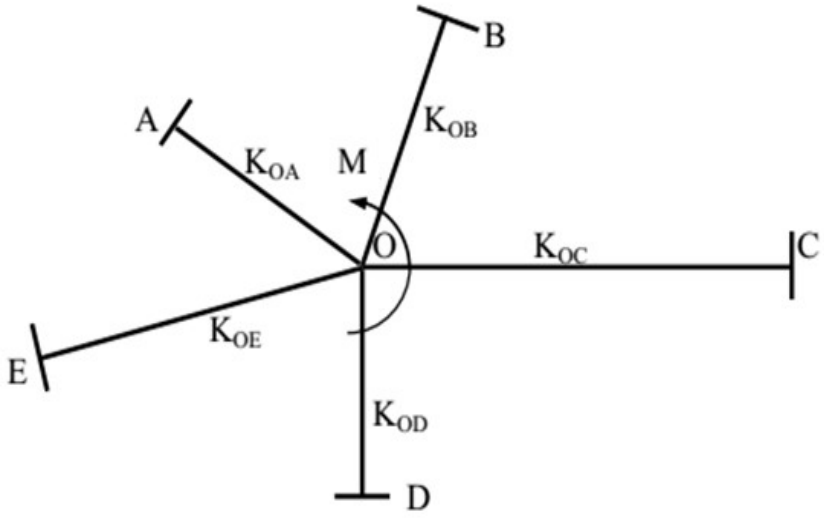
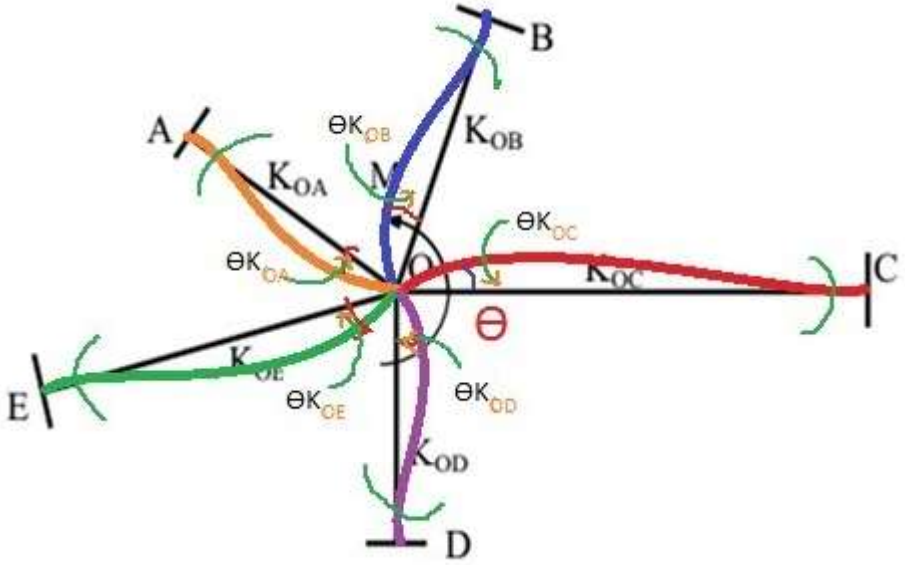
MOMENT DISTRIBUTION METHOD

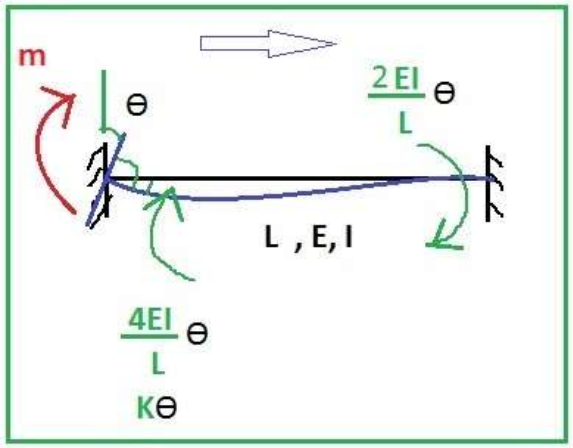
(Indeterminate Structure Analysis)

MOMENT DISTRIBUTION METHOD

Rotation of a Joint and Moment Distribution Factors:







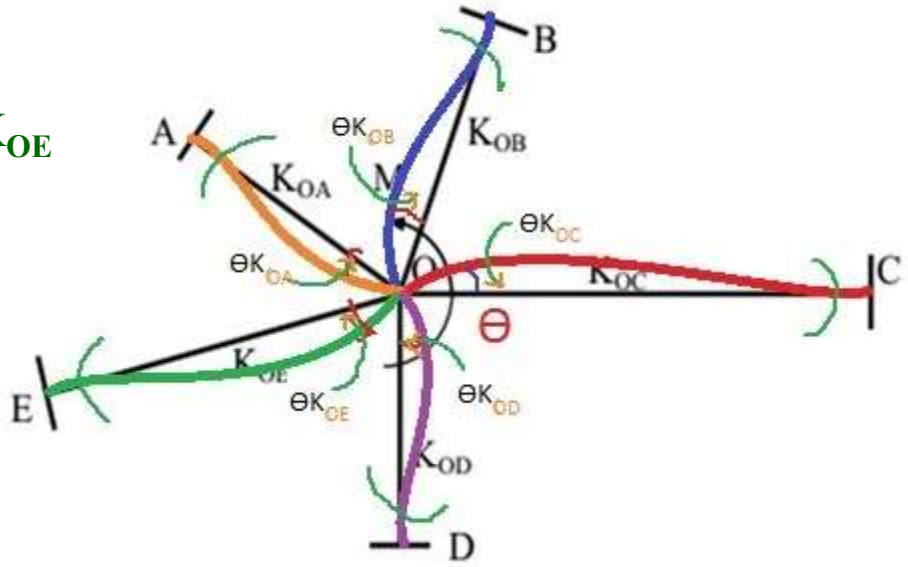
- **Joint Equilibrium:**

  $M = m_{OA}+m_{OB}+m_{OC}+m_{OD}+m_{OE}$
- **Assumption: Rigid Joints**
- **No joint translation**
- **Joint rotation Θ**

MOMENT DISTRIBUTION METHOD

Rotation of a Joint and Moment Distribution Factors:

- $M = m_{OA} + m_{OB} + m_{OC} + m_{OD} + m_{OE}$
  $= \Theta K_{OA} + \Theta K_{OB} + \Theta K_{OC} + \Theta K_{OD} + \Theta K_{OE}$
  $= \Theta (K_{OA} + K_{OB} + K_{OC} + K_{OD} + K_{OE})$
  $= \Theta \Sigma K$



$M = \Theta \Sigma K \qquad m_{OA} = \Theta K_{OA}$

$$\frac{m_{OA}}{M} = \frac{\Theta K_{OA}}{\Theta \Sigma K} \qquad \boxed{m_{OA} = \frac{K_{OA}}{\Sigma K} M}$$

$M_{OA} = [K_{OA} / \Sigma K]\ M$

$M_{OB} = [K_{OB} / \Sigma K]\ M$

$M_{OC} = [K_{OC} / \Sigma K]\ M$

$M_{OD} = [K_{OD} / \Sigma K]\ M$

$M_{OE} = [K_{OE} / \Sigma K]\ M$

- **Distribution factor = $(K_i / \Sigma K)$**
- **Distributed moment, $m_i = (K_i / \Sigma K)\ M$**

Moment applied at a joint is distributed among members in proportion to their stiffness.

MOMENT DISTRIBUTION METHOD

Moment Distribution Method: Cosider cloclwise positive

Moment distribution:

1. Wich moment should be distributed?

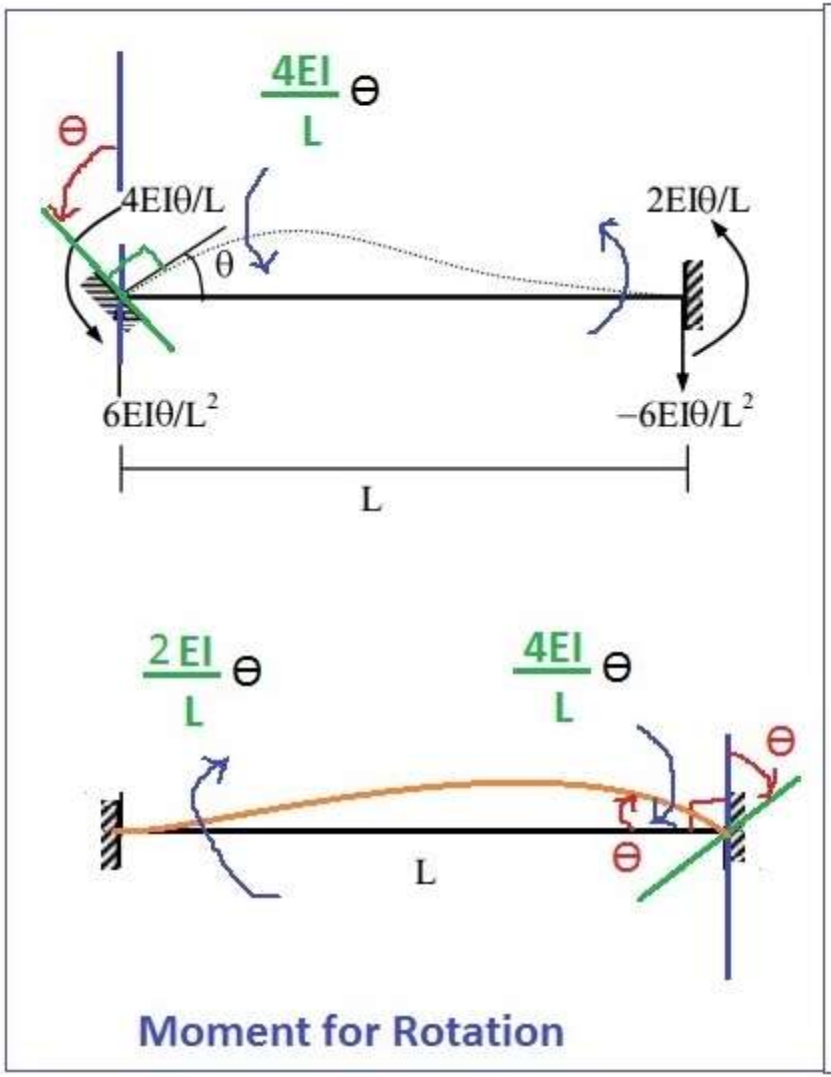
> moment due to applied load (Fixed End Moment)

> Moment due to rotation or translation

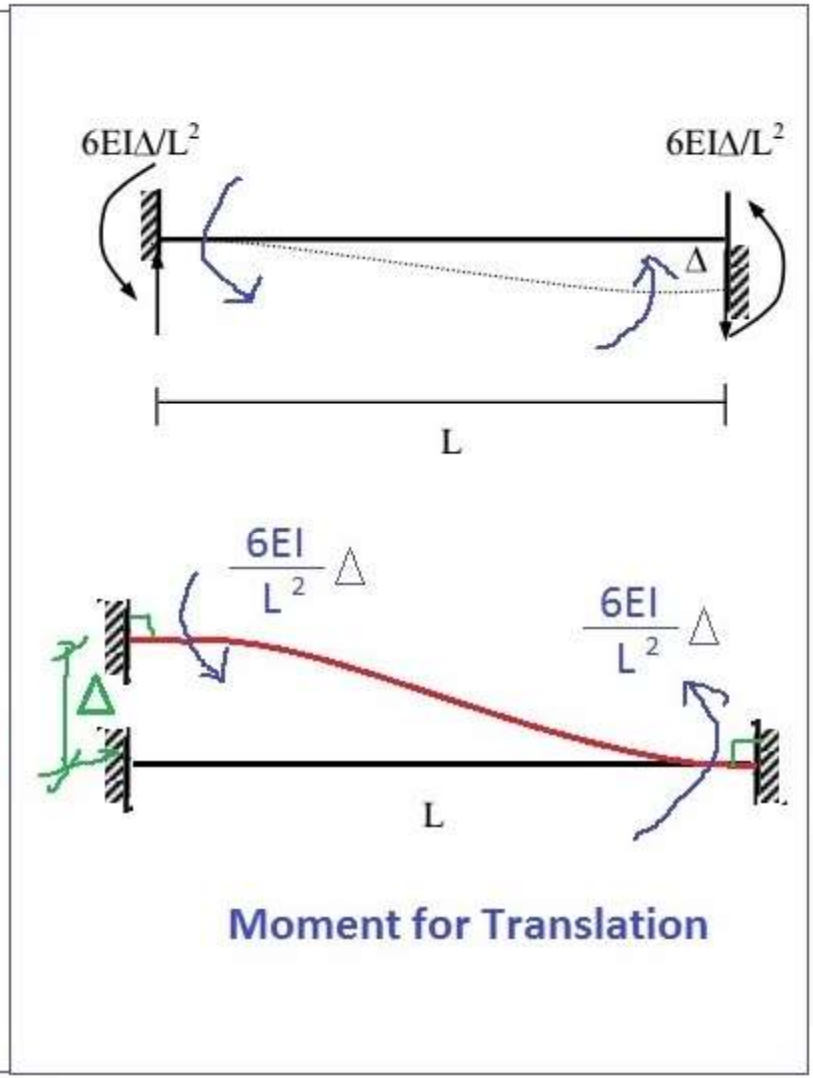
2. How the moments should be distributed?

