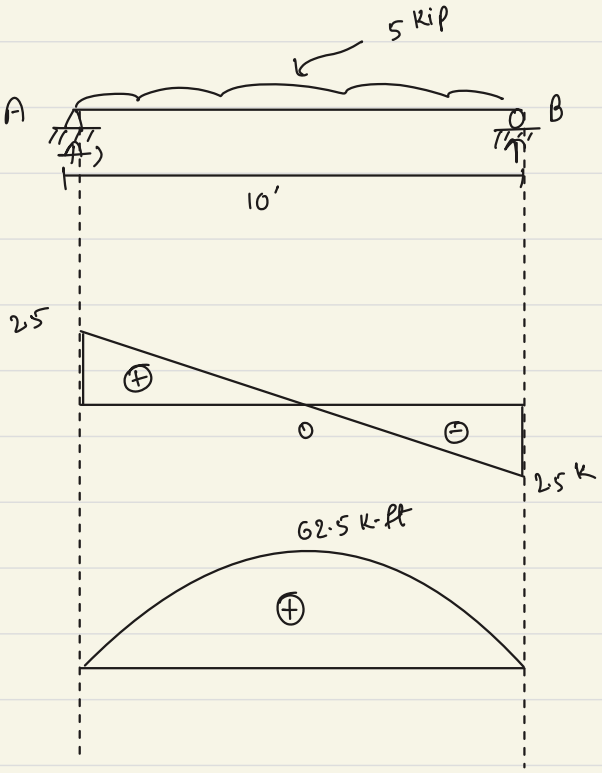
EWCE 411 - SB

Indeterminate structure

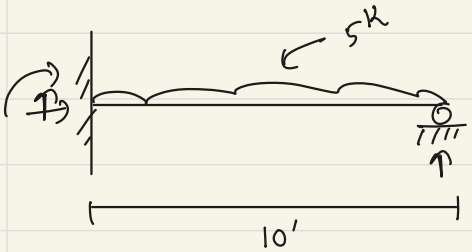


$\overset{+}{\uparrow} \Sigma F_y = 0 \Rightarrow R_{Ay} - 5 \times 10 + R_{By} = 0 \Rightarrow R_{By} = 50 - R_{Ay}$

$\overset{\circlearrowleft +}{\Sigma M_A = 0} \Rightarrow 5 \times 10 \times 5 - 10 R_{By} = 0$

$\Rightarrow 250 = 500 - 10 R_{Ay} \Rightarrow R_{Ay} = 2.5^K$

$R_{By} = 2.5^K$

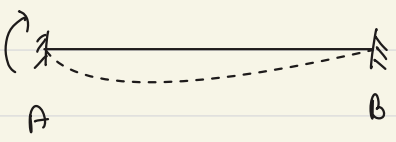


$DOI = (3+1) - 3 = 1^\circ$

$R + 3m - 3j - R_n = 4 + 3 \times 1 - 3 \times 2 - 0 = 1^\circ$

Moment Distribution method — Upto 3° DOI (manually)

with assumptions (joint rigid/fixed, ...)

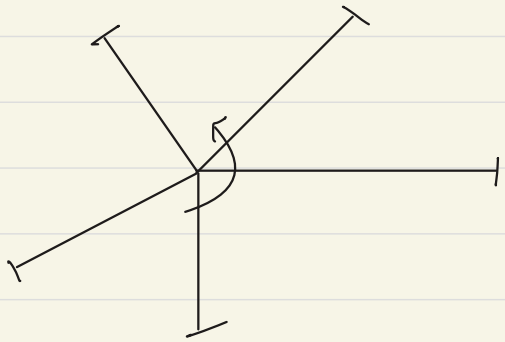


Stiffness

$$K = \frac{4EI}{L}$$

$$m_{AB} = K\theta$$

$$= \frac{4EI}{L}(\theta_A)$$



$$M = m_{OA} + m_{OB} + m_{OC} + m_{OD} + m_{OE}$$

$$= \theta(K_{OA} + K_{OB} + \cdots + K_{OE})$$

$$= \theta \Sigma K$$

$$m_{OA} = \theta K_{OA}$$

$$\frac{m_{OA}}{M} = \frac{\theta K_{OA}}{\theta \Sigma K}$$

Distribution factor (D.F)

