# *EWCE 411*

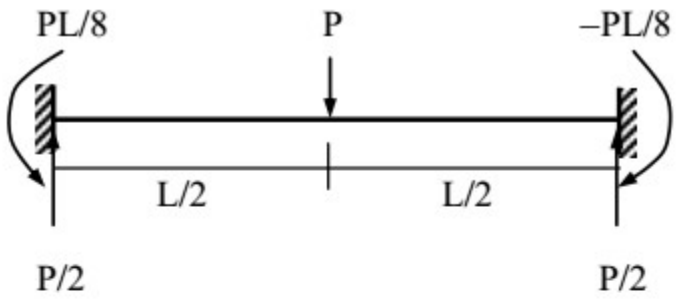
# *Method of Slope Deflection*

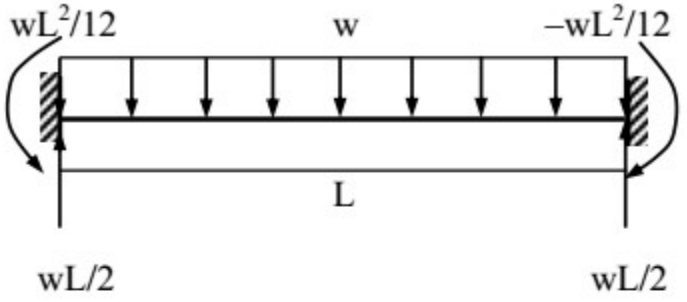
# SLOPE DEFLECTION METHOD

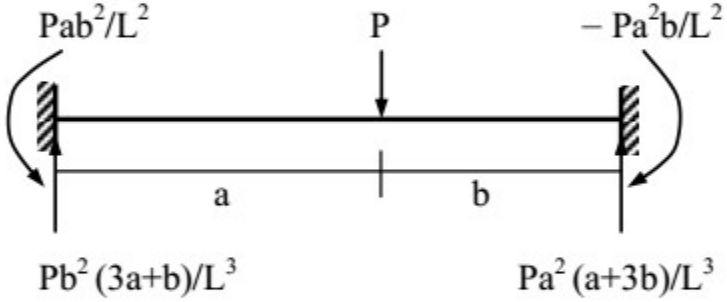
*(Indeterminate Structure Analysis)*

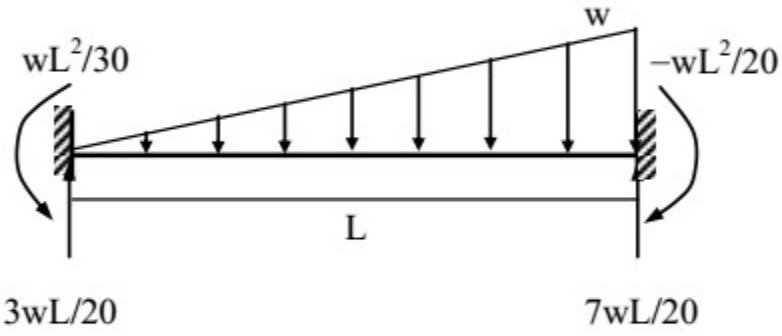
# SLOPE DEFLECTION METHOD

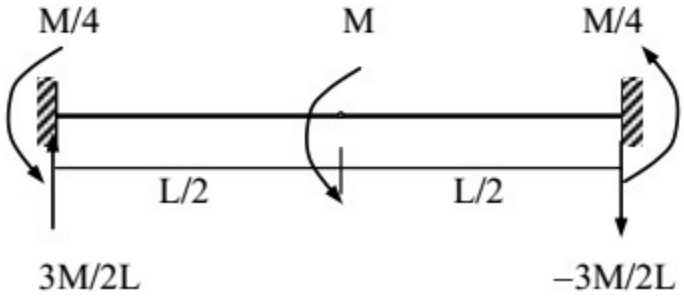
## Fixed End Reactions for One-dimensional Members under Typical Loadings

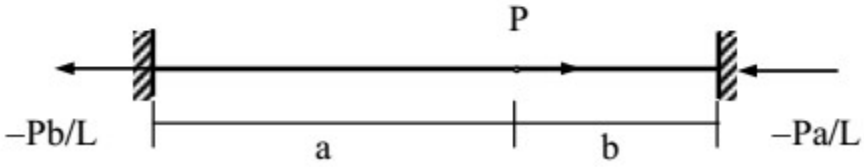


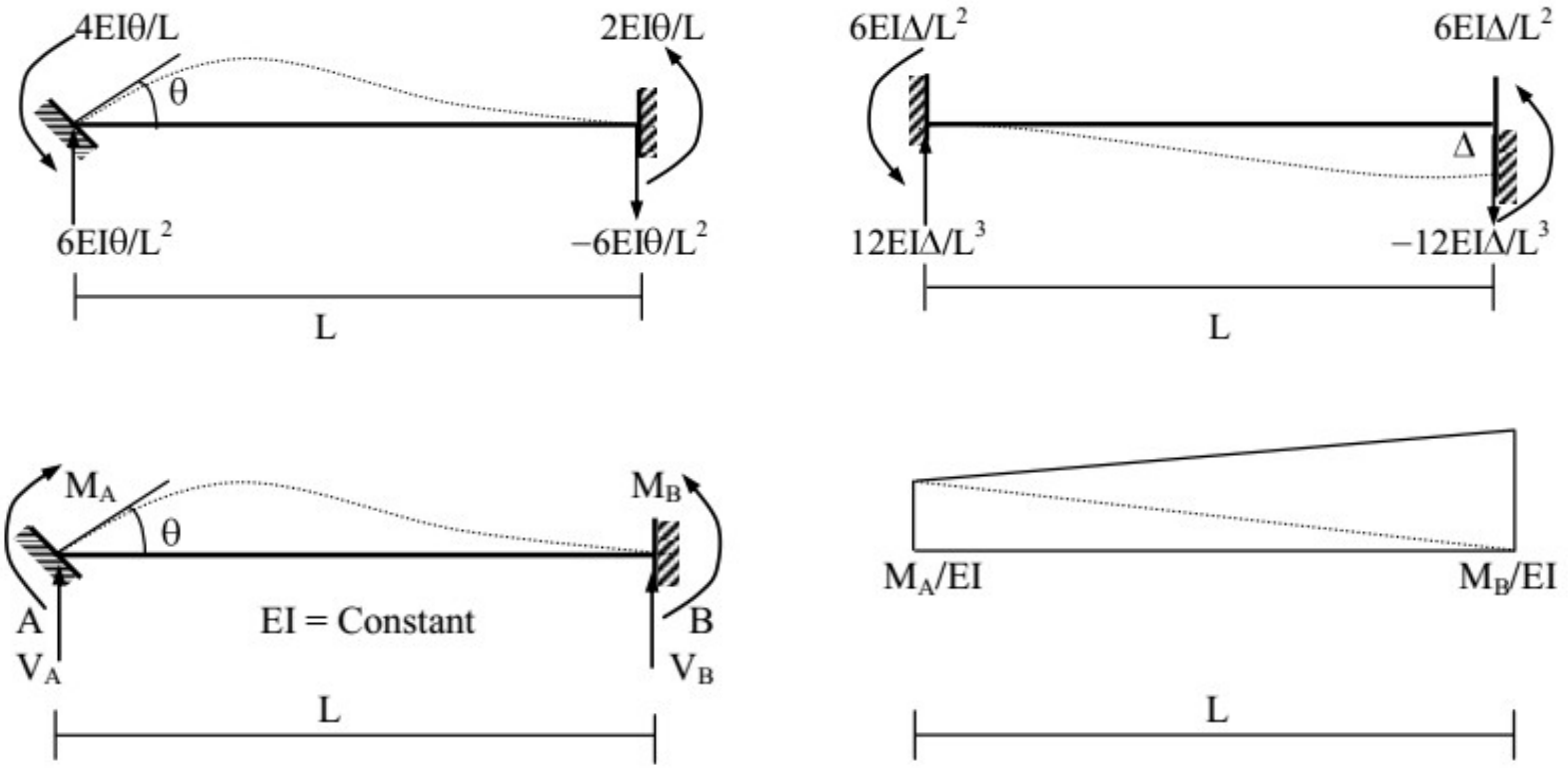



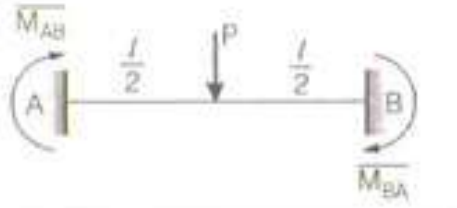
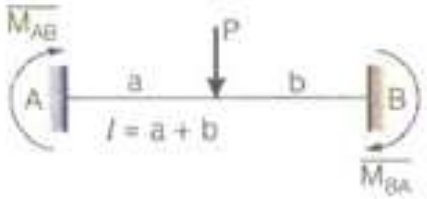
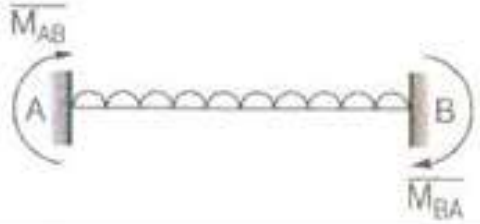
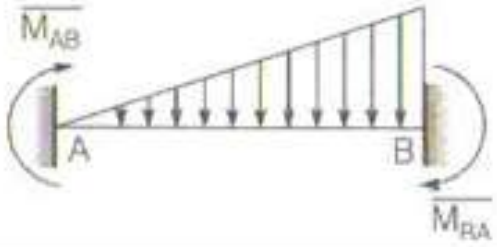



# SLOPE DEFLECTION METHOD

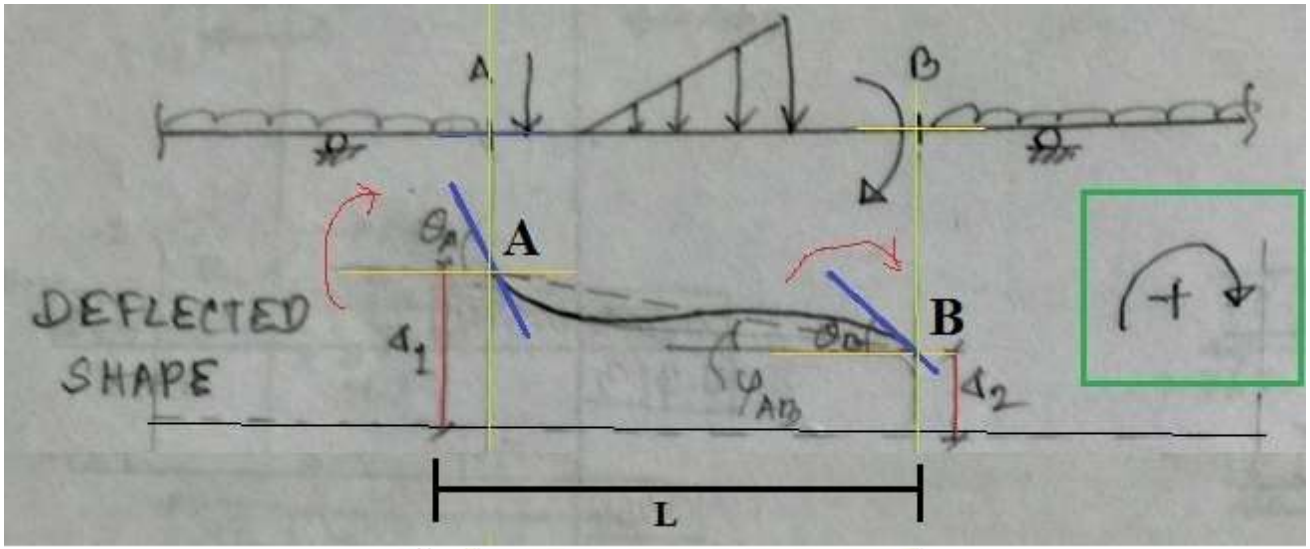
## End Rotation and Rotational Stiffness of Fixed Ended Members