> according to the ratio of member stiffness (distribution factors)

MOMENT DISTRIBUTION METHOD

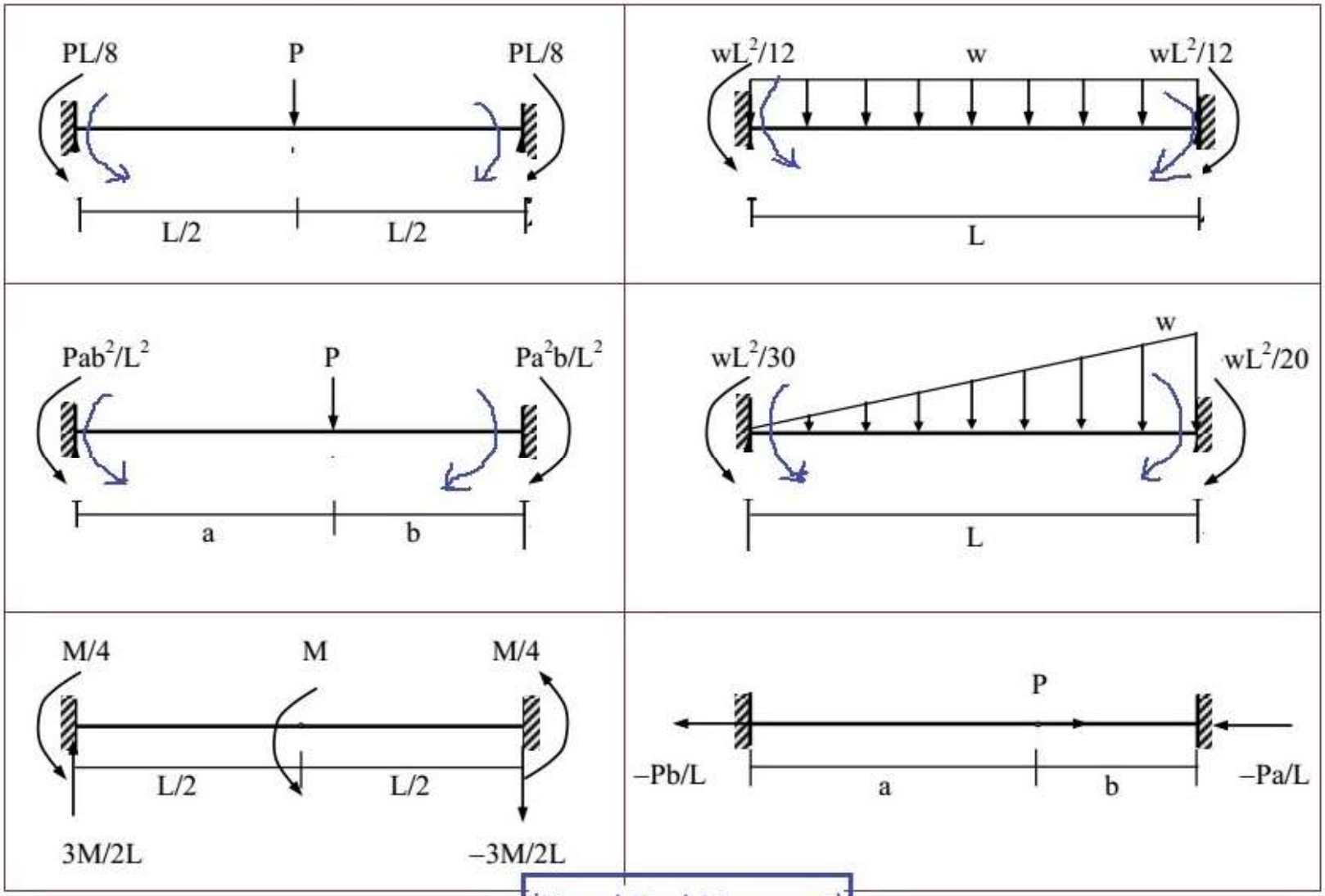


Moment for Rotation



Moment for Translation

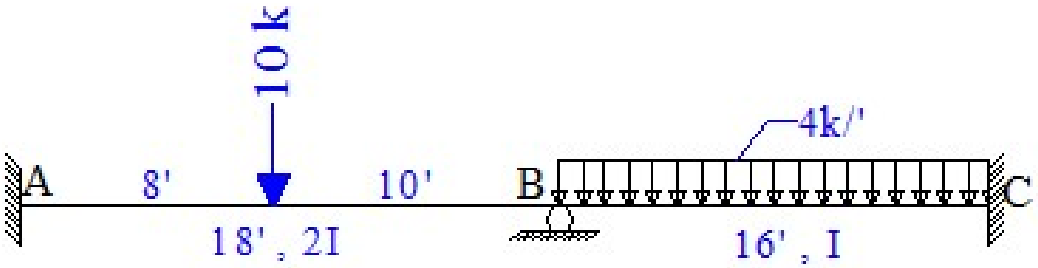
MOMENT DISTRIBUTION METHOD

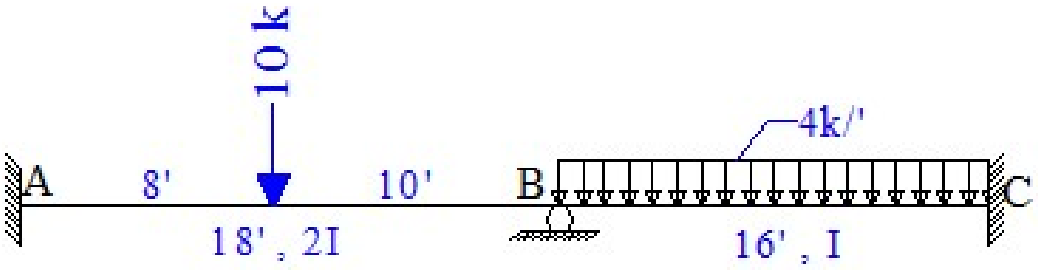


(Fixed End Moment)

MOMENT DISTRIBUTION METHOD

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows:**





- **Support at A rotate clockwise 0.03 radian.**
- **Support at A moves upward 0.15 ft.**
- **Support at B settles down by 0.2 ft.**
- **EI= 4500 K-ft^2**

MOMENT DISTRIBUTION METHOD

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding.**

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

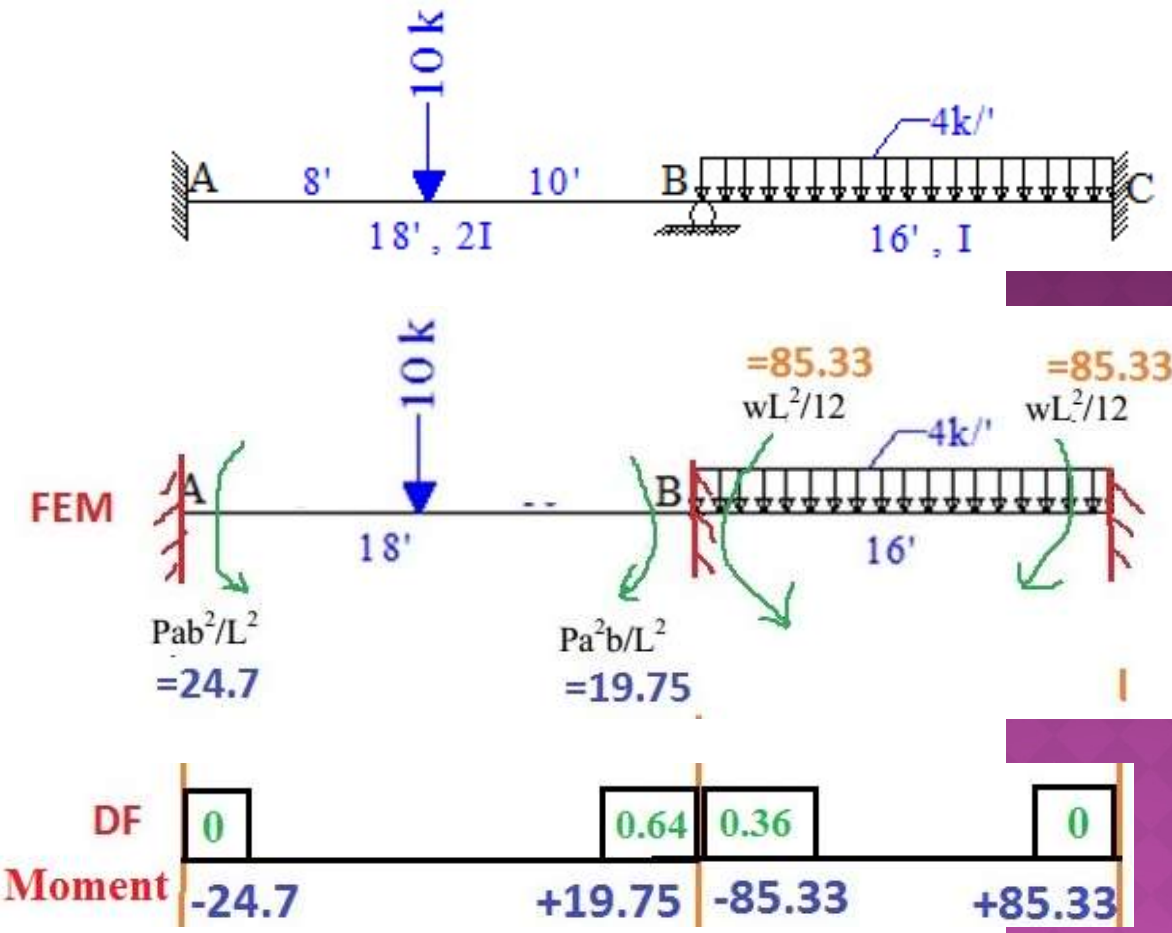
$$= \frac{\frac{4*2EI}{18}}{\frac{4*2EI}{18} + \frac{4EI}{16}}$$