$$\Rightarrow m_{OA} = \boxed{\frac{K_{OA}}{\Sigma K}} * M$$

Distributed moment

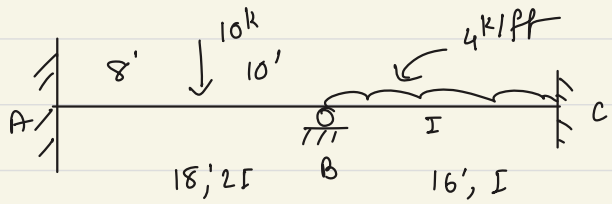
$$m_{OB} = \boxed{\frac{K_{OB}}{\Sigma K} * M}$$

$$DF_{AB} = \frac{\frac{4\times 2I}{18}}{\frac{4\times 2I}{18} + \frac{4I}{16}}$$

$$= 0.64$$

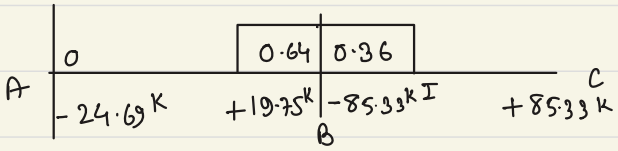
$$DF_{BC} = \frac{\frac{4\times I}{18}}{\frac{4\times 2I}{18} + \frac{4I}{16}}$$

$$= 0.36$$



D.f for fixed support = 0

D.f for pin support = 1



$$R_{Ay} = \frac{Pab^2}{L^2} = \frac{10\times 8\times 10^2}{18^2}$$

$$= 24.69^{k} \quad (-)$$

$$R_{By_L} = \frac{Pba^2}{L^2} = \frac{10\times 10\times 8^2}{18^2}$$

$$= 19.75^{k} \quad (+)$$

$$R_{By_R} = \frac{WL^2}{12} = \frac{4\times 16^2}{12} = 85.33^{k} \quad (-)$$

$$R_{Cy} = \frac{WL^2}{12} = 85.33^{k} \quad (+)$$

Moment distribution method

$$DF_{AB} = \frac{\frac{4\times 2I}{18}}{\frac{4\times 2I}{18} + \frac{4I}{16}}$$