# SLOPE DEFLECTION METHOD

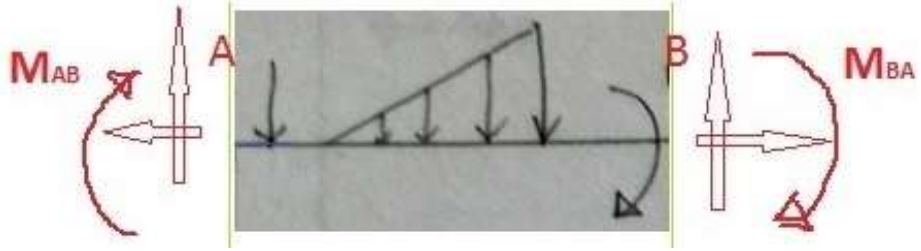
| | $\overline{M_{AB}}$ | $\overline{M_{BA}}$ |
|---|---|---|
|  | $\frac{-Pl}{8}$ | $\frac{Pl}{8}$ |
|  | $\frac{-Pab^2}{l^2}$ | $\frac{Pa^2b}{l^2}$ |
|  | $\frac{-wl^2}{12}$ | $\frac{wl^2}{12}$ |
|  | $\frac{-wl^2}{30}$ | $\frac{wl^2}{20}$ |

# SLOPE DEFLECTION METHOD

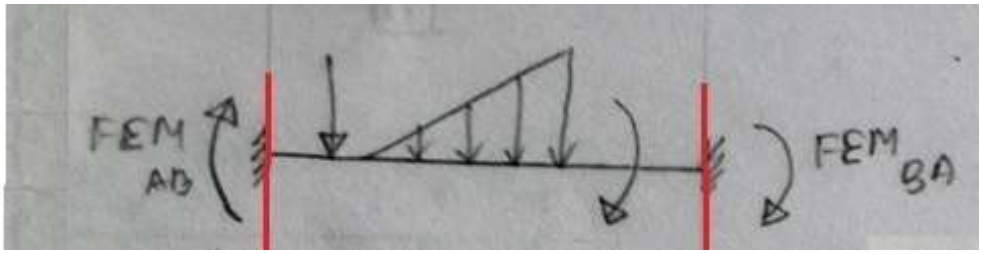



(Clockwise positive)

**Free body Diagram**




**Fixed end moment**



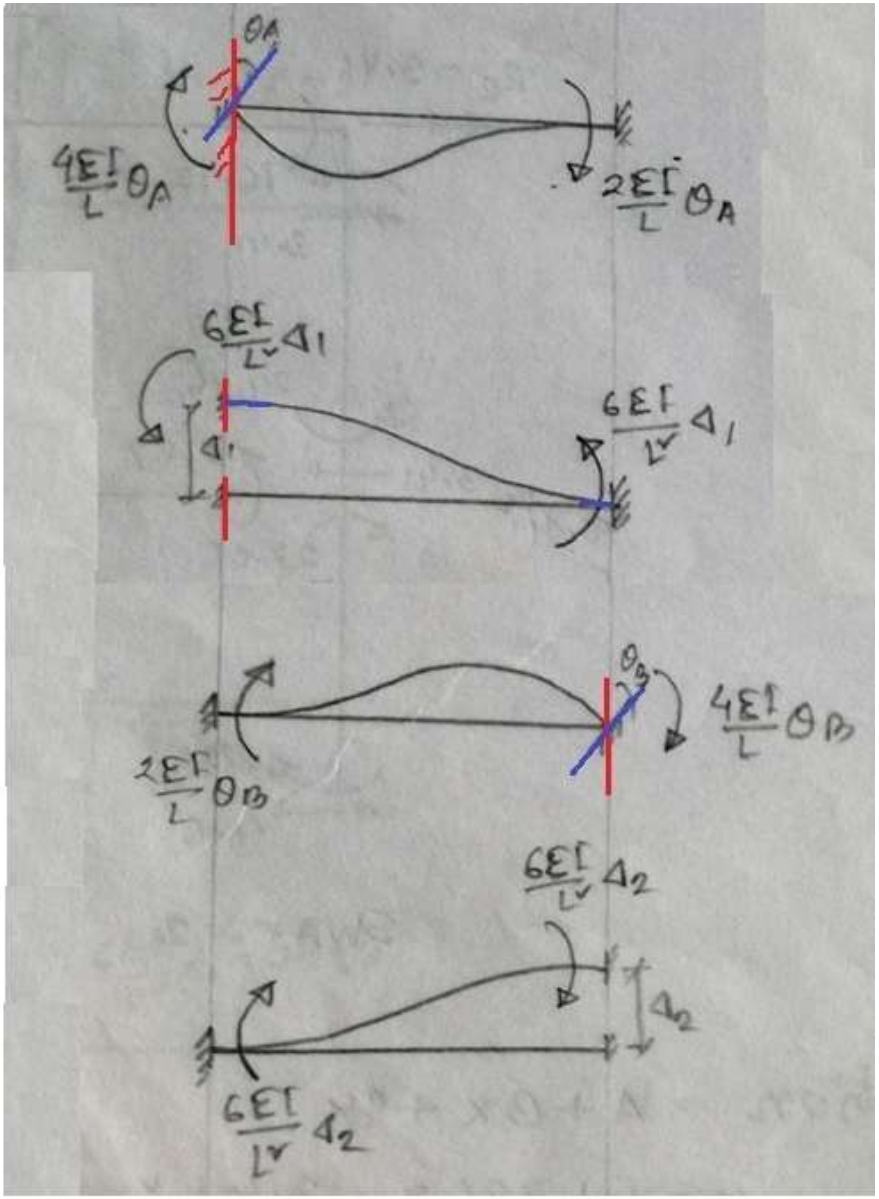
# SLOPE DEFLECTION METHOD

rotation of left end

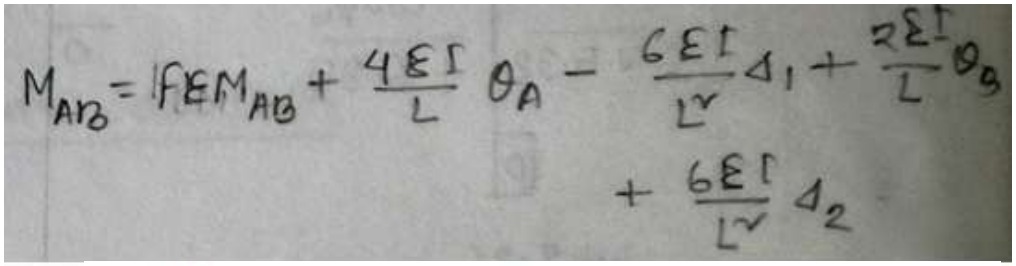
translation of left end

Rotation of right end

Translation of right end



# SLOPE DEFLECTION METHOD



$$M_{AB} = FEM_{AB} + \frac{4EI}{L}\theta_A - \frac{6EI}{L^2}\Delta_1 + \frac{2EI}{L}\theta_B + \frac{6EI}{L^2}\Delta_2$$

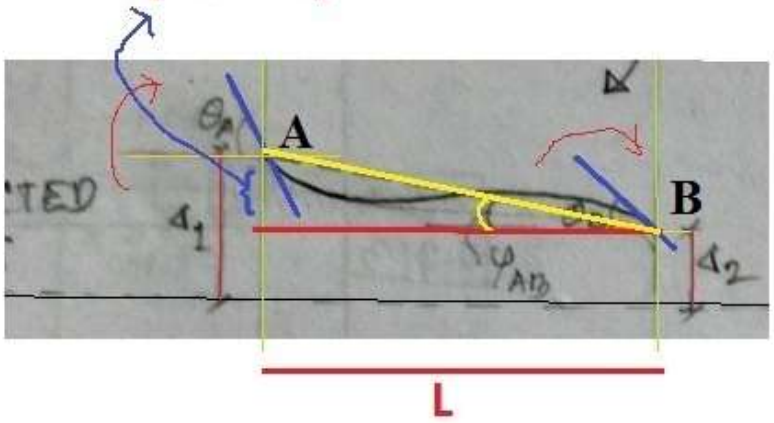
$$= \frac{2EI}{L}\{2\theta_A + \theta_B - \frac{3}{L}(\Delta 1 - \Delta 2)\} + FEM_{AB}$$

**Slope Deflection Equations**

$$M_{AB} = \frac{2EI}{L}\{2\theta_A + \theta_B - 3\psi_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}\{\theta_A + 2\theta_B - 3\psi_{AB}) + FEM_{BA}$$

$(\Delta 1 - \Delta 2)$




$\psi_{AB} = \psi_{BA}$ = Relative rotation of A and B point

$$\psi_{AB} = \psi_{BA} = \left(\frac{|\Delta_A - \Delta_B|}{L}\right)$$

Relative gap between A and B point

gap is always positive.

$$\tan\psi_{AB} = \left(\frac{\Delta 1 - \Delta 2}{L}\right)$$

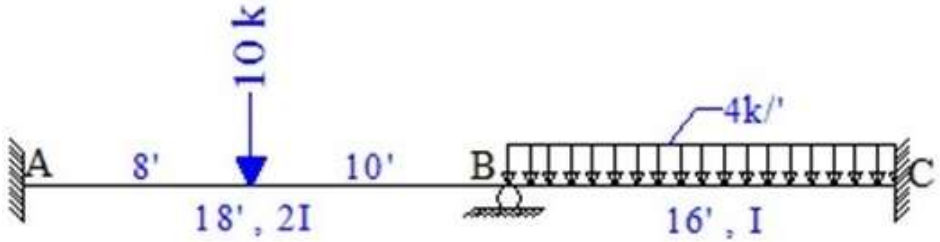
$$\psi_{AB} = \left(\frac{\Delta 1 - \Delta 2}{L}\right)$$

# SLOPE DEFLECTION METHOD

**Problem-1**

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding, use slope deflection method.**