$$= 0.64$$

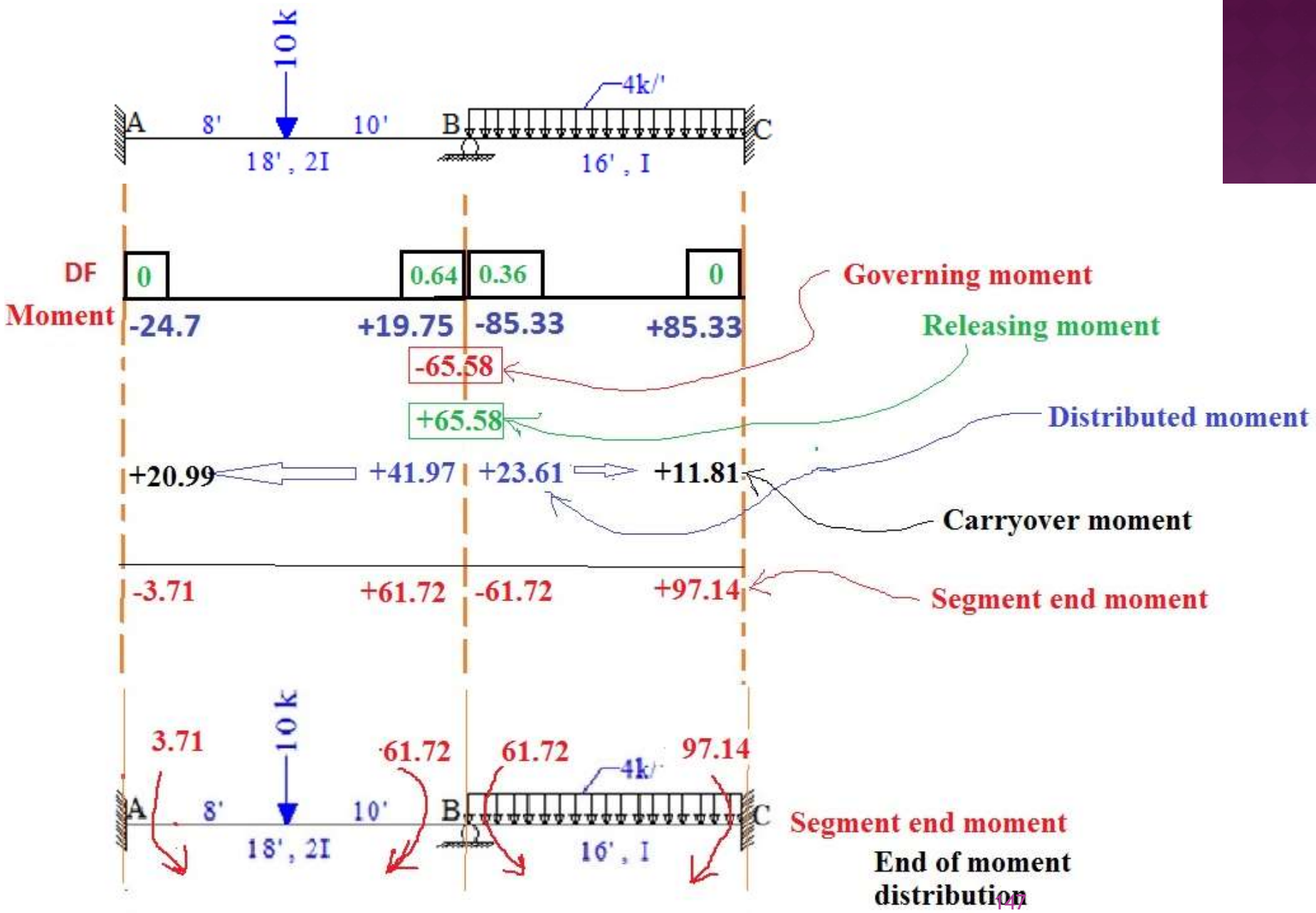
$$DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}}$$

$$= \frac{\frac{4EI}{16}}{\frac{4*2EI}{18} + \frac{4EI}{16}}$$

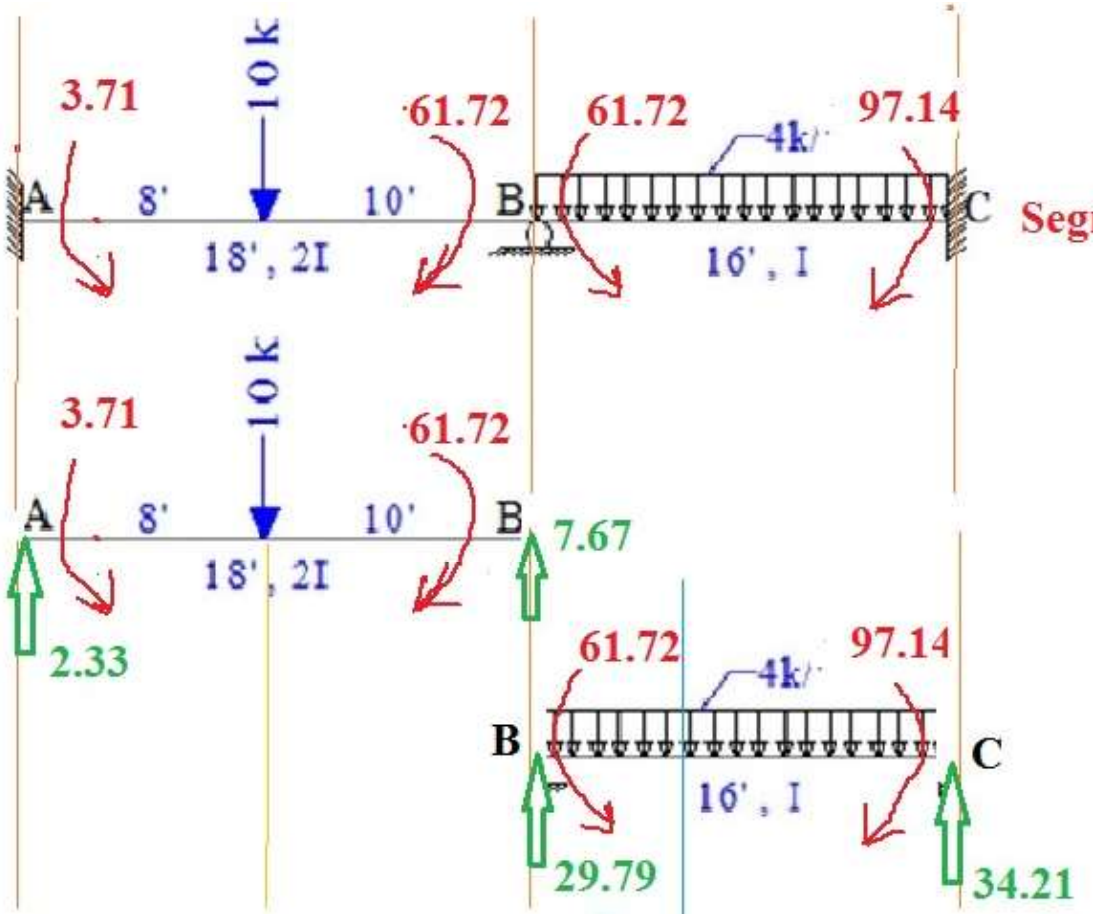
$$= 0.36$$



MOMENT DISTRIBUTION METHOD



MOMENT DISTRIBUTION METHOD



Segment end moment

End of moment distribution

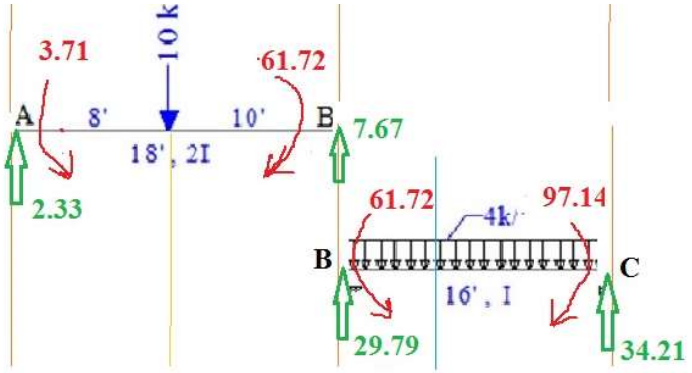
Segment AB:

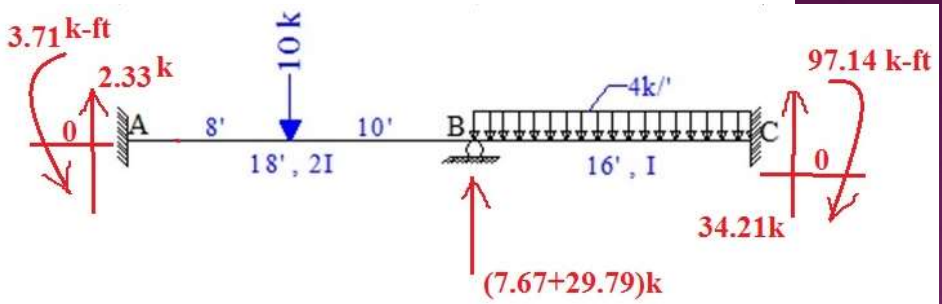
$\Sigma M_A = 0$

$-3.71+10*8+61.72-V_B*18=0$

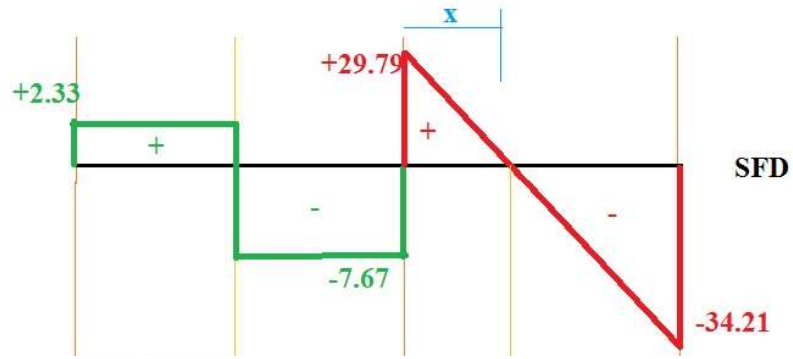
$-61.72+4*16*8+97.14-V_C*16=0$

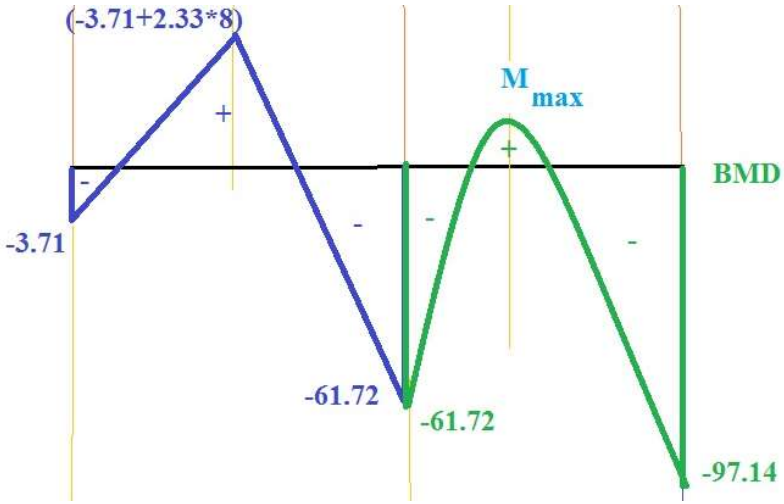
MOMENT DISTRIBUTION METHOD





Support reactions





M max where V=0

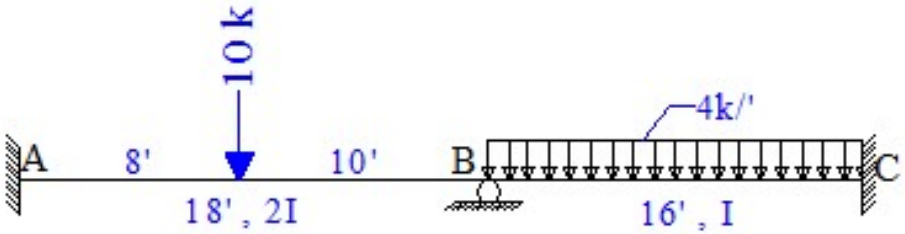
at x distance V=0

29.79-4*x=0

x= 7.45

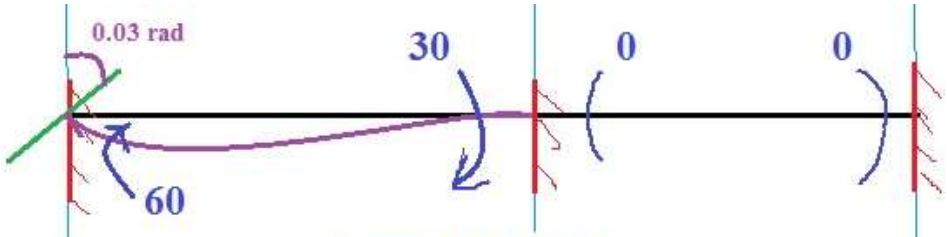
M_{max} = 29.79*x-61.72-4*x*x/2

MOMENT DISTRIBUTION METHOD



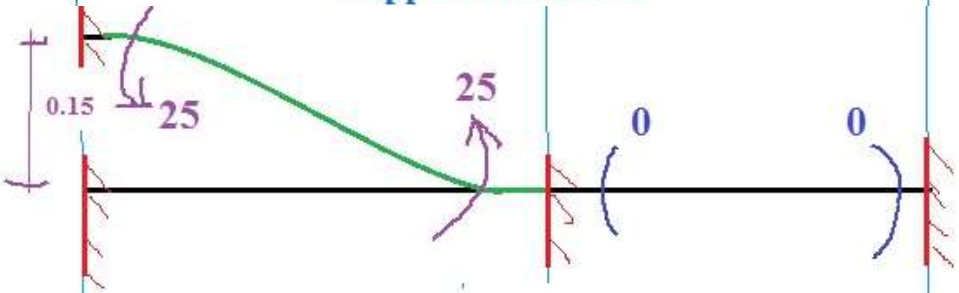
- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows:**