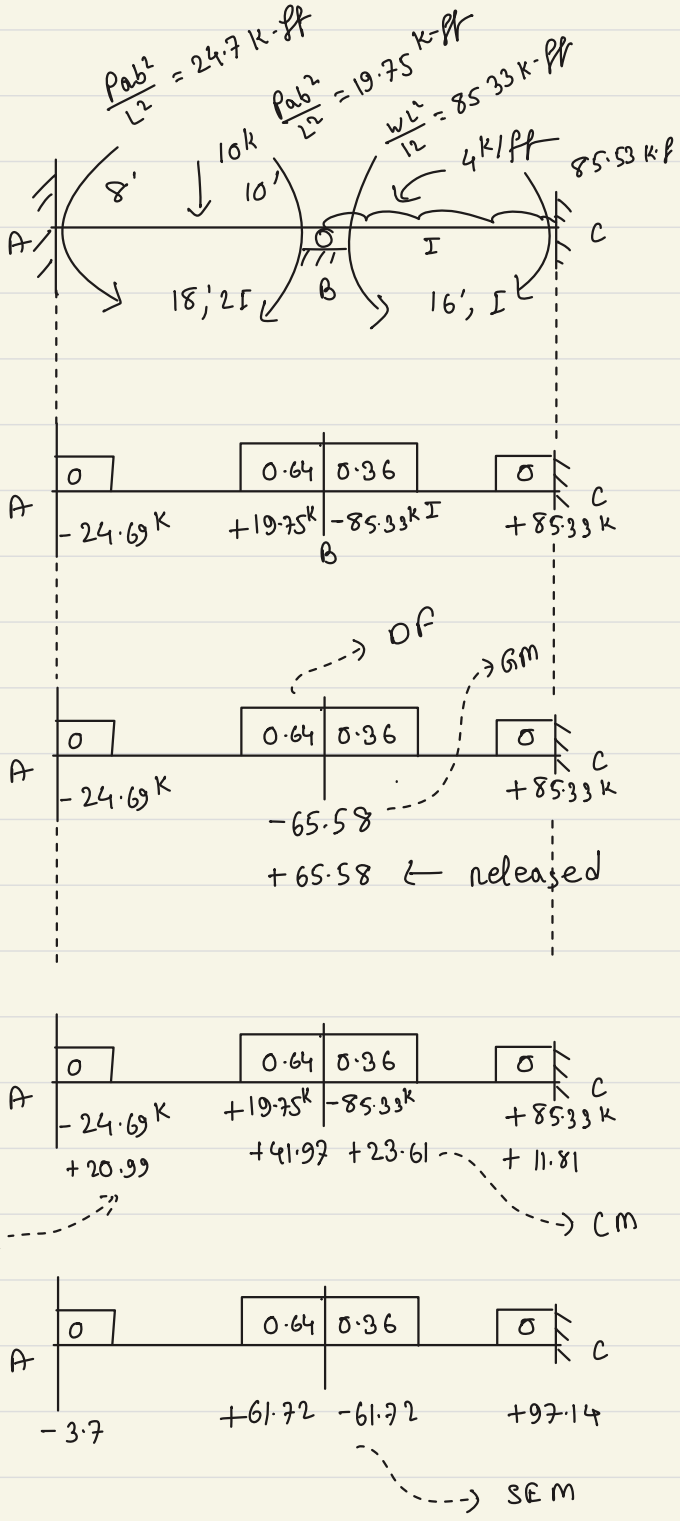
$$= 0.64$$

$$DF_{BC} = \frac{\frac{4\times I}{18}}{\frac{4\times 2I}{18} + \frac{4I}{16}}$$

$$= 0.36$$

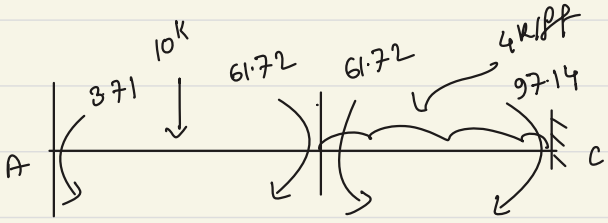
Interconnecting joint → B

Governing moment → Release (−1 × sign)



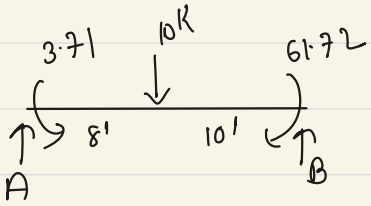
$\overset{\curvearrowright +}{\Sigma M_A = 0} \Rightarrow -3.71 + 80 + 61.72 - 18V_B = 0$

$\Rightarrow V_B = 7.67 \text{ Kip}$



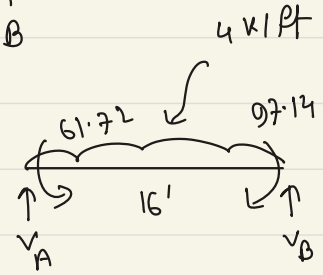
$\overset{\uparrow +}{\Sigma F_y = 0} \Rightarrow V_A - 10 + 7.67 = 0$