**1. No translation $\Psi=0$**
**2. fix rotations as per the support type**



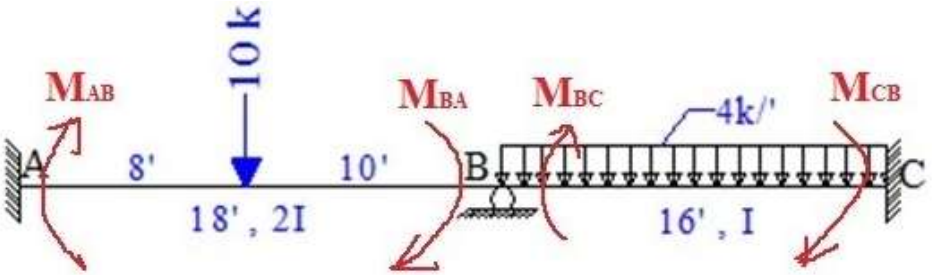

$\Theta_A = 0$ (fixed support)

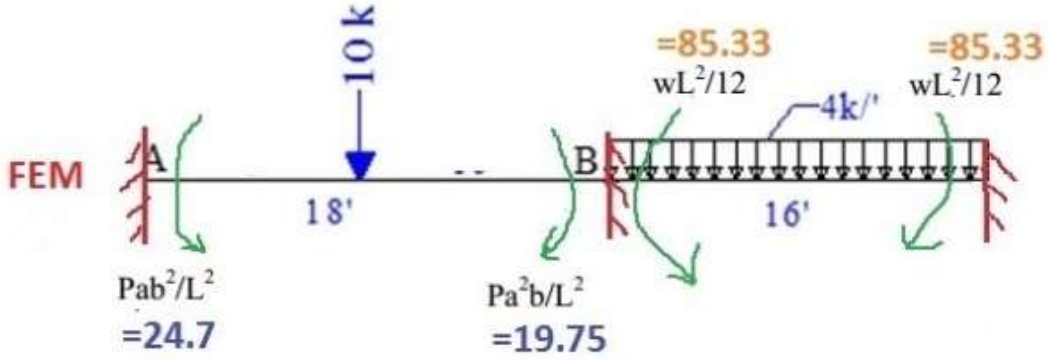
$\Theta_B$ = ?? (not zero)

$\Theta_C = 0$

$Y_{AB} = 0$

$Y_{BC} = 0$




$FEM_{AB} = -24.7$

$FEM_{BA} = +19.7$

$FEM_{BC} = -85.33$

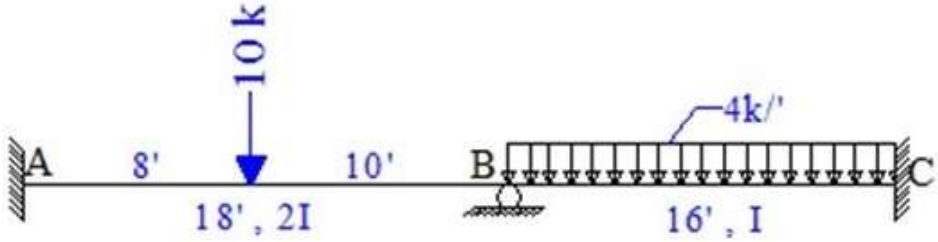
$FEM_{CB} = +85.33$

# SLOPE DEFLECTION METHOD

**Problem-1**

- Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding, use slope deflection method.

1. No translation $\Psi=0$
2. fix rotations as per the support type




$\Theta_A = 0$ (fixed support)

$\Theta_B =$ ?? (not zero)

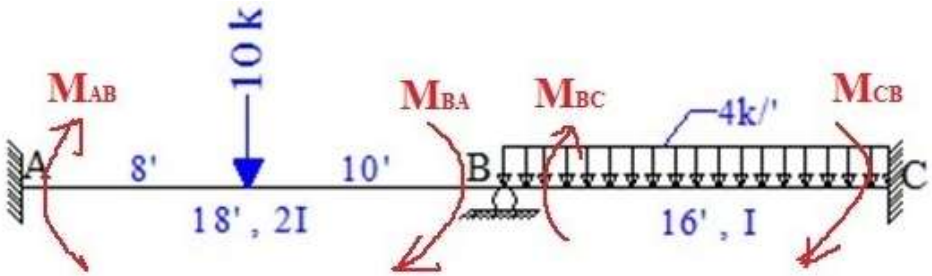
$\Theta_C = 0$

$Y_{AB} = 0$

$Y_{BC} = 0$

Consider point load = 20Kip and UDL acting is 8K/ft and rest of them remaining same as it is.

# SLOPE DEFLECTION METHOD




**Segment end moments**

$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB} \quad ; M_{AB} = (4EI/18)(\Theta_B) - 24.7 = 4EI\Theta_B/18 - 24.7$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA} \quad ; M_{BA} = 8EI\Theta_B/18 + 19.7$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC} \quad ; M_{BC} = 4EI\Theta_B/16 - 85.33$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB} \quad ; M_{CB} = 2EI\Theta_B/16 + 85.33$$

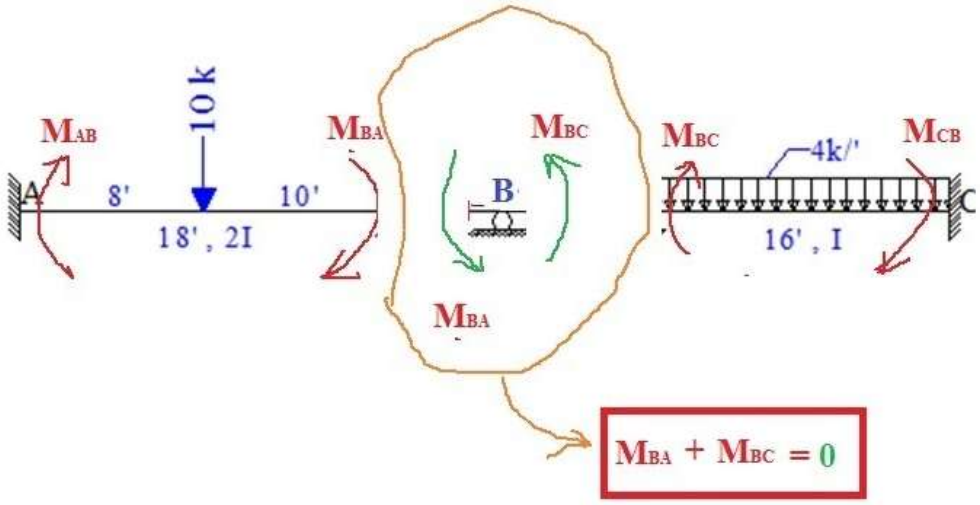
**Calculate $\Theta_B$ by joint moment equilibrium:**

$$M_{BA} + M_{BC} = 0$$

$$8EI\Theta_B/18 + 19.7 + 4EI\Theta_B/16 - 85.33 = 0$$

$$8EI\Theta_B/18 + 4EI\Theta_B/16 = 65.63$$

$$EI\Theta_B = +94.51$$



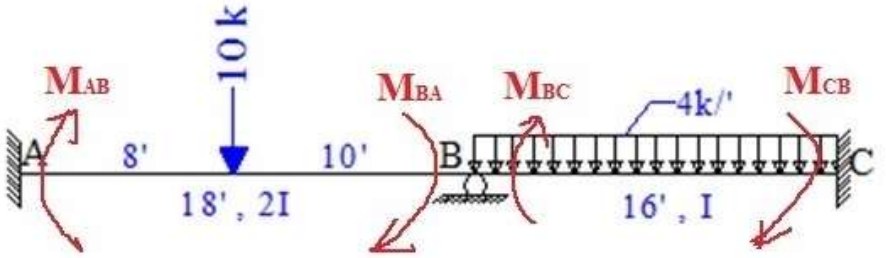
# SLOPE DEFLECTION METHOD

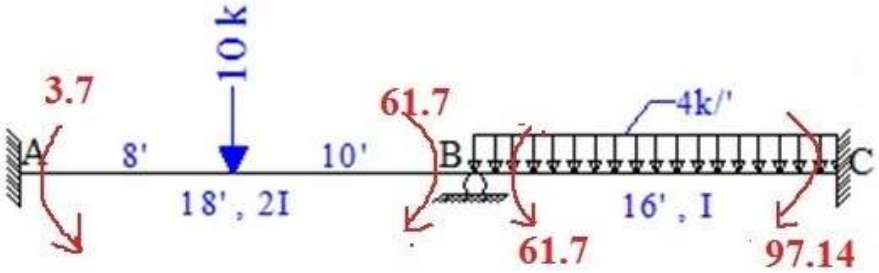
$M_{AB} = 4EI\Theta_B / 18 - 24.7 = -3.7$

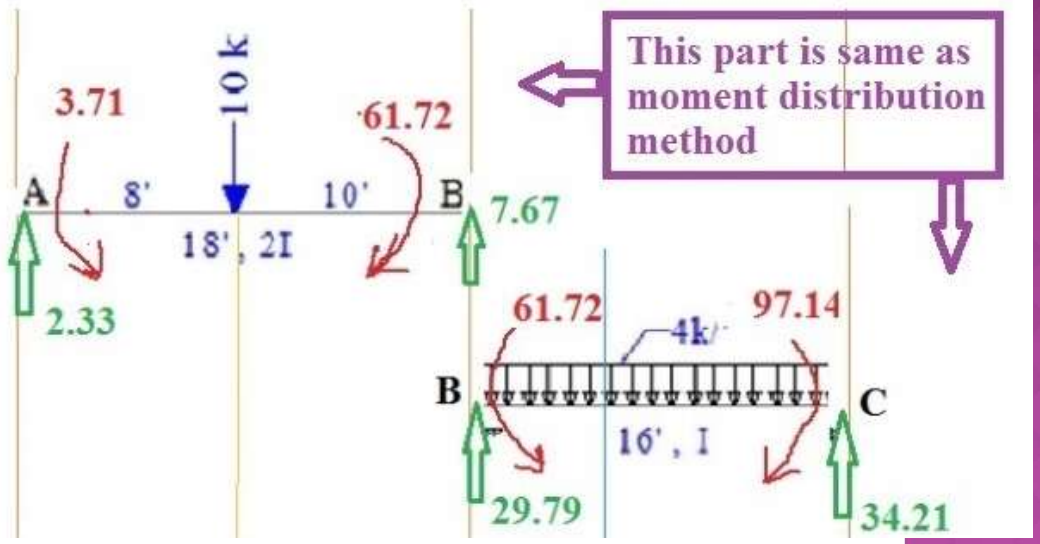
$M_{BA} = 8EI\Theta_B / 18 + 19.7 = +61.7$