- **Support at A rotate clockwise 0.03 radian.**
- **Support at A moves upward 0.15 ft.**
- **Support at B settles down by 0.2 ft.**
- **EI= 4500 K-ft²**



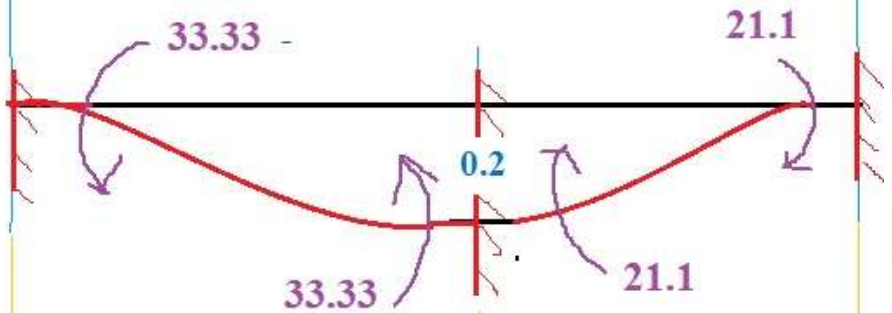
Support at A rotate

$$\frac{4EI}{L}\theta = \frac{4*2*4500}{18} \times 0.03 = 60$$



Support at A moves upward 0.15 ft.

$$\frac{6EI}{L^2}\Delta = \frac{6*2*4500}{18^2} \times 0.15 = 25$$

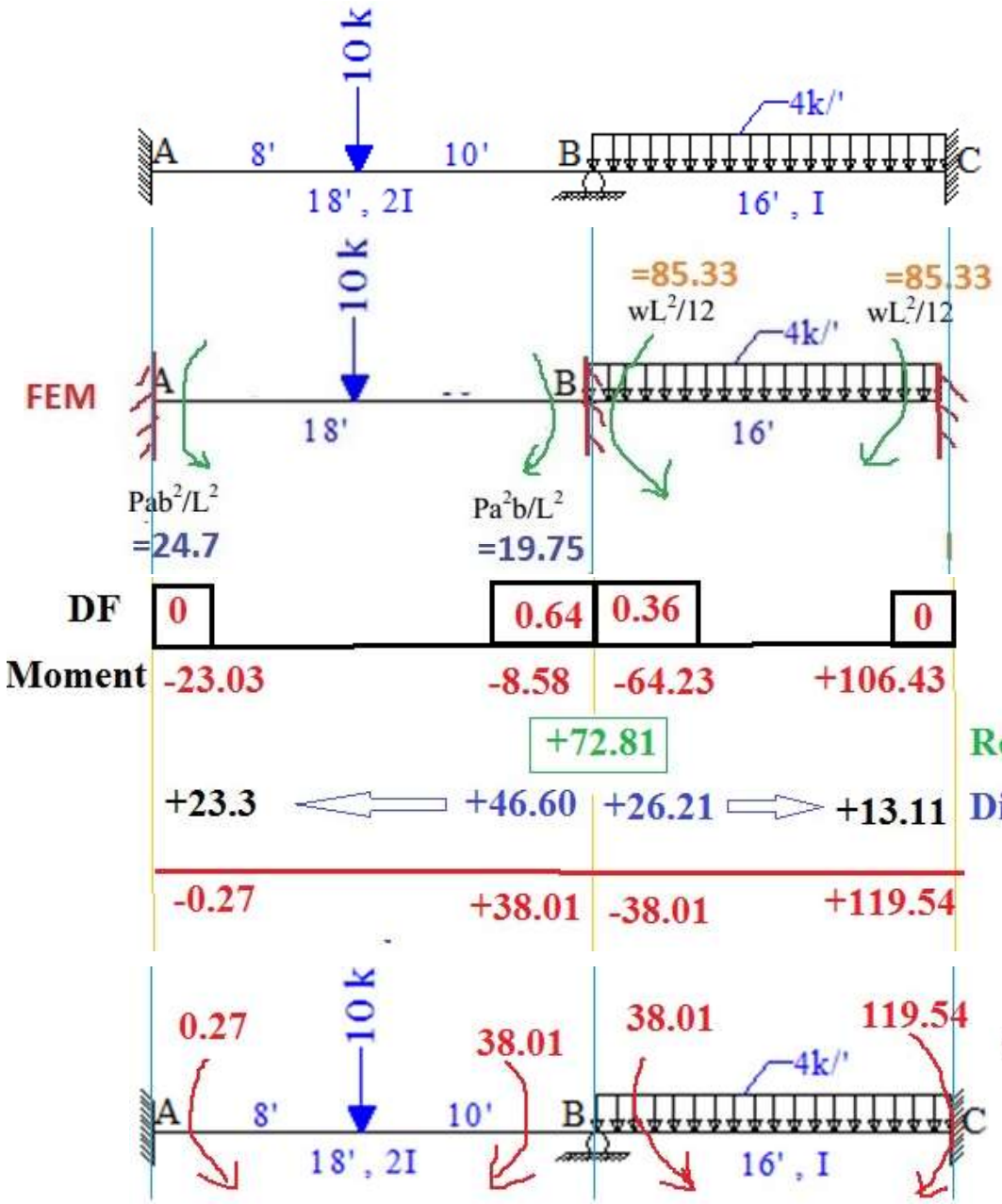


Support at B settles down by 0.2 ft.

$$\left(\frac{6EI}{L^2}\Delta\right)_{AB} = \frac{6*2*4500}{18^2} \times 0.2 = 33.33$$

$$\left(\frac{6EI}{L^2}\Delta\right)_{BC} = \frac{6*1*4500}{16^2} \times 0.2 = 21.1$$

MOMENT DISTRIBUTION METHOD



$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{\frac{4*2EI}{18}}{\frac{4*2EI}{18} + \frac{4EI}{16}} = 0.64$$

Draw SFD, BMD, Calculate support reaction

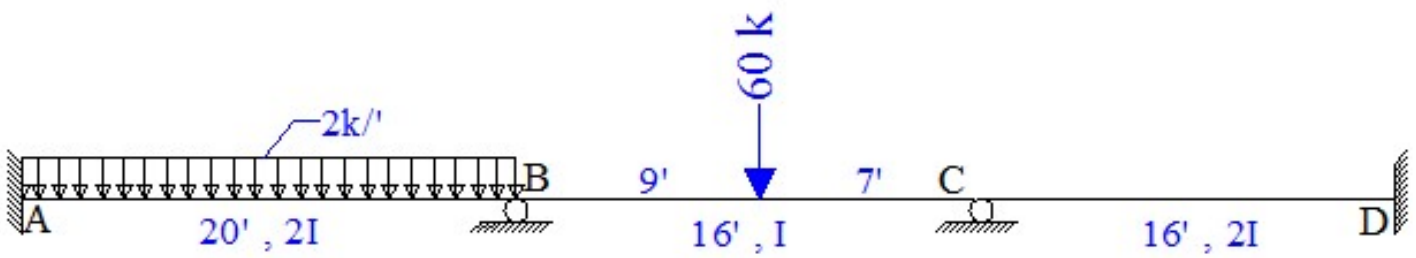
MOMENT DISTRIBUTION METHOD

Assignment-03: CEN 410 (Moment distribution method)

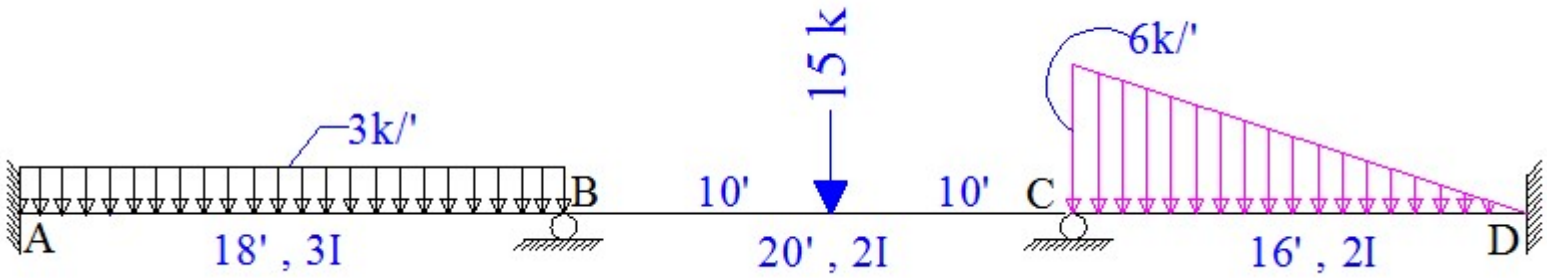
- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielded as follows:**

Assignment

- **EI= 3000 K-ft^2**

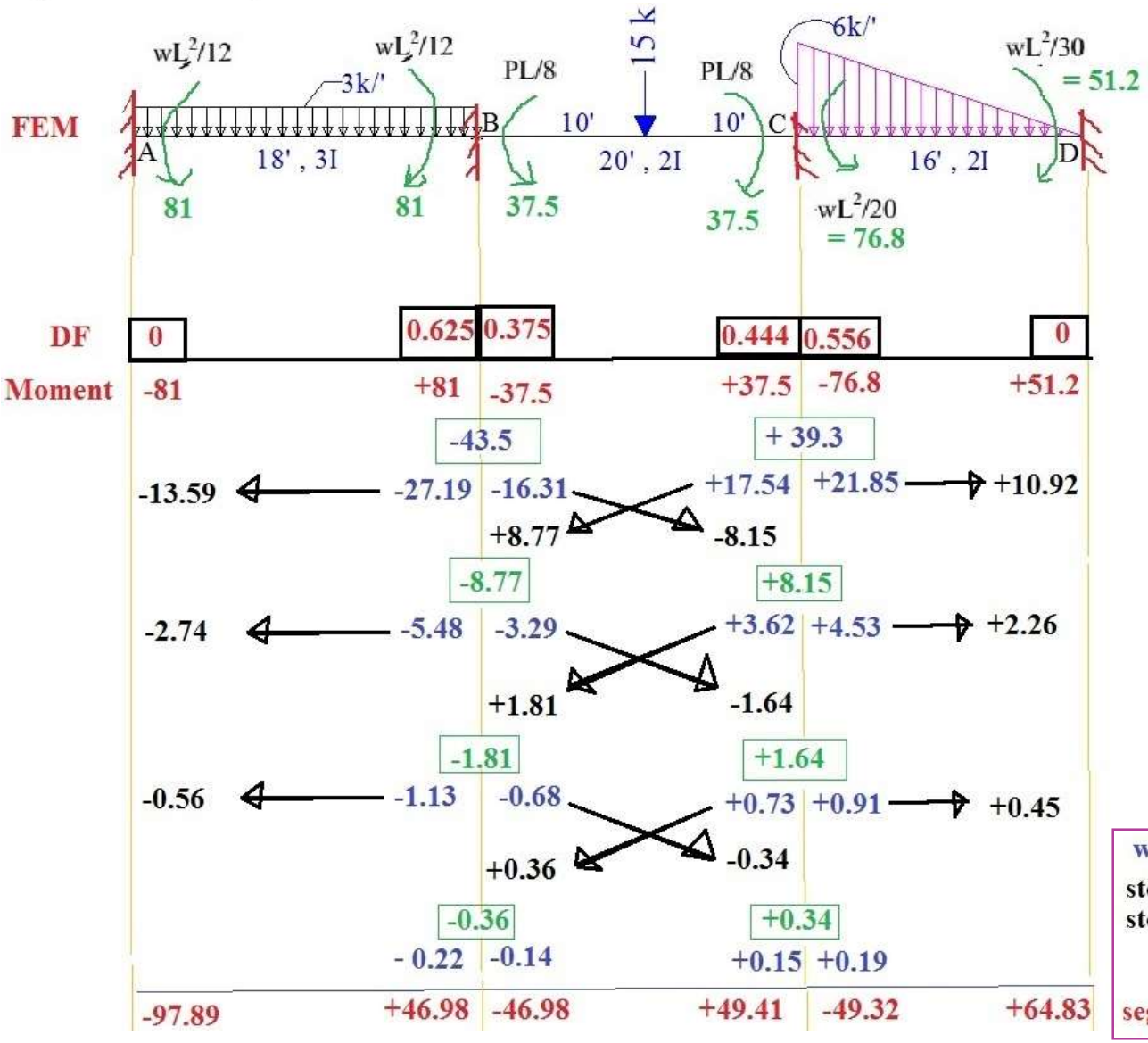


Compute the reactions and draw shear and bending moment diagram for the beam, supports are unyielding (Use Moment Distribution Method)