$\Rightarrow V_A = 2.33 \text{ Kip}$



$\overset{\curvearrowright +}{\Sigma M_A} \Rightarrow -61.72 + 4 \times 16 \times 8 + 97.14 + 16V_B = 0$

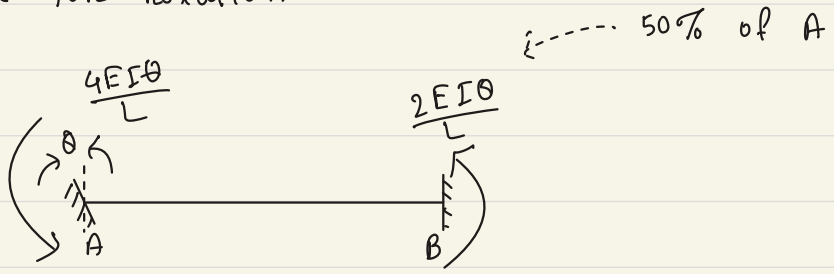
$\Rightarrow V_B = 34.21 \text{ Kip}$



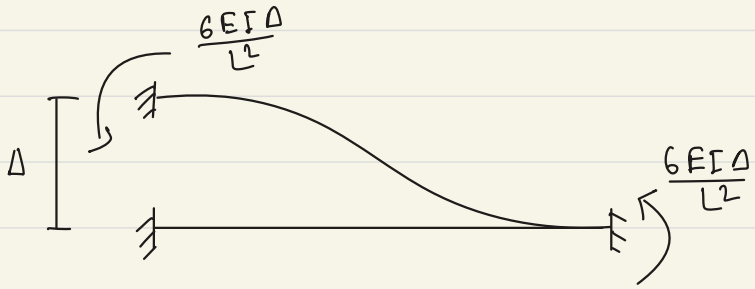
$\overset{\uparrow +}{\Sigma F_y = 0} \Rightarrow V_A - 4 \times 16 + 34.21$

$\Rightarrow V_A = 29.79 \text{ Kip}$

Moment for rotation



Moment for translation $\left(\frac{6EI\Delta}{L^2}\right)$

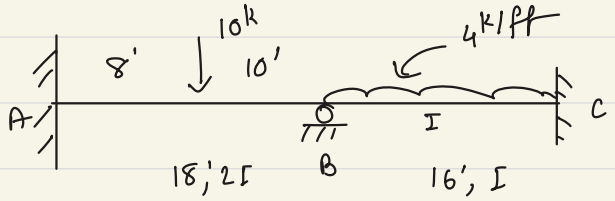


Support A rotates 0.03^r ↷

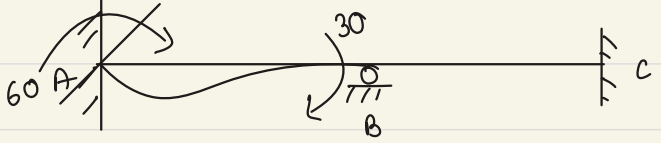
A moves 0.15' ↑

B settles 0.2' ↓

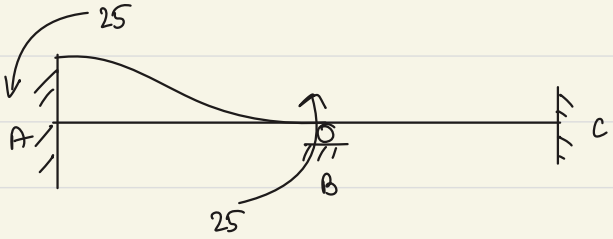
$EI = 4500\ k\text{-}ft^2$



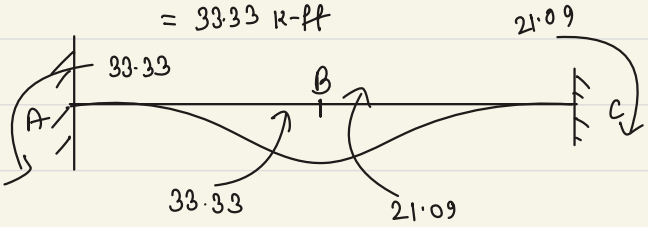
$\frac{4EI\theta}{L} = \frac{4 \times 4500 \times 2 \times 0.03}{18} = 60\ k\text{-}ft$