$M_{BC} = 4EI\Theta_B / 16 - 85.33 = -61.7$

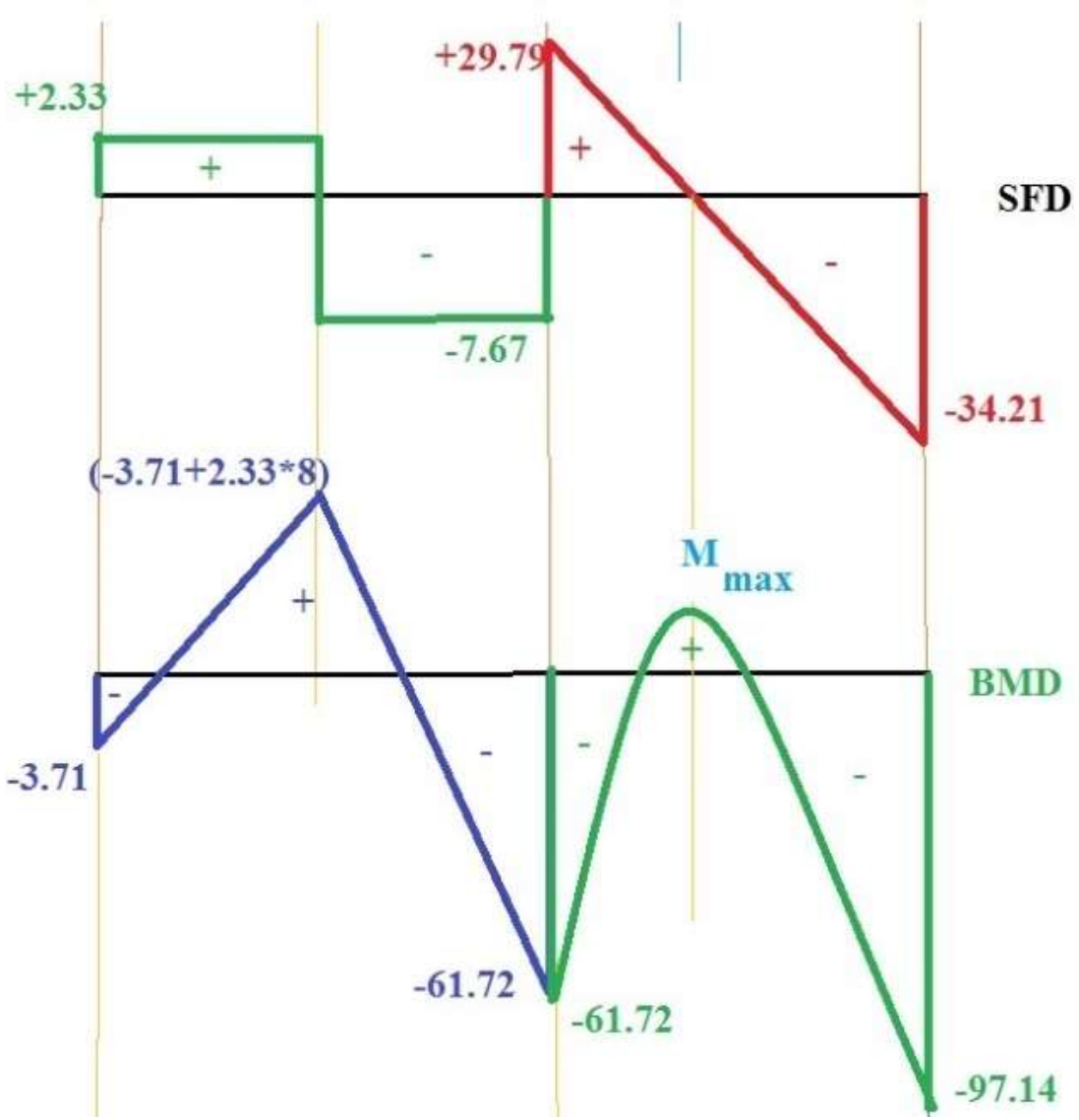
$M_{CB} = 2EI\Theta_B / 16 + 85.33 = +97.14$



# SLOPE DEFLECTION METHOD



# *Method of Slope Deflection (Beam Analysis)*

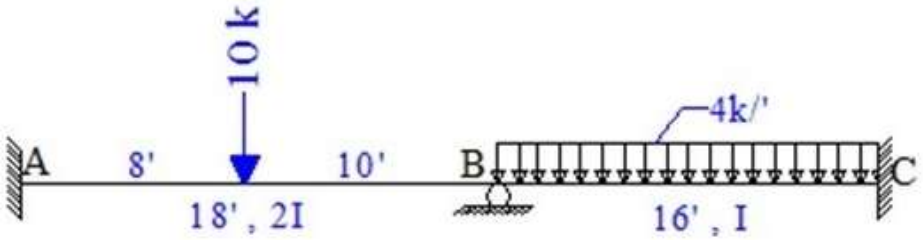
# SLOPE DEFLECTION METHOD

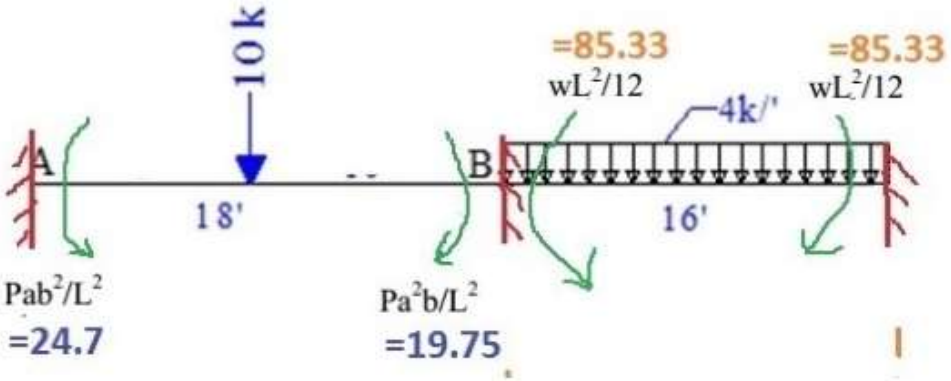
**Problem- 2**

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows. use slope deflection method.**

- **Support at A rotate clockwise 0.03 radian.**
- **Support at A moves upward 0.15 ft.**
- **Support at B settles down by 0.2 ft.**

**EI= 4500 K-ft²**




$FEM_{AB} = -24.7$

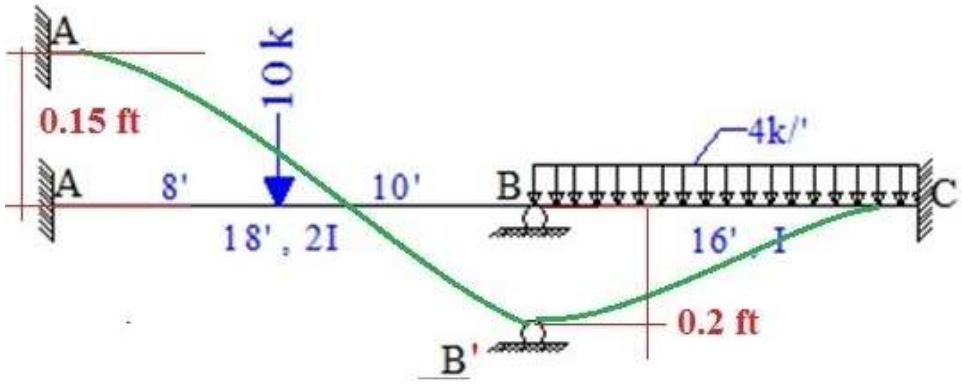
$FEM_{BA} = +19.7$

$FEM_{BC} = -85.33$

$FEM_{CB} = +85.33$

# SLOPE DEFLECTION METHOD

1. **Support at A rotate clockwise 0.03 radian.**
2. **Support at A moves upward 0.15 ft.**
3. **Support at B settles down by 0.2 ft.**




$\Theta_A$ = +0.03 rad [1]

$\Theta_B$ = ?? (not zero)

$\Theta_C$ = 0 (fixed support)

$$Y_{AB} = \frac{|\Delta_A - \Delta_B|}{L} = \frac{(0.15+0.2)}{18} = +0.0194$$

$$Y_{BC} = \frac{|\Delta_B - \Delta_C|}{L} = \frac{(0.2+0)}{16} = -0.011 \text{ ??}$$

$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$ (+0.03, +0.0194)

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$ (+0.03, +0.0194)

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$ (0, -0.011)

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$ (0, -0.011)

**if the deflected shape rotates clockwise then Y will be positive.** here, $Y_{AB}$ pisitive

**if the deflected shape rotates anticlockwise then Y will be negative.** here, $Y_{BC}$ negative

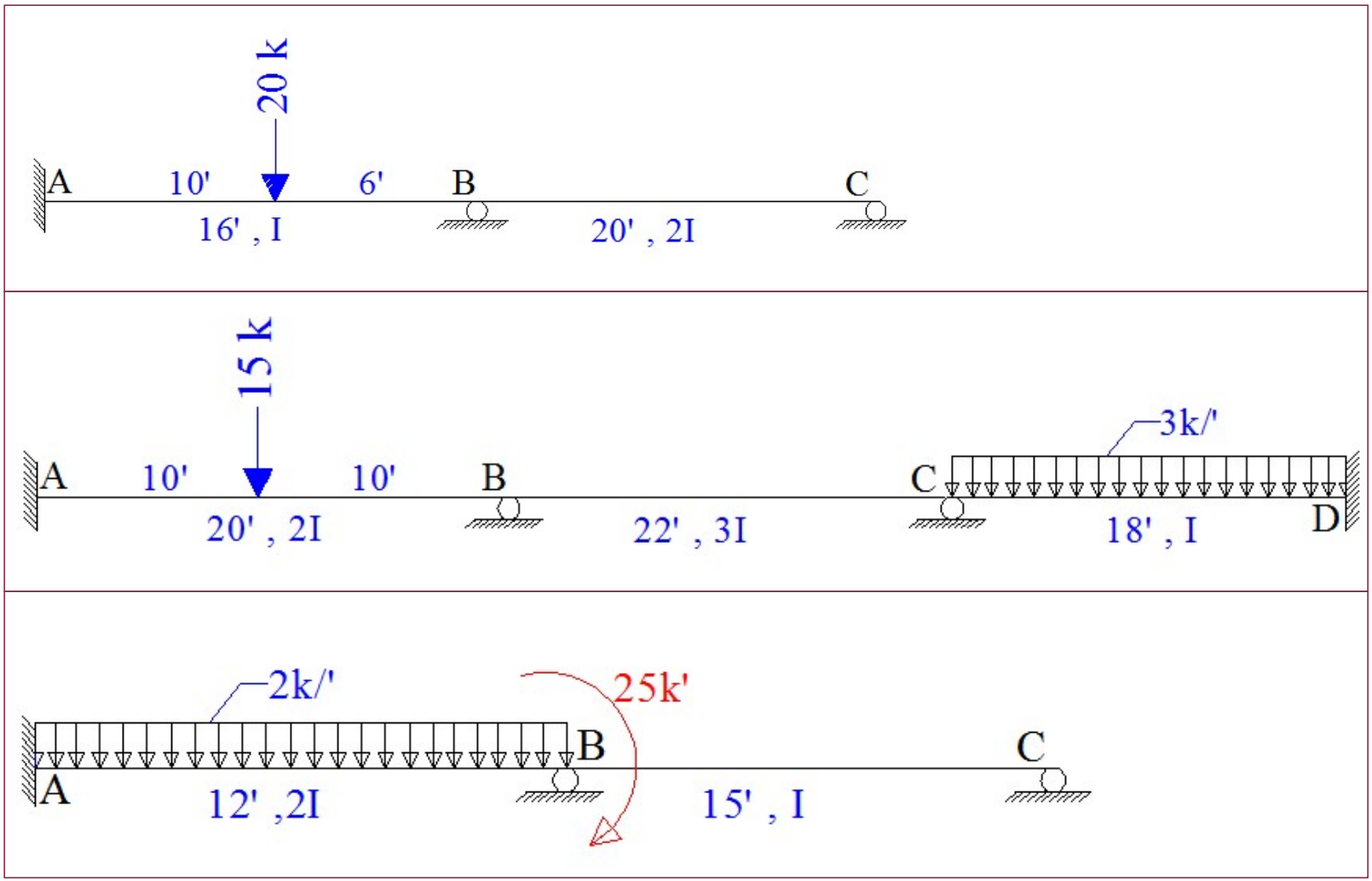
**when deflection (rotation, translation) will be given, must use I value**