MOMENT DISTRIBUTION METHOD

Problem-3



$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= \frac{12EI/18}{(12EI/18)+(8EI/20)}$$

$$= 0.625$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}}$$

$$= \frac{8EI/20}{8EI/20 + 8EI/16}$$

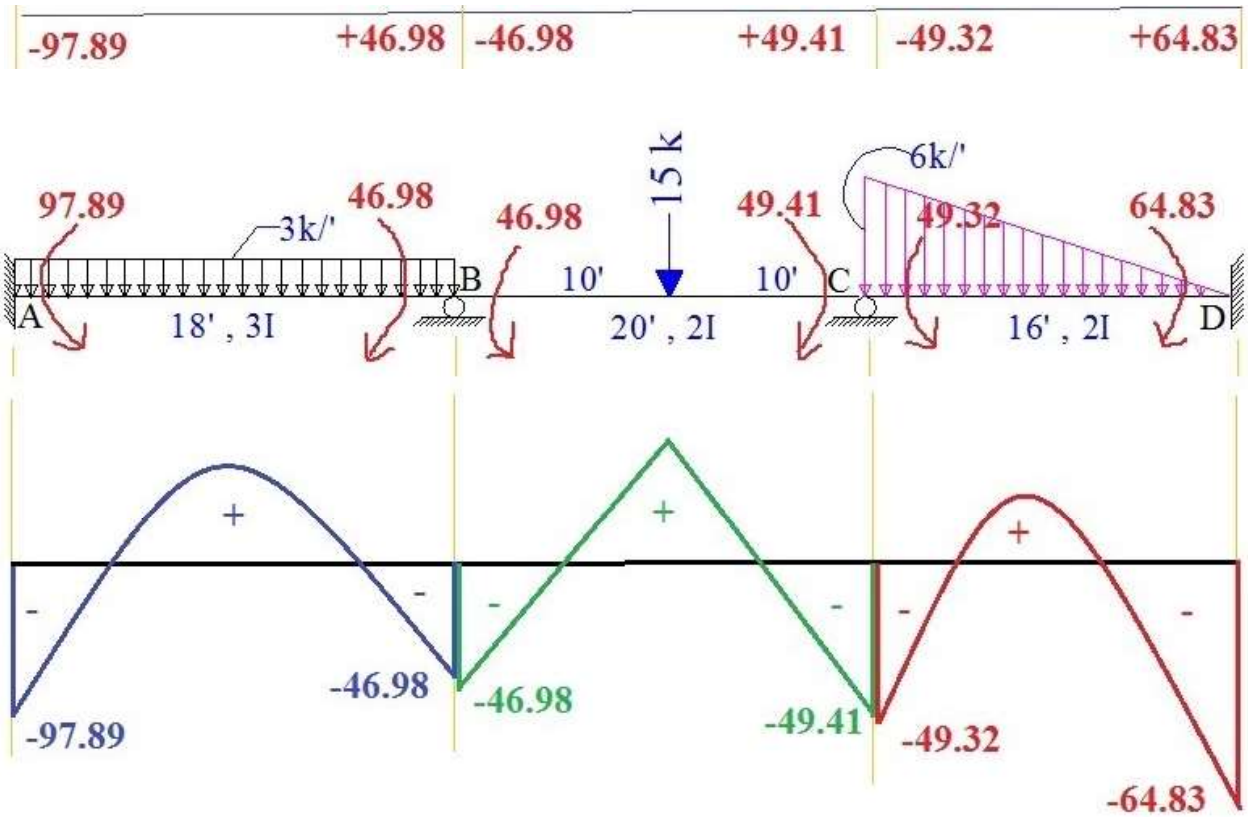
$$= 0.444$$

when moment reduces to 1% stop distribution

stop after distribution/ not to stop after carryover

segment end moments

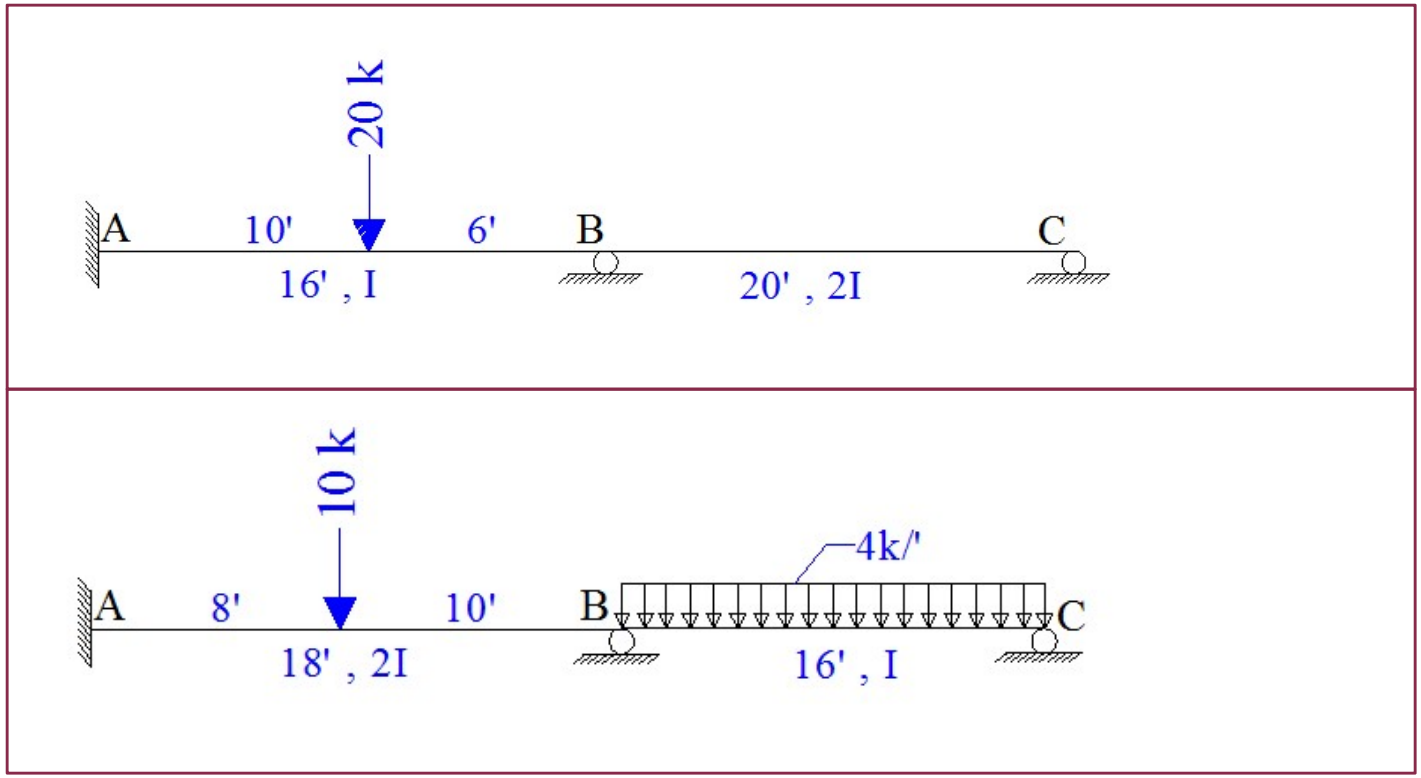
MOMENT DISTRIBUTION METHOD



do SFD and support reaction calculation by calculating segment shears

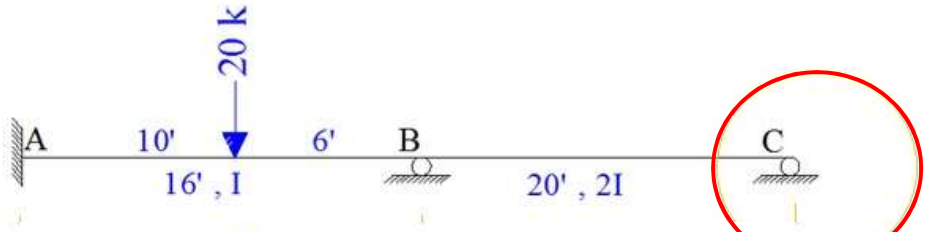
MOMENT DISTRIBUTION METHOD

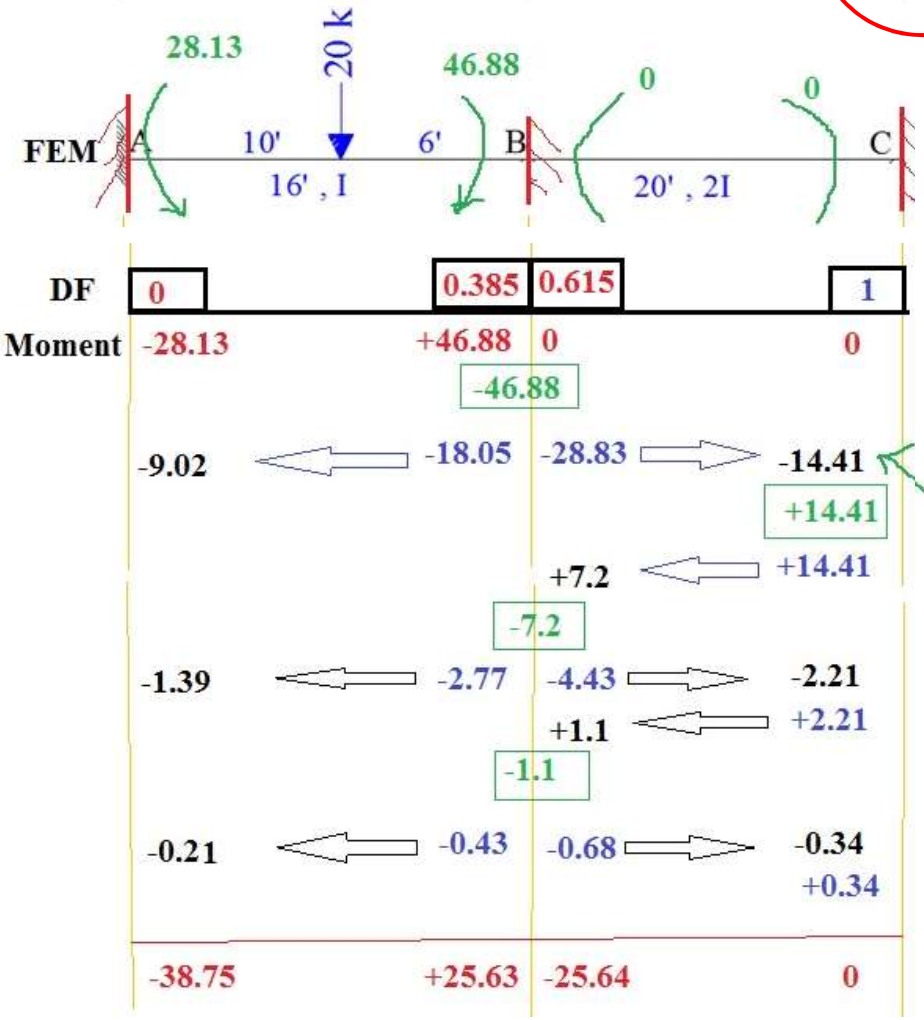
- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows:**