$\frac{6EI\Delta}{L^2} = \frac{6 \times 4500 \times 2 \times 0.15}{18^2} = 25\ k\text{-}ft$



$\frac{6EI\Delta}{L^2} = \frac{6 \times 4500 \times 2 \times 0.2}{18^2} = 33.33\ k\text{-}ft$; $\frac{6 \times 4500 \times 0.2}{16^2} = 21.09\ k\text{-}ft$



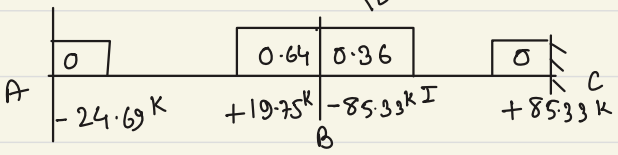
$$DF_{AB} = \frac{\frac{4 \times 2I}{18}}{\frac{4 \times 2I}{18} + \frac{4I}{16}}$$

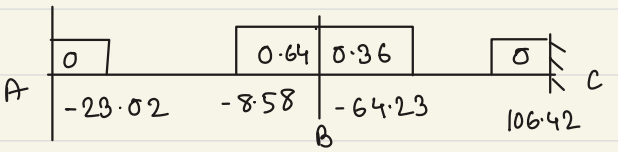
$$= 0.64$$

$$DF_{BC} = \frac{\frac{4 \times I}{18}}{\frac{4 \times 2I}{18} + \frac{4I}{16}}$$

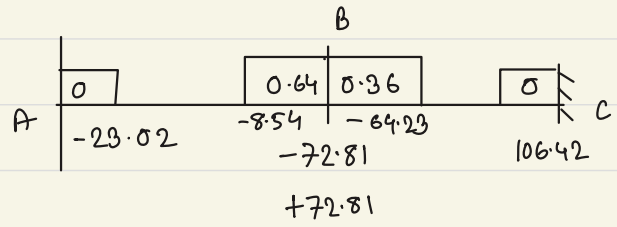
$$= 0.36$$

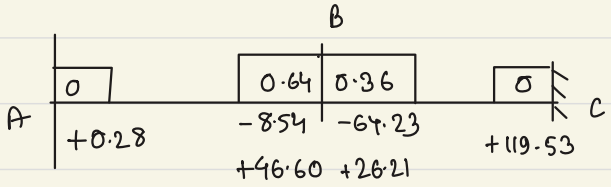
$\frac{Pab^2}{L^2} = 24.7\ k\text{-}ft$ $\frac{Pab^2}{L^2} = 19.75\ k\text{-}ft$ $\frac{wL^2}{12} = 85.33\ k\text{-}ft$

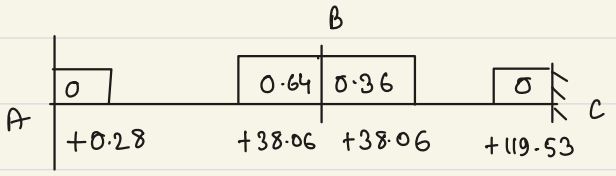




CT - Next cls
After 1hr cls

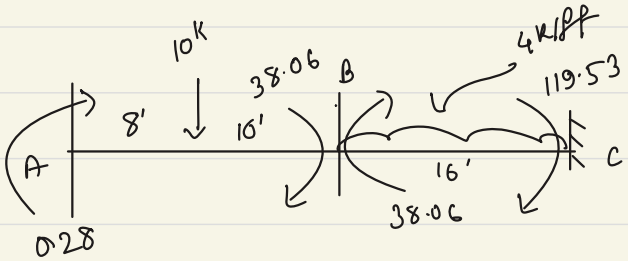






$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \Rightarrow 0.28 + 8 \times 10 + 38.06 - 18 V_B = 0$