**Calculate $\Theta_B$ by joint moment equilibrium:**

$M_{BA} + M_{BC} = 0$

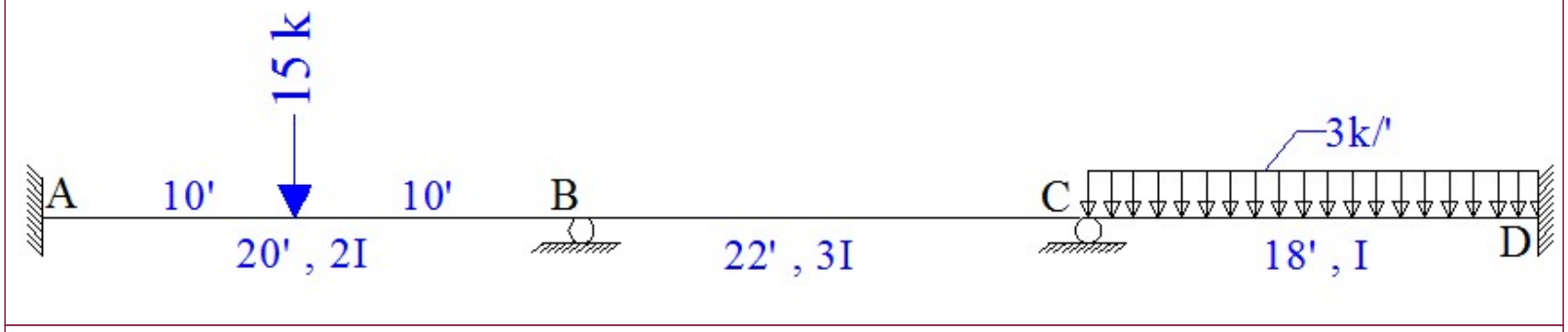
# SLOPE DEFLECTION METHOD EXERCISE

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding.**



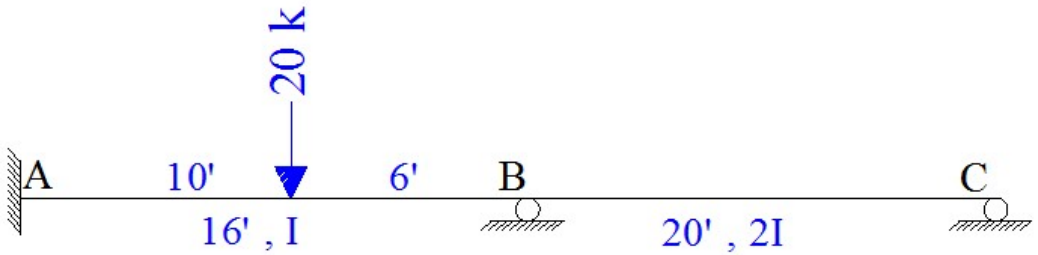
# SLOPE DEFLECTION METHOD EXERCISE

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding.**



# SLOPE DEFLECTION METHOD EXERCISE

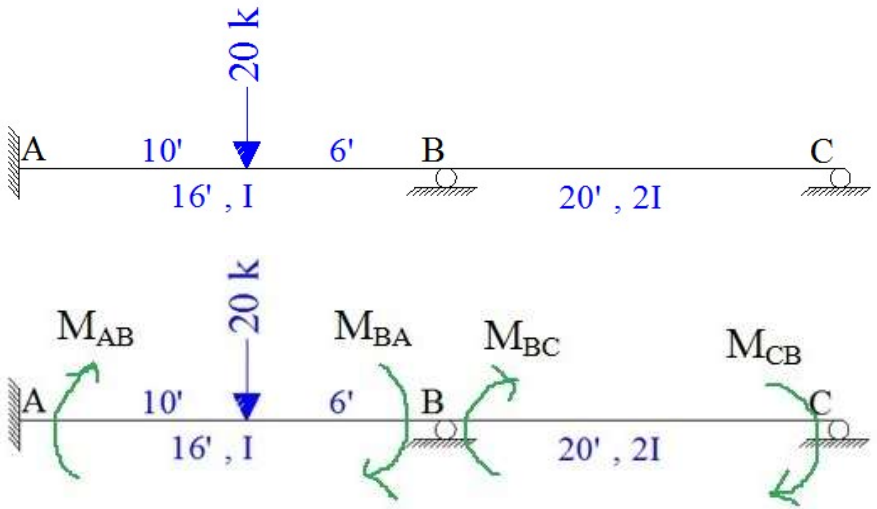
- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding.**



# SLOPE DEFLECTION METHOD EXERCISE

**Problem-3**

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports are unyielding.**




$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$

**calculate FEM**

$\Theta_A = 0$

$\Theta_B = ??$

$\Theta_C = ??$

$Y_{AB} = 0$

$Y_{BC} = 0$

$FEM_{AB} =$

$FEM_{BA} =$

$FEM_{BC} = 0$

$FEM_{CB} = 0$

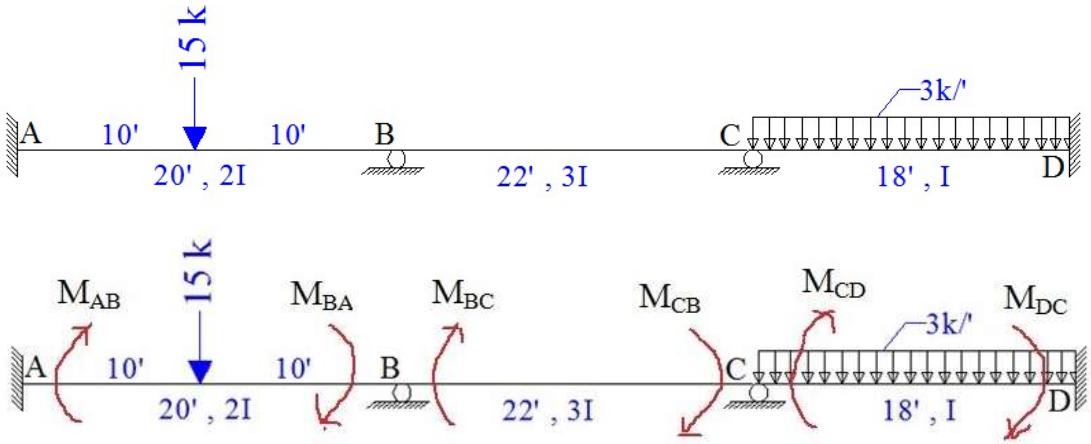
Calculate $\Theta_B$ , $\Theta_C$ by joint **moment** equilibrium:

joint B: $M_{BA} + M_{BC} = 0$

joint C: $M_{CB} = 0$

# SLOPE DEFLECTION METHOD EXERCISE

**Problem- 4**




$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$

$$M_{CD} = \frac{2EI}{L}(2\Theta_C + \Theta_D - 3Y_{CD}) + FEM_{CD}$$

$$M_{DC} = \frac{2EI}{L}(2\Theta_D + \Theta_C - 3Y_{CD}) + FEM_{DC}$$

$\Theta_A = 0$

$\Theta_B = ??$

$\Theta_C = ??$

$\Theta_D = 0$

$Y_{AB} = 0$

$Y_{BC} = 0$

$Y_{CD} = 0$

$FEM_{AB} =$

$FEM_{BA} =$

$FEM_{BC} = 0$

$FEM_{CB} = 0$

$FEM_{CD} =$

$FEM_{DC} =$

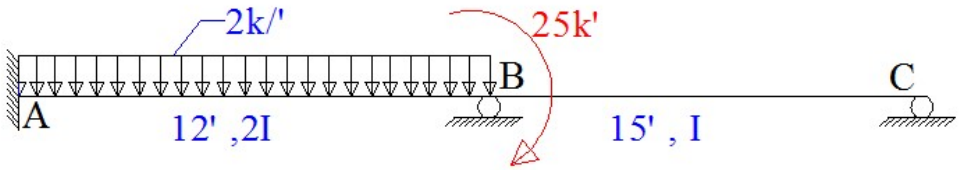
Calculate $\Theta_B$ , $\Theta_C$ by joint moment equilibrium:

joint B: $M_{BA} + M_{BC} = 0$

joint C: $M_{CB} + M_{CD} = 0$

# SLOPE DEFLECTION METHOD EXERCISE

**Problem- 5**

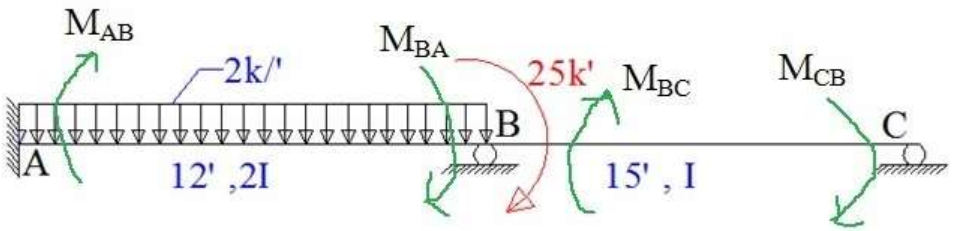



$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$




$\Theta_A = 0$

$\Theta_B = ??$

$\Theta_C = ??$

$Y_{AB} = 0$

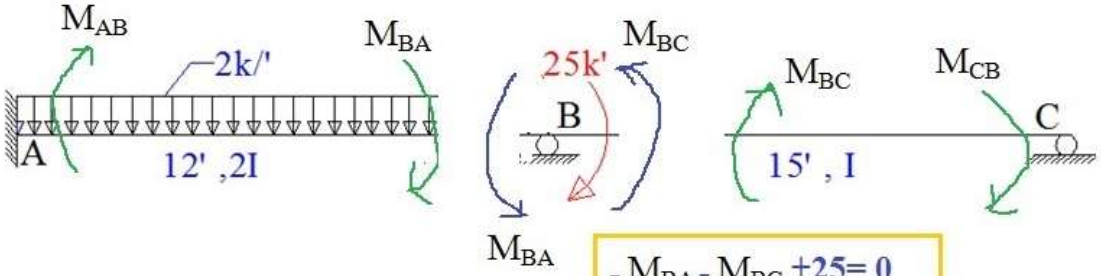
$Y_{BC} = 0$

$FEM_{AB} =$

$FEM_{BA} =$

$FEM_{BC} = 0$

$FEM_{CB} = 0$




$$-M_{BA} - M_{BC} + 25 = 0$$

$$M_{BA} + M_{BC} = +25$$

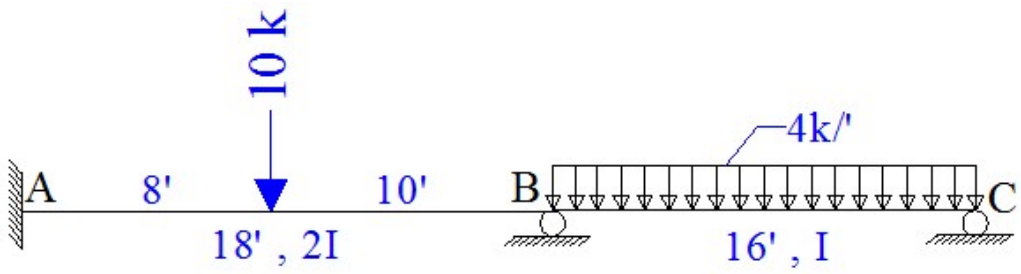
**Calculate $\Theta_B$ , $\Theta_C$ by joint moment equilibrium:**