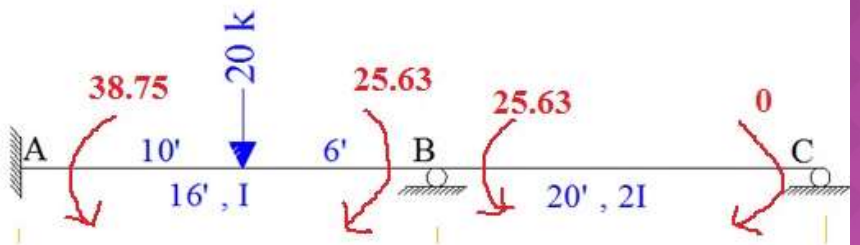
MOMENT DISTRIBUTION METHOD

Exterior End roller or hinge support

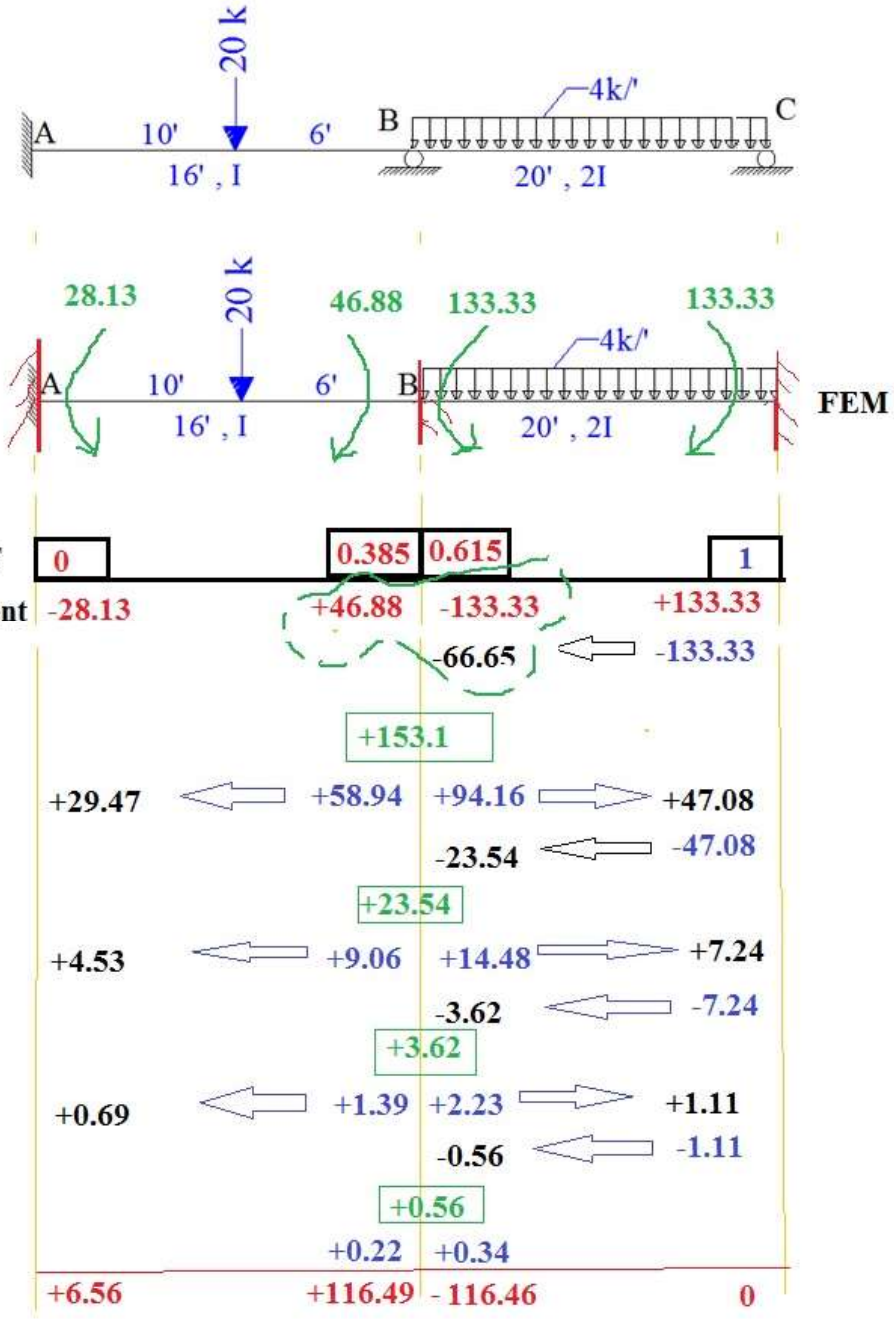




$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{\frac{4EI}{16}}{\frac{4EI}{16} + \frac{8EI}{20}}$$

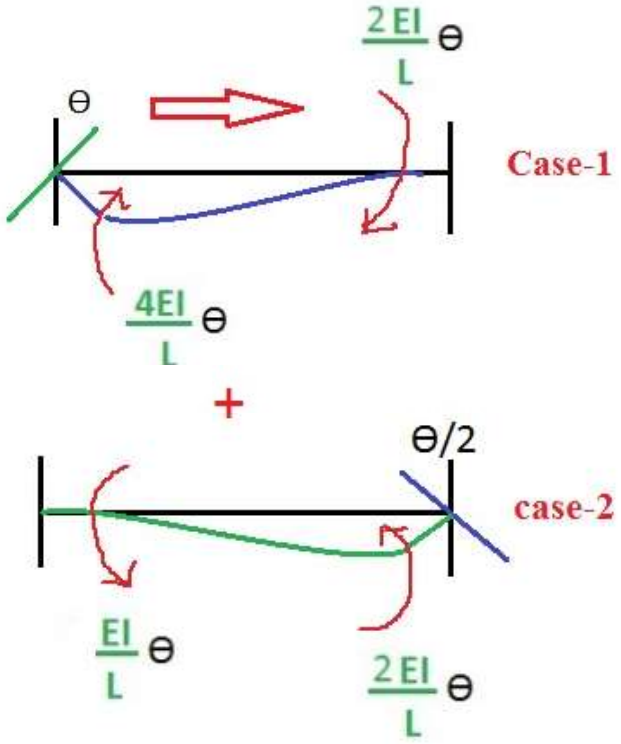


MOMENT DISTRIBUTION METHOD



MOMENT DISTRIBUTION METHOD: MODIFIED STIFFNESS

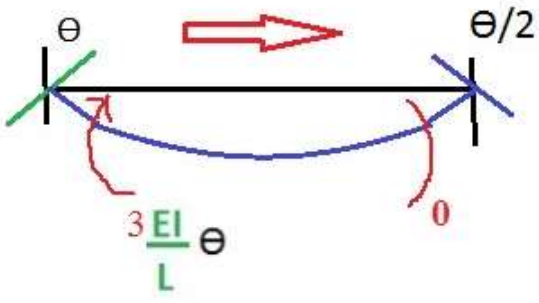
Exterior End roller or hinge support (Modified Stiffness)



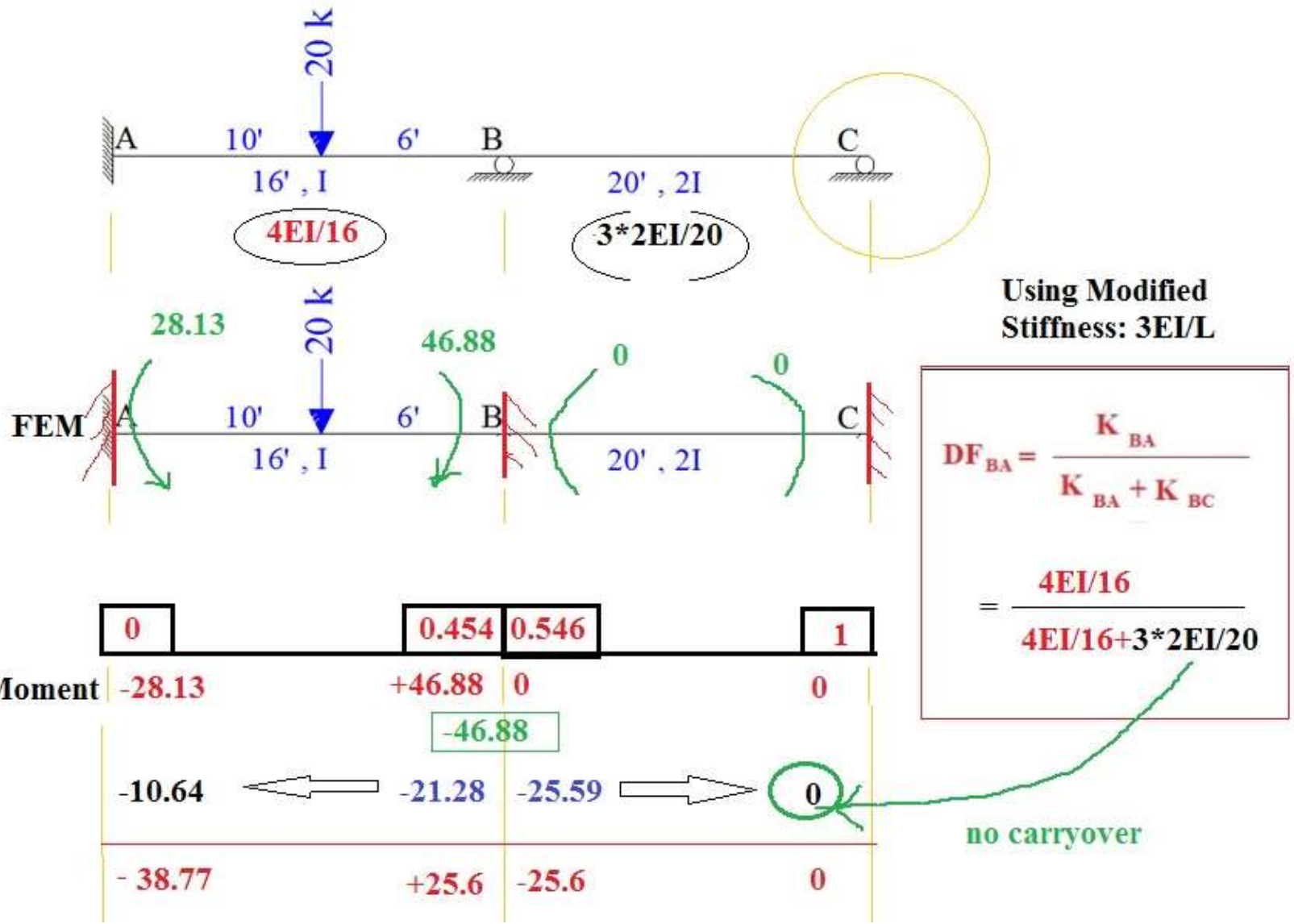
$$\text{Regular Stiffness} = K = \frac{4EI}{L}$$

If in the problem, there is exterior end roller/hinge, then, if we choose a modified stiffness 3EI/L instead of regular stiffness 4EI/L at the interior end, then there will be no carryover moment at the exterior end.