$\Rightarrow V_B = 6.57\ K$



$\overset{+}{\uparrow} \Sigma F_y = 0 \Rightarrow V_A - 10 + 6.57 = 0$

$\Rightarrow V_A = 3.43\ K$

$\overset{+}{\circlearrowleft} \Sigma M_B = 0 \Rightarrow 38.06 + 4 \times 16 \times 8 + 119.53 - 16 V_C = 0$

$\Rightarrow V_C = 41.85\ K$

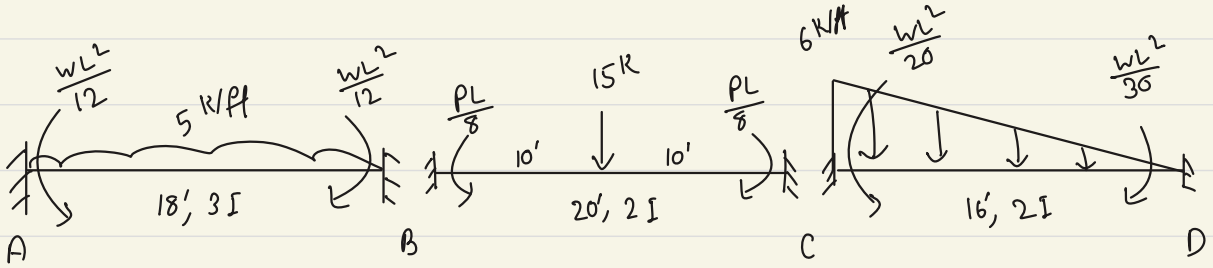
$\overset{+}{\uparrow} \Sigma F_y = 0 \Rightarrow V_B - 4 \times 16 + 41.85 = 0$

$\Rightarrow V_B = 22.15\ K$

LO4

16/07

Problem 3:



$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{\frac{4EX3I}{18}}{\frac{4EX3I}{18} + \frac{4EX2I}{20}} = 0.625$$

$$D_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{\frac{4EX2I}{20}}{\frac{4EX3I}{18} + \frac{4EX2I}{20}} = 0.375$$

$0.625 + 0.375 = 1$

$$D_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{\frac{4EX2I}{20}}{\frac{4EX2I}{20} + \frac{4EX2I}{16}} = 0.444$$

$$D_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD}} = \frac{\frac{4EX2I}{16}}{\frac{4EX2I}{20} + \frac{4EX2I}{16}} = 0.555$$