**joint B:** $M_{BA} + M_{BC} = +25$

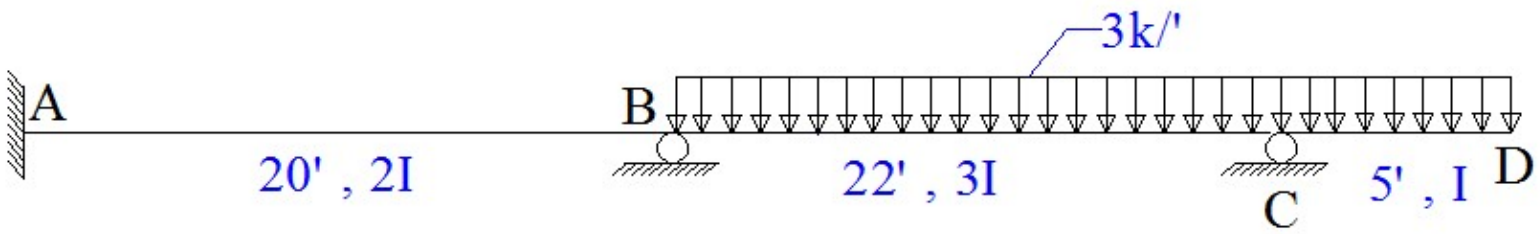
**joint C:** $M_{CB} = 0$

# SLOPE DEFLECTION METHOD EXERCISE

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows:**




- **Support at A rotate clockwise 0.03 radian.**
- **Support at B settles down by 0.2 ft.**

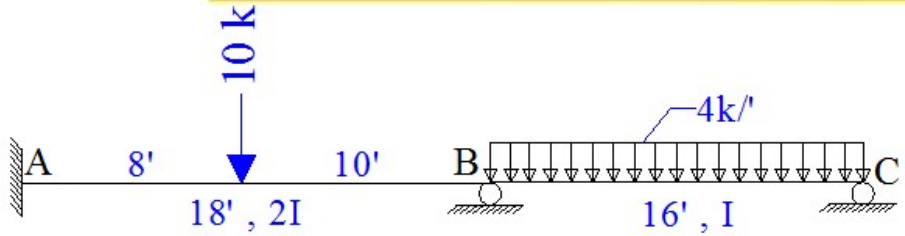


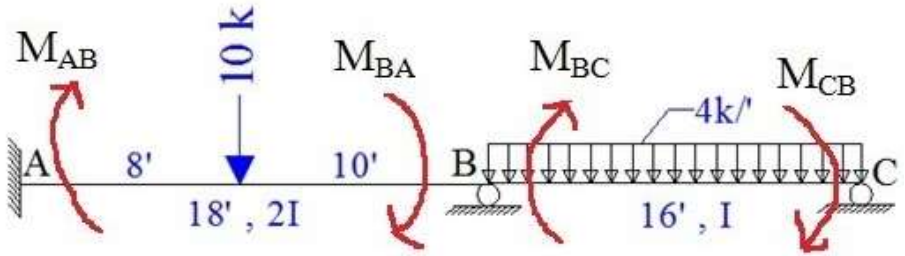

- **Support at C moves upward by 0.15 ft.**

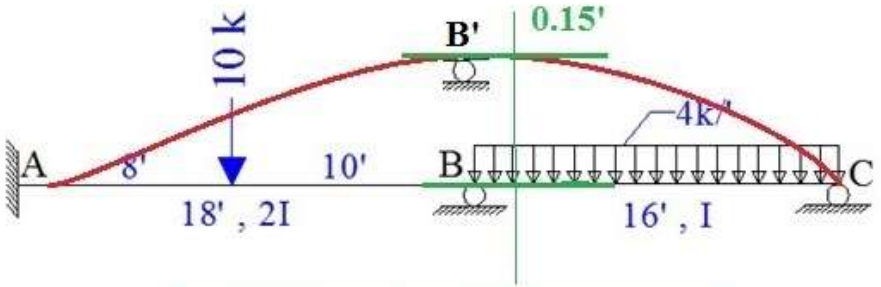
# SLOPE DEFLECTION METHOD EXERCISE

**Problem-6**

- Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows: EI= 5000 k-ft$^2$




Support B moves upward 0.15 ft

- Support at A rotate anti-clockwise 0.02 radian.
- Support at B moves upward by 0.15 ft.

$\Theta_A$ = - 0.02 radian

$\Theta_B$ = ??

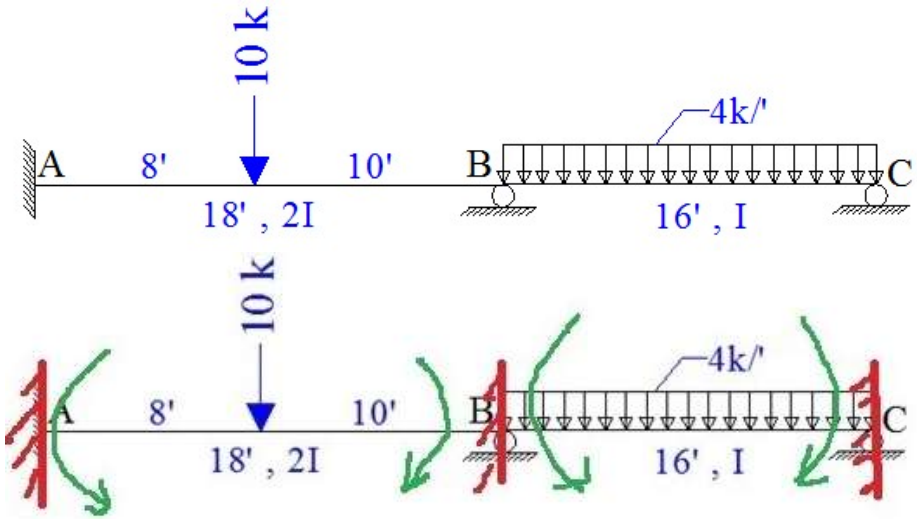
$\Theta_C$ = ??

$$Y_{AB} = -\frac{0.15}{18}$$

$$Y_{BC} = +\frac{0.15}{16}$$

# SLOPE DEFLECTION METHOD EXERCISE

**Problem-6**




$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$

**FEM**

$FEM_{AB}$ =

$FEM_{BA}$ =

$FEM_{BC}$ =

$FEM_{CB}$ =

Calculate $\Theta_B$ , $\Theta_C$ by joint moment equilibrium:

joint B: $M_{BA} + M_{BC} = 0$

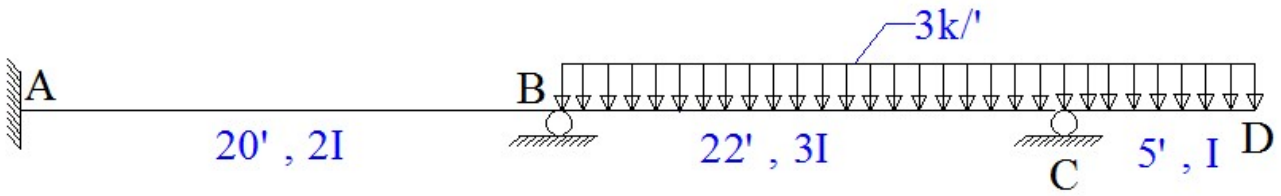
joint C: $M_{CB} = 0$

# SLOPE DEFLECTION METHOD EXERCISE

**Problem-7**

- Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows: EI= 5000 k-ft$^2$

➢ Support at C moves upward by 0.15 ft.



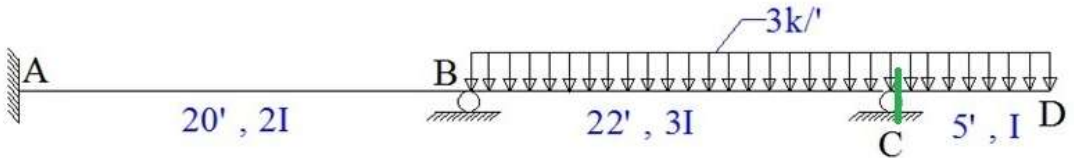

1. How many segments?
2. How many segment end moments?
3. What will happen to CD segment?
4. Solution of this problem by moment distribution method.