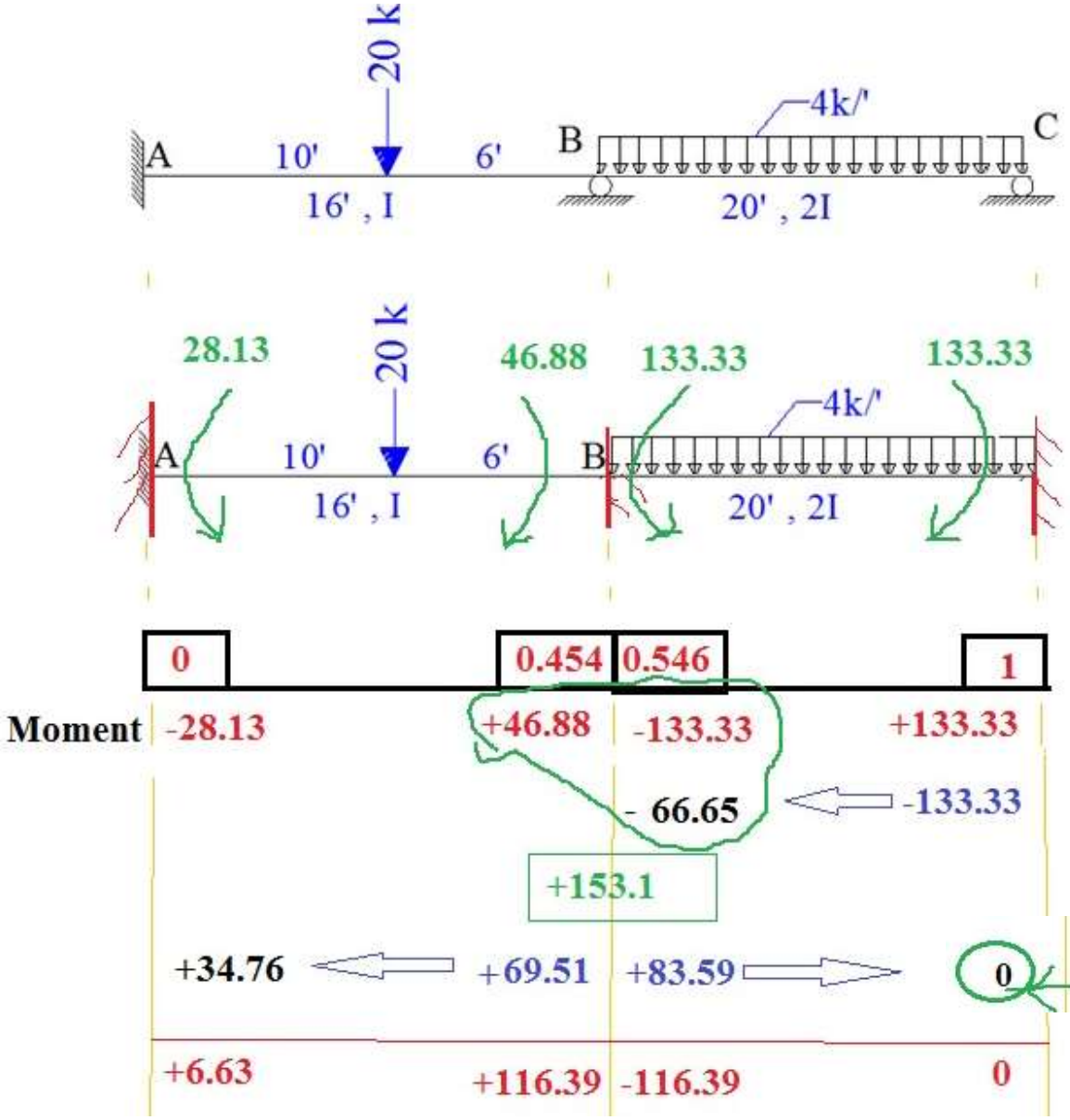
Modified stiffness for exterior end roller/Hinge= 3EI/L



MOMENT DISTRIBUTION METHOD: MODIFIED STIFFNESS



MOMENT DISTRIBUTION METHOD: MODIFIED STIFFNESS



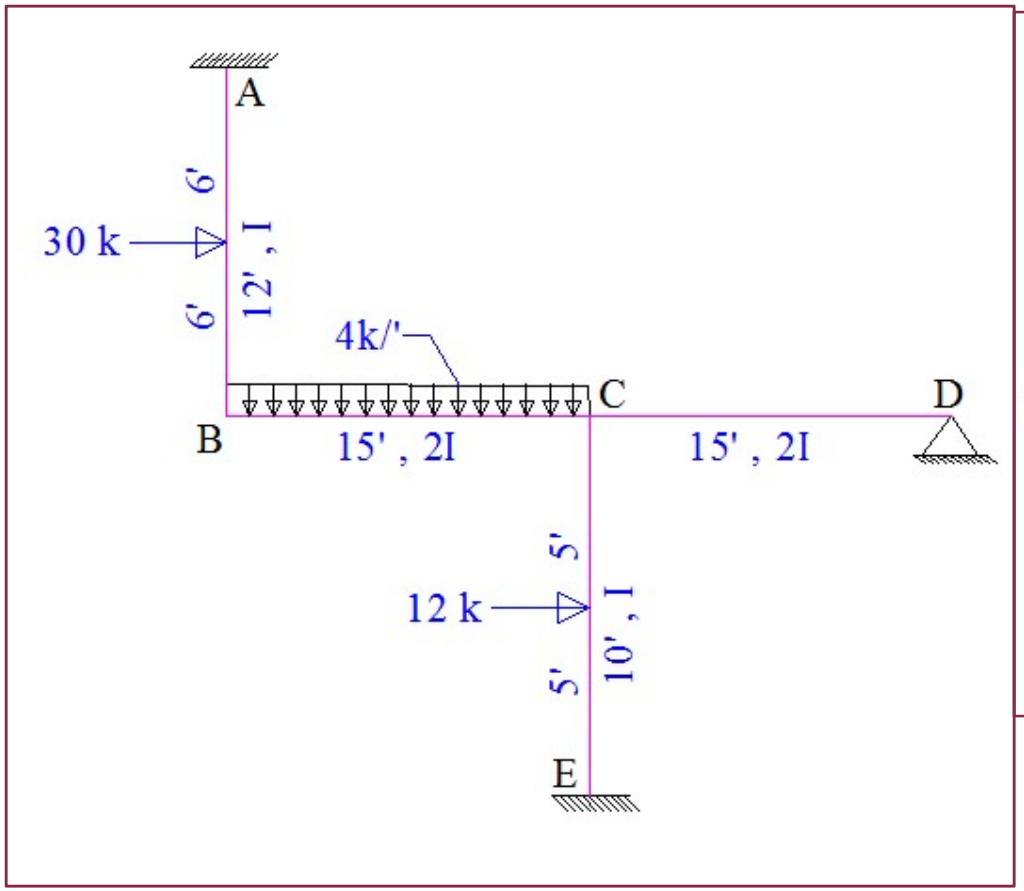
Using Modified Stiffness: 3EI/L

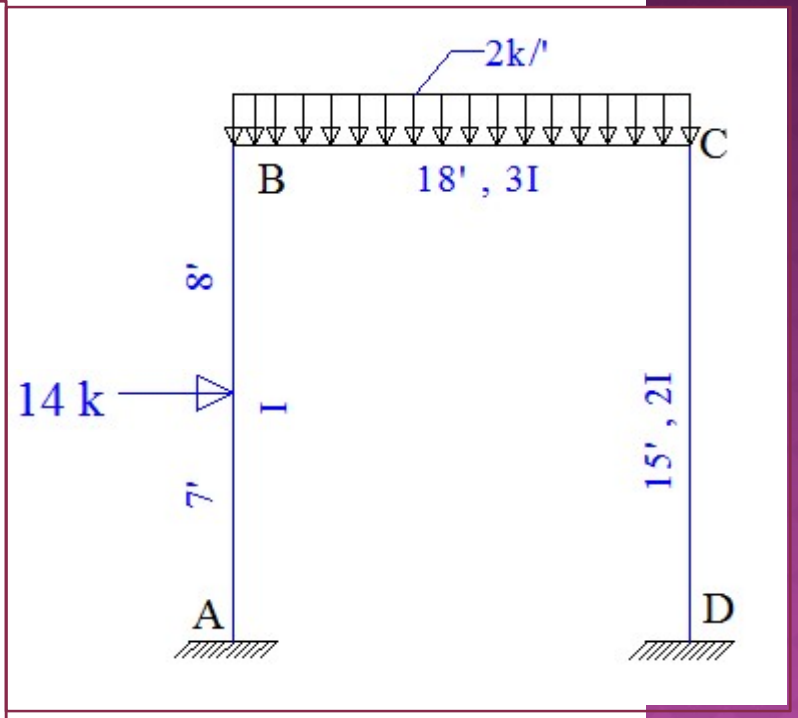
$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= \frac{4EI/16}{4EI/16 + 3*2EI/20}$$

MOMENT DISTRIBUTION METHOD

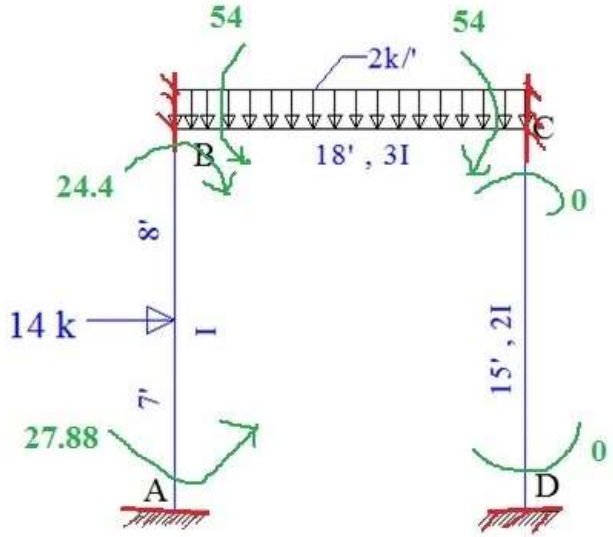
- **Compute the reactions and draw shear and bending moment diagram for the frame. Supports yield as follows:**