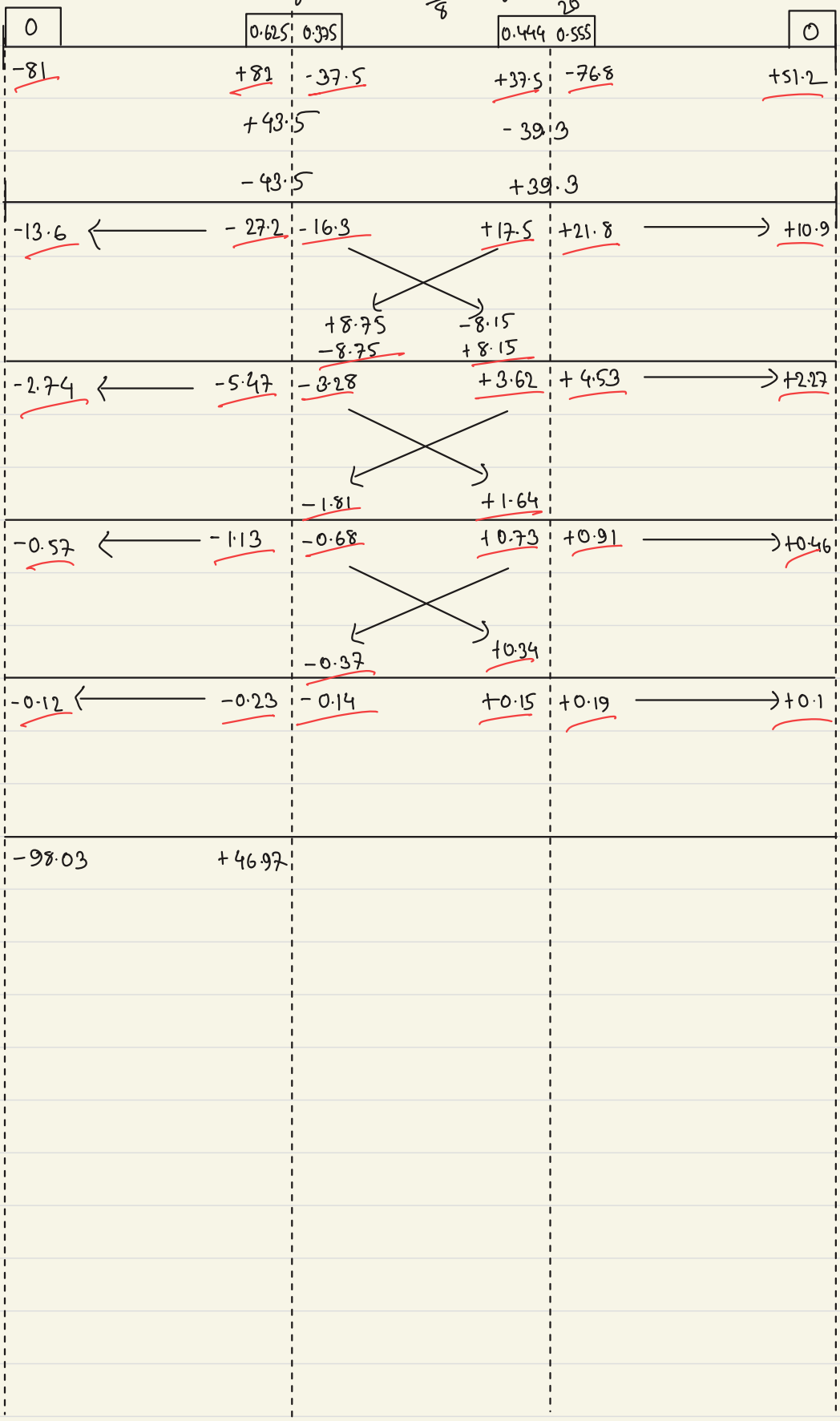
$0.444 + 0.555 \approx 1$

$\frac{wL^2}{12} = \frac{3 \times 18^2}{12}$

$\frac{PL}{8} = \frac{15 \times 20}{8}$

$\frac{PL}{8} = \frac{15 \times 20}{8}$

$\frac{wL^2}{26} = \frac{6 \times 16^2}{20}$



CT Next wk

10:00 A.M

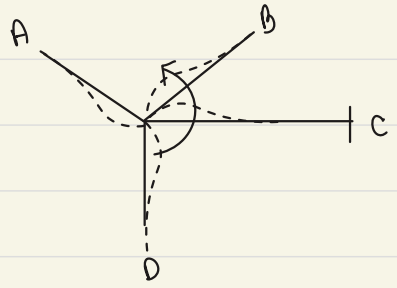
Rough/ CT-1

23/07

Moment distribution method

Assumptions :

1. Rigid joint
2. No translation
3. Rotation = θ



Joint equilibrium: $m = m_A + m_B + m_C + m_D$

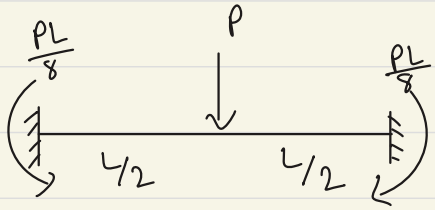
$= \theta (K_A + K_B + K_C + K_D)$; $m_x = K_x \theta$