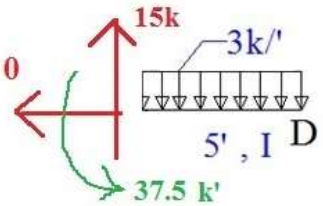
# SLOPE DEFLECTION METHOD EXERCISE

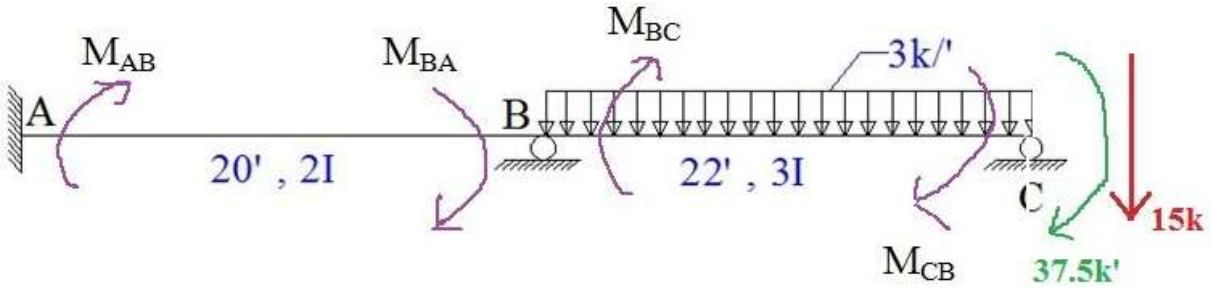
**Problem-7**

- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows:** $EI = 5000\ k\text{-}ft^2$

➢ **Support at C moves upward by 0.15 ft.**



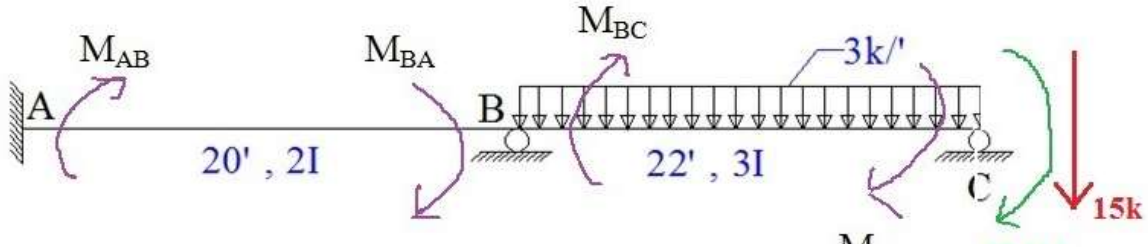



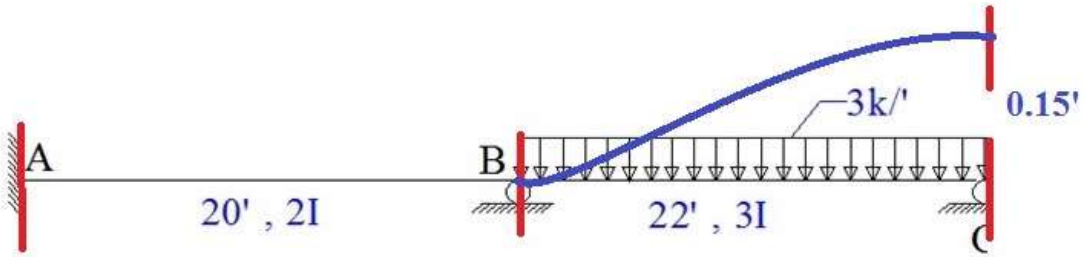




**In an indeterminate problem: if any cantilever part is there, that part is a determinate one.**
**So, split the cantilever portion from the indeterminate part for solution.**

# SLOPE DEFLECTION METHOD EXERCISE

**Problem-7**




$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$

**Support at C moves upward by 0.15 ft.**

$\Theta_A = 0$

$\Theta_B = ??$

$\Theta_C = ??$

$Y_{AB} = 0$

$Y_{BC} = -0.15/22$

$FEM_{AB} = 0$

$FEM_{BA} = 0$

$FEM_{BC} = ?$

$FEM_{CB} = ?$

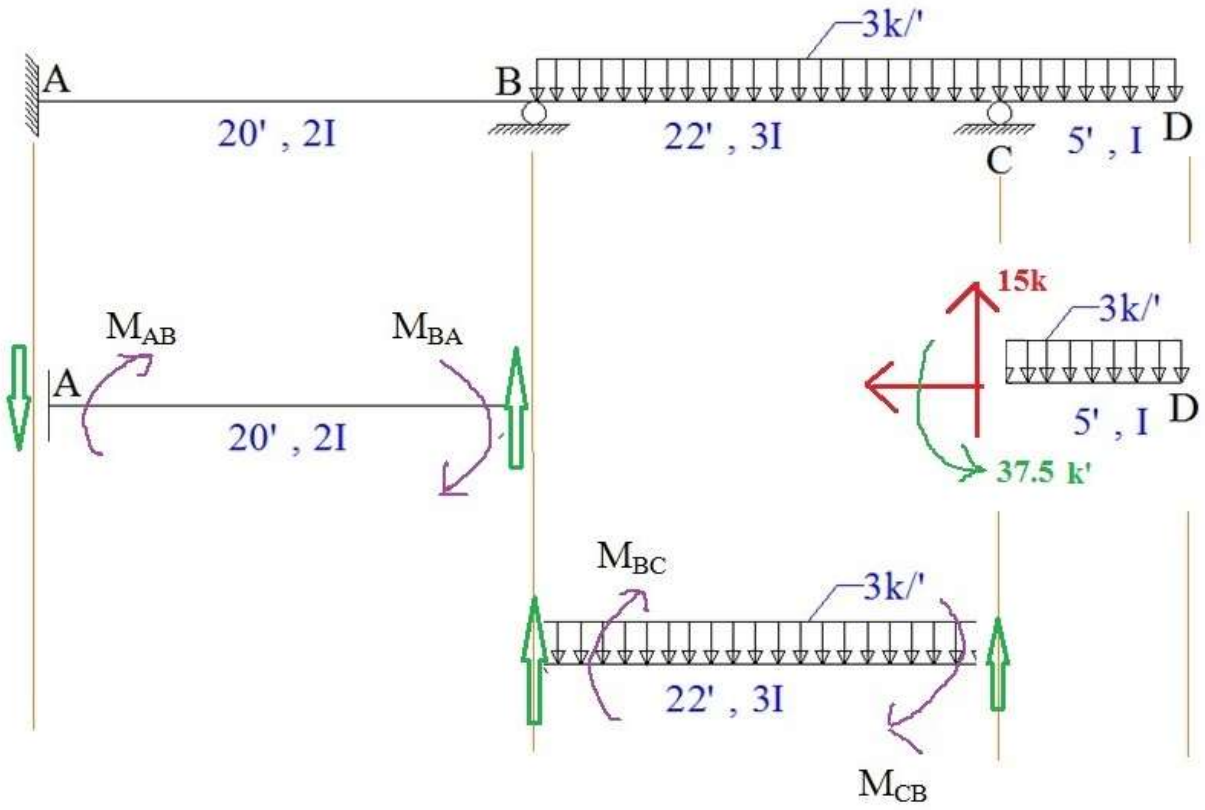
Calculate $\Theta_B$ , $\Theta_C$ by joint moment equilibrium:

joint B: $M_{BA} + M_{BC} = 0$

joint C: $M_{CB} = +37.5$

# SLOPE DEFLECTION METHOD EXERCISE

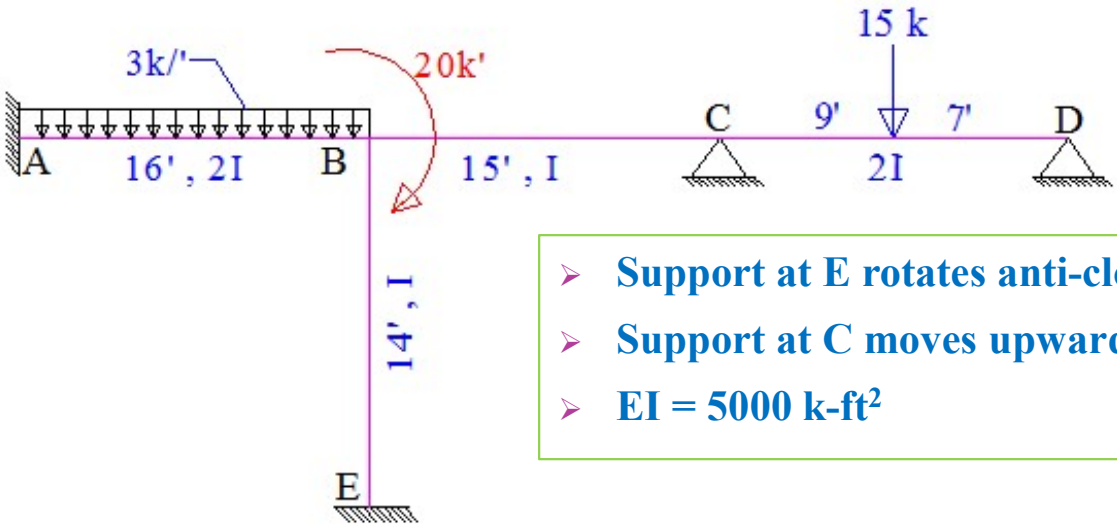
**Problem-7**



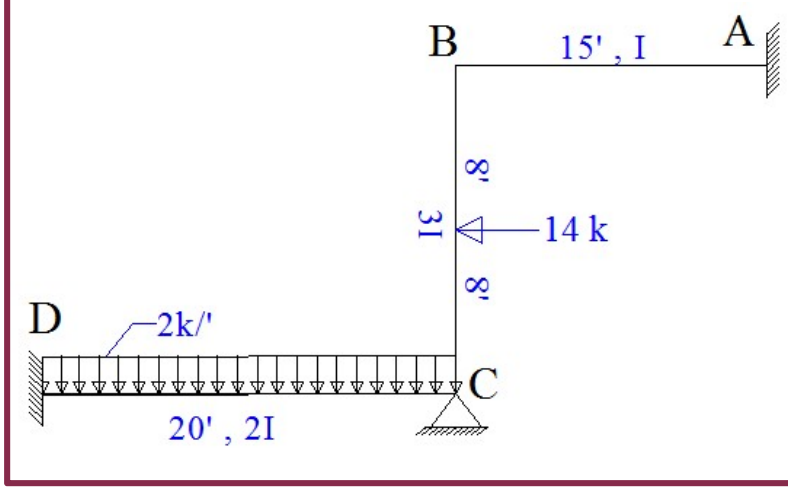
# *Method of Slope Deflection (Frame Analysis)*

# SLOPE DEFLECTION METHOD EXERCISE

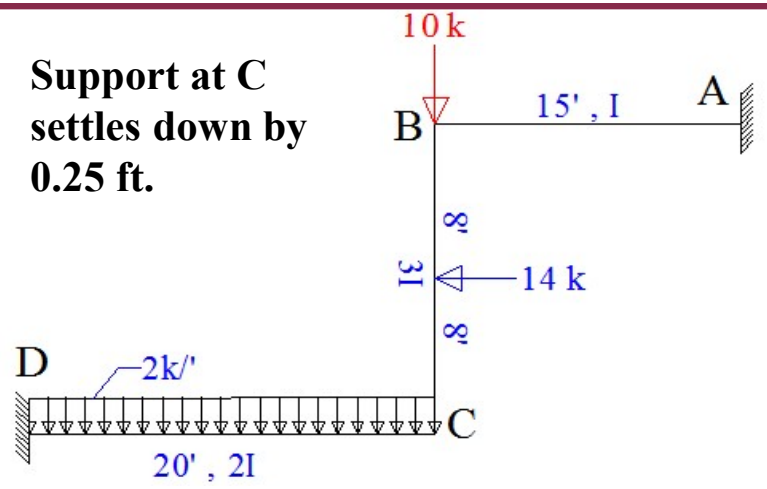
- **Compute the reactions and draw shear and bending moment diagram for the beam. Supports yield as follows:**




- **Support at E rotates anti-clockwise 0.01 radian.**
- **Support at C moves upward by 0.15 ft.**
- **EI = 5000 k-ft$^2$**