MOMENT DISTRIBUTION METHOD

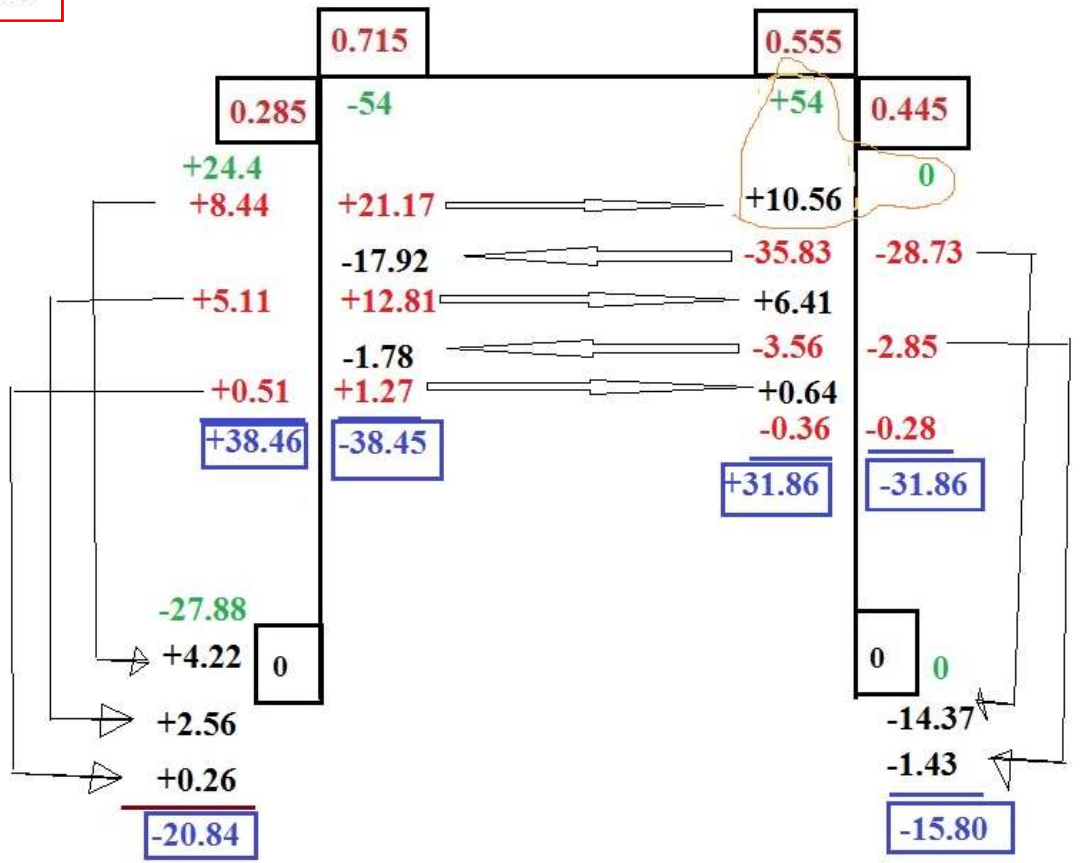
Single Bay Frame / Single Portal Frame



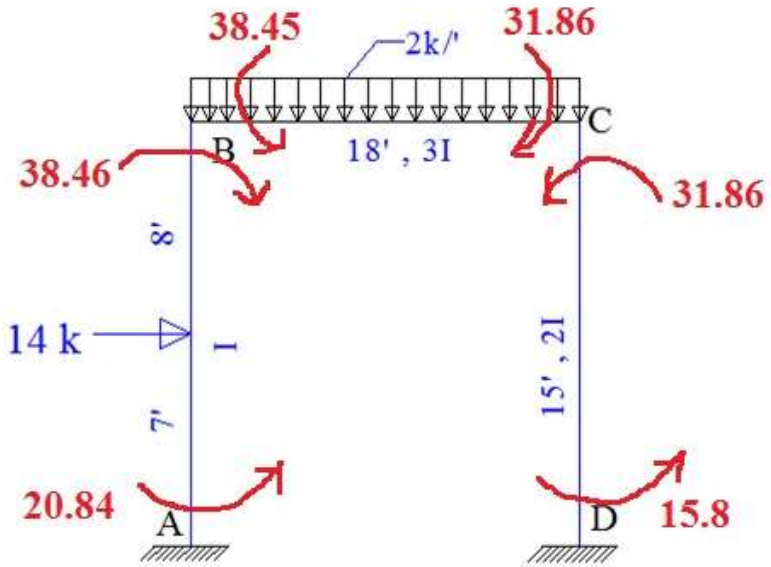
FEM

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{4EI/15}{4EI/15+12EI/18}$$

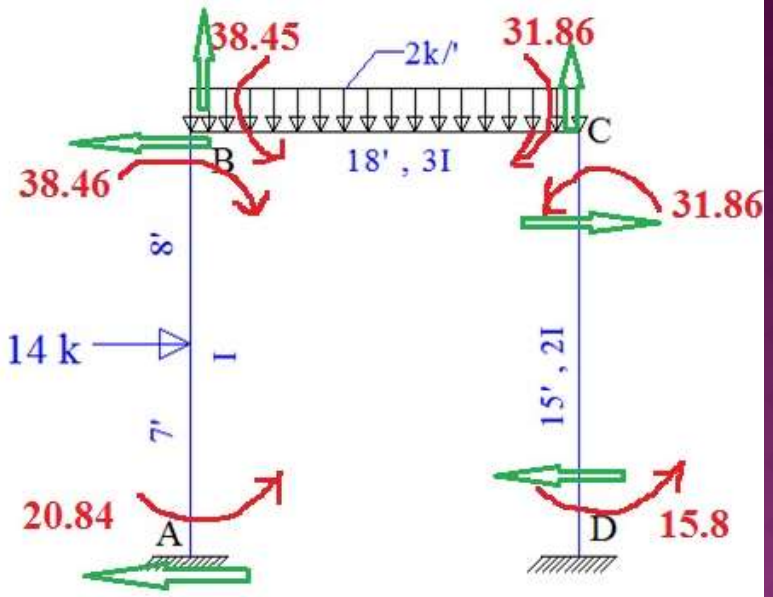
$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{12EI/18}{12EI/18 + 8EI/15}$$



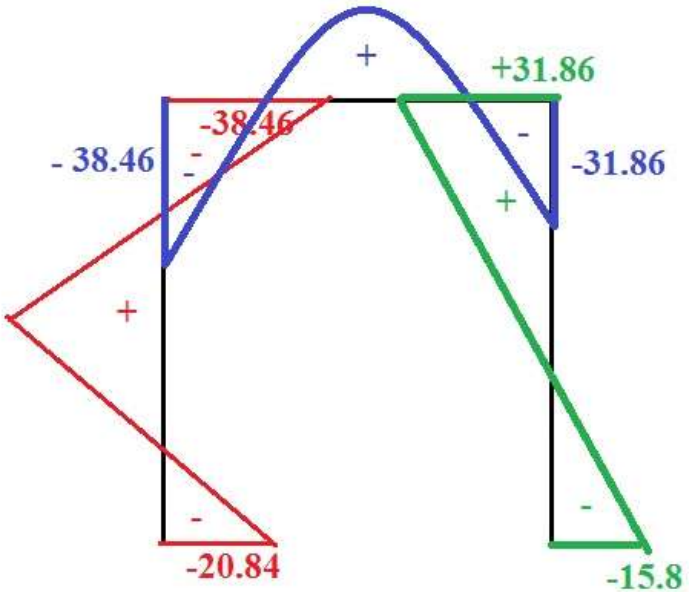
MOMENT DISTRIBUTION METHOD



Segment End Moments

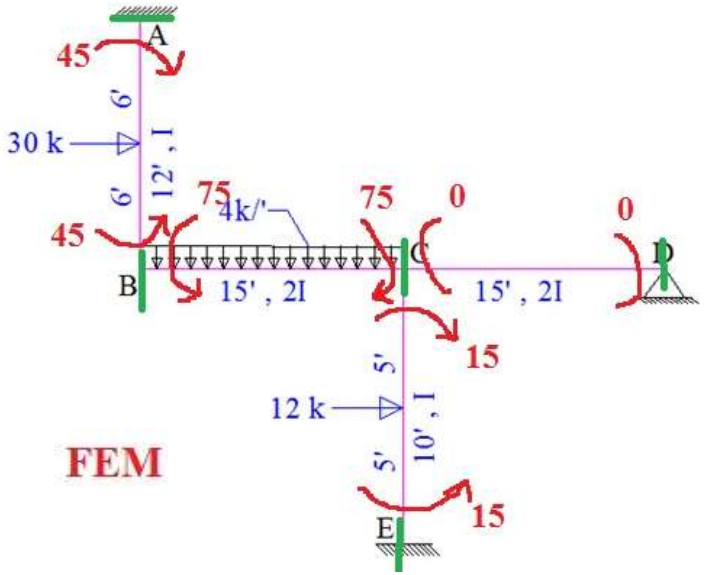


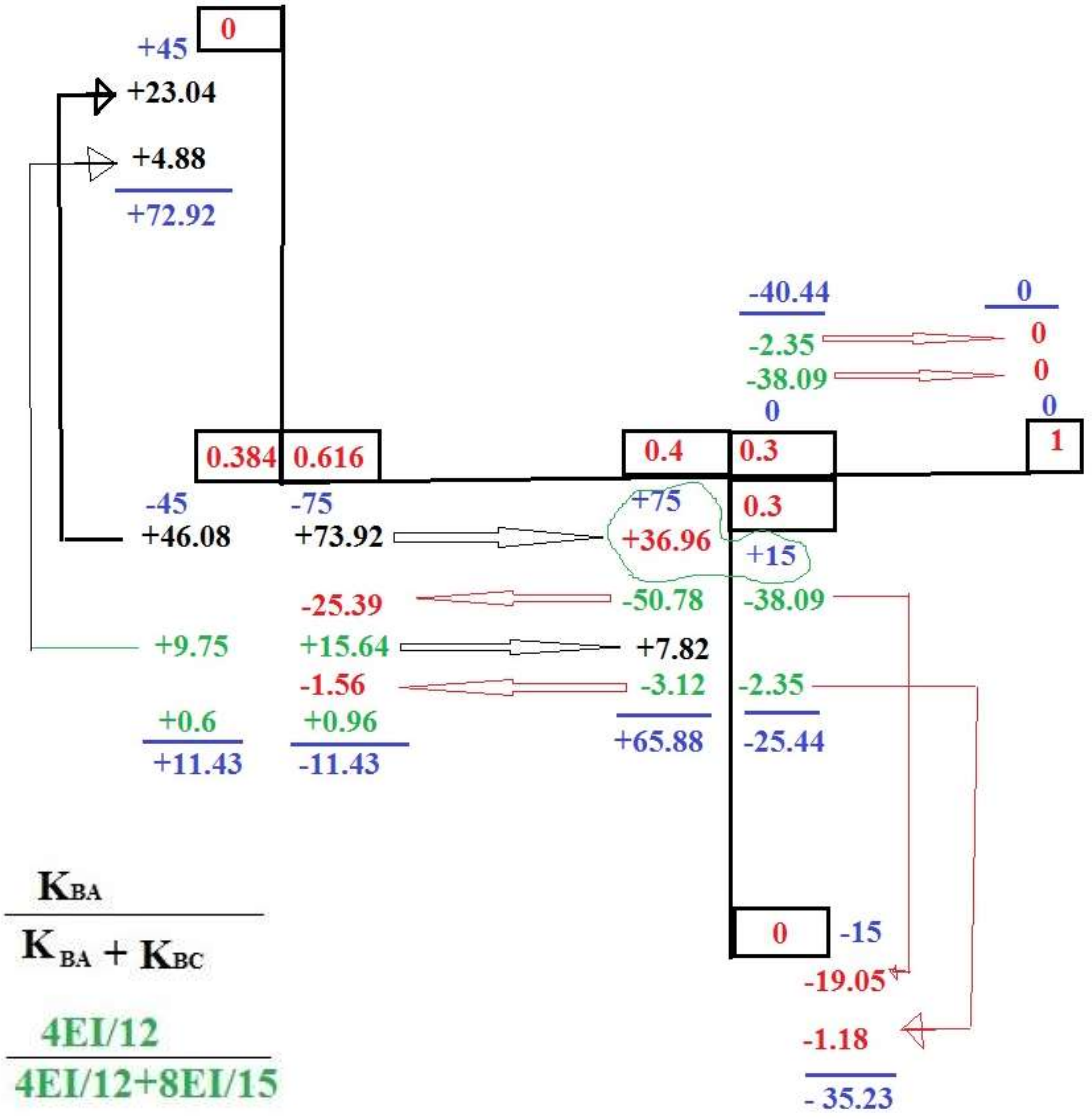
Segment End Moments



BMD

MOMENT DISTRIBUTION METHOD





Modified Stiffness on CD

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CE}}$$

$$= \frac{8EI/15}{8EI/15+6EI/15+4EI/10}$$

$$DF_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD} + K_{CE}}$$

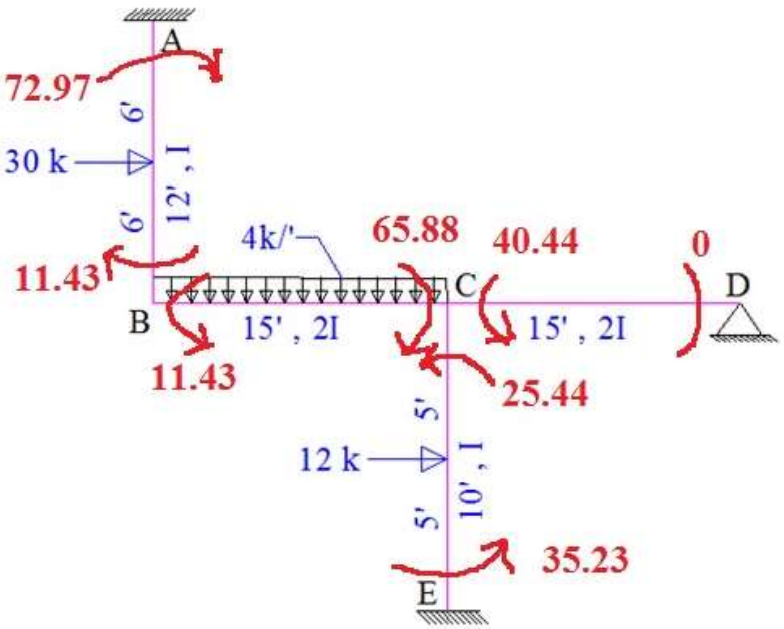
$$= \frac{6EI/15}{8EI/15+6EI/15+4EI/10}$$

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= \frac{4EI/12}{4EI/12+8EI/15}$$

$$= 0.384$$

MOMENT DISTRIBUTION METHOD



Segment End Moments