**Support at C settles down by 0.25 ft.**

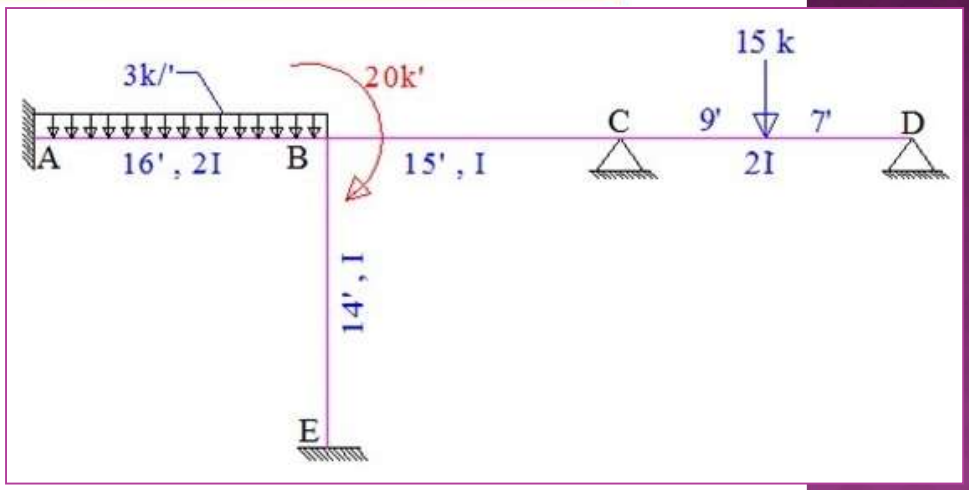


# SLOPE DEFLECTION METHOD EXERCISE

**Problem-8:**

- **Compute the reactions and draw shear and bending moment diagram for the Frame. Supports yield as follows:**

- **Support at E rotates clockwise 0.01 radian.**
- **Support at C settles down by 0.1 ft.**
- **EI = 5000 k-ft²**




$\Theta_A = 0$ $\quad \psi_{AB} = 0$

$\Theta_B = ??$ $\quad \psi_{BC} =$

$\Theta_C = ??$

$\Theta_D = ??$ $\quad \psi_{CD} =$

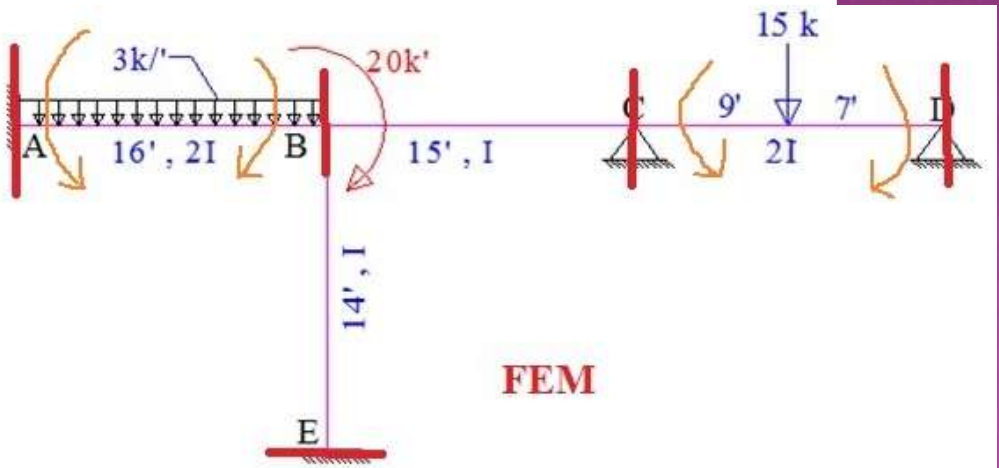
$\Theta_E = +0.01$ $\quad \psi_{BE} = 0$

$FEM_{AB} =$ $\quad FEM_{CD} =$

$FEM_{BA} =$ $\quad FEM_{DC} =$

$FEM_{BC} =$ $\quad FEM_{BE} =$

$FEM_{CB} =$ $\quad FEM_{EB} =$



# SLOPE DEFLECTION METHOD EXERCISE

**Problem-8:**

$\Theta_A = 0$

$\Theta_B = ??$

$\Theta_C = ??$

$\Theta_D = ??$

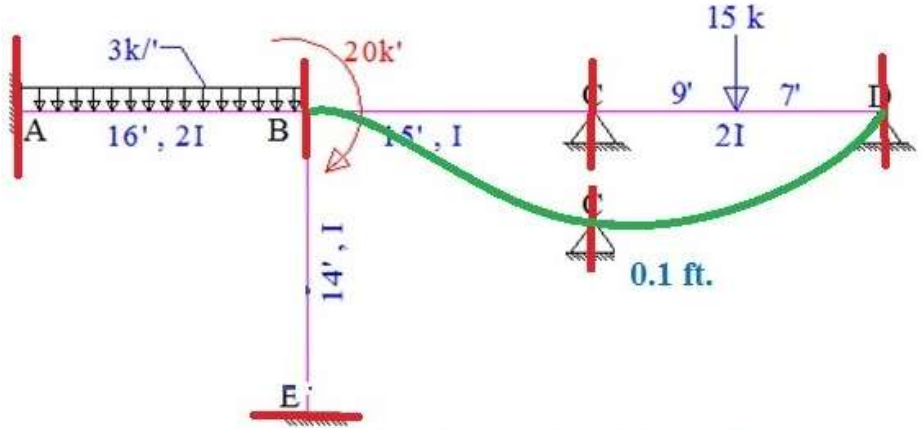
$\Theta_E = +0.01$ rad.

$Y_{AB} = 0$

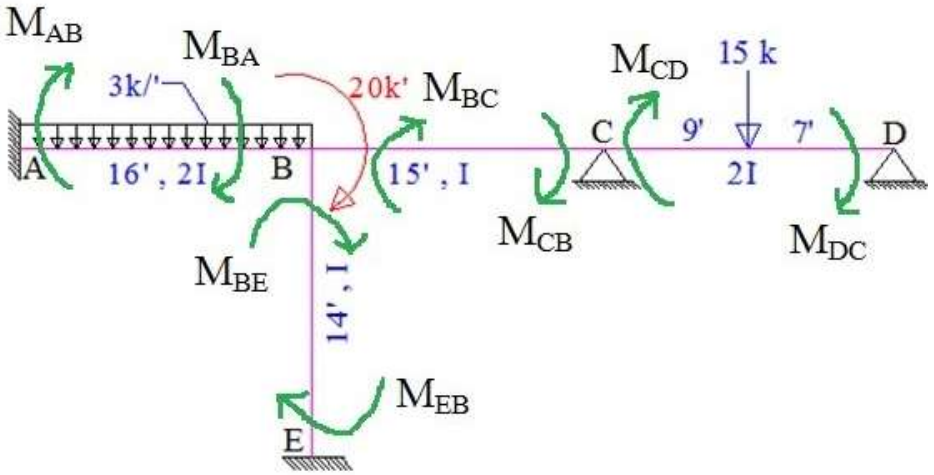
$Y_{BC} = +\frac{0.1}{15}$

$Y_{CD} = -\frac{0.1}{16}$

$Y_{BE} = 0$




Support at C settles down by 0.1 ft.




$$M_{AB} = \frac{2EI}{L}(2\Theta_A + \Theta_B - 3Y_{AB}) + FEM_{AB}$$

$$= 4EI\,\Theta_B/16 - (\quad)$$

$$M_{BA} = \frac{2EI}{L}(\Theta_A + 2\Theta_B - 3Y_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\Theta_B + \Theta_C - 3Y_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(\Theta_B + 2\Theta_C - 3Y_{BC}) + FEM_{CB}$$

$$M_{CD} = \frac{2EI}{L}(2\Theta_C + \Theta_D - 3Y_{CD}) + FEM_{CD}$$

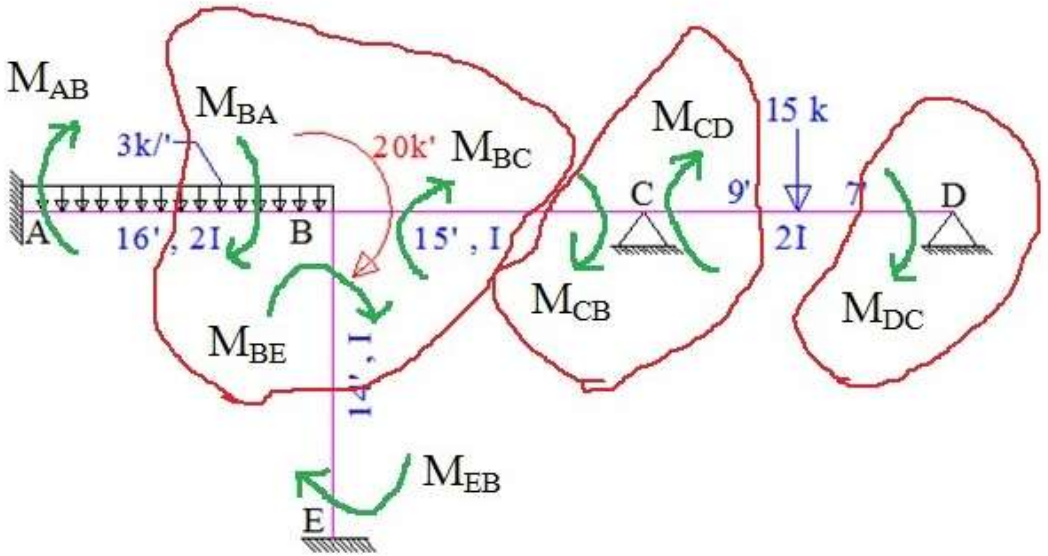
$$M_{DC} = \frac{2EI}{L}(2\Theta_D + \Theta_C - 3Y_{CD}) + FEM_{DC}$$

$$M_{BE} = \frac{2EI}{L}(2\Theta_B + \Theta_E - 3Y_{BE}) + FEM_{BE}$$

$$M_{EB} = \frac{2EI}{L}(2\Theta_E + \Theta_B - 3Y_{BE}) + FEM_{EB}$$

# SLOPE DEFLECTION METHOD EXERCISE

**Problem-8:**




Calculate $\Theta_B, \Theta_C, \Theta_D$ by joint moment equilibrium:

**Joint B,** $M_{BA} + M_{BC} + M_{BE} = 20$

**Joint C,** $M_{CB} + M_{CD} = 0$

**Joint D,** $M_{DC} = 0$

# SLOPE DEFLECTION METHOD

**SUMMARY:**

- Indeterminate structure/ Fixed End Moment (applied load +rotation +translation) / Method of Slope Deflection/ Slope Deflection Equations
- Analysis of indeterminate beams
- Analysis of indeterminate frames
- Analysis of indeterminate truss – Not possible.
- Reference: ***Article 9.5, Chapter-09, "Elementary Structural Analysis and Design", Wilbur & Norris, Fourth Edition.***