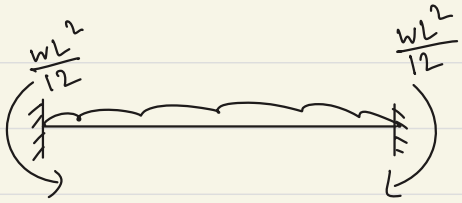
$\therefore \theta \Sigma K$

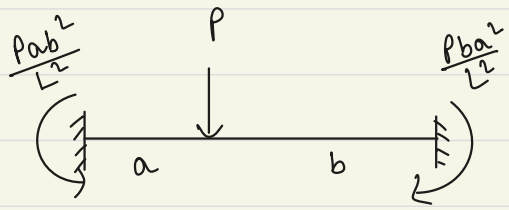
$\therefore m = \theta \Sigma K$; $m_A = \theta K_A$

$$\frac{m_A}{m} = \frac{K_A}{\Sigma K} \Rightarrow \boxed{m_A = \frac{K_A}{\Sigma K} m}$$

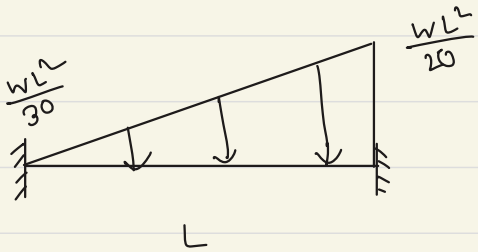
Fixed end moments:



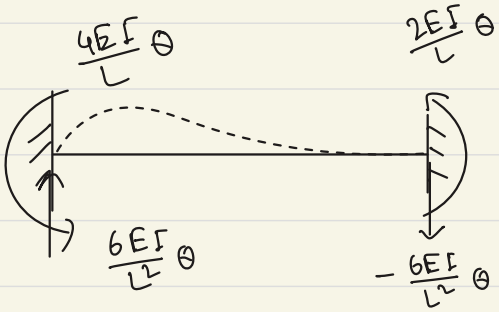




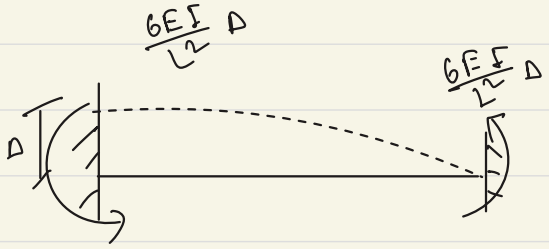
$L = a + b$



Moment for rotation:



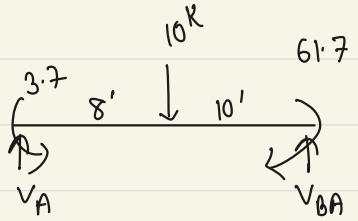
Moment for translation:



Example 1:

$$D_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{\frac{4\times 2EI}{18}}{\frac{4\times 2EI}{18} + \frac{4EI}{16}} = 0.64$$

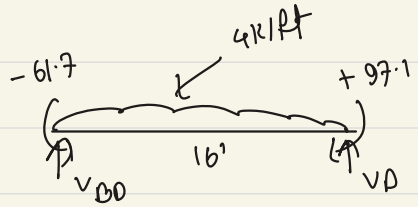
$$D_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{\frac{4\times EI}{16}}{\frac{4\times 2EI}{18} + \frac{4EI}{16}} = 0.36$$



$\overset{+}{\circlearrowleft} \sum M_A = 0 \Rightarrow -3.7 + 10\times 8 + 61.7 - 18V_{BA} = 0$

$\Rightarrow V_{BA} = 7.7\ K$

$\overset{+}{\uparrow} \sum F_y = 0 \Rightarrow V_A - 10 + 7.7 \Rightarrow V_A = 2.3\ K$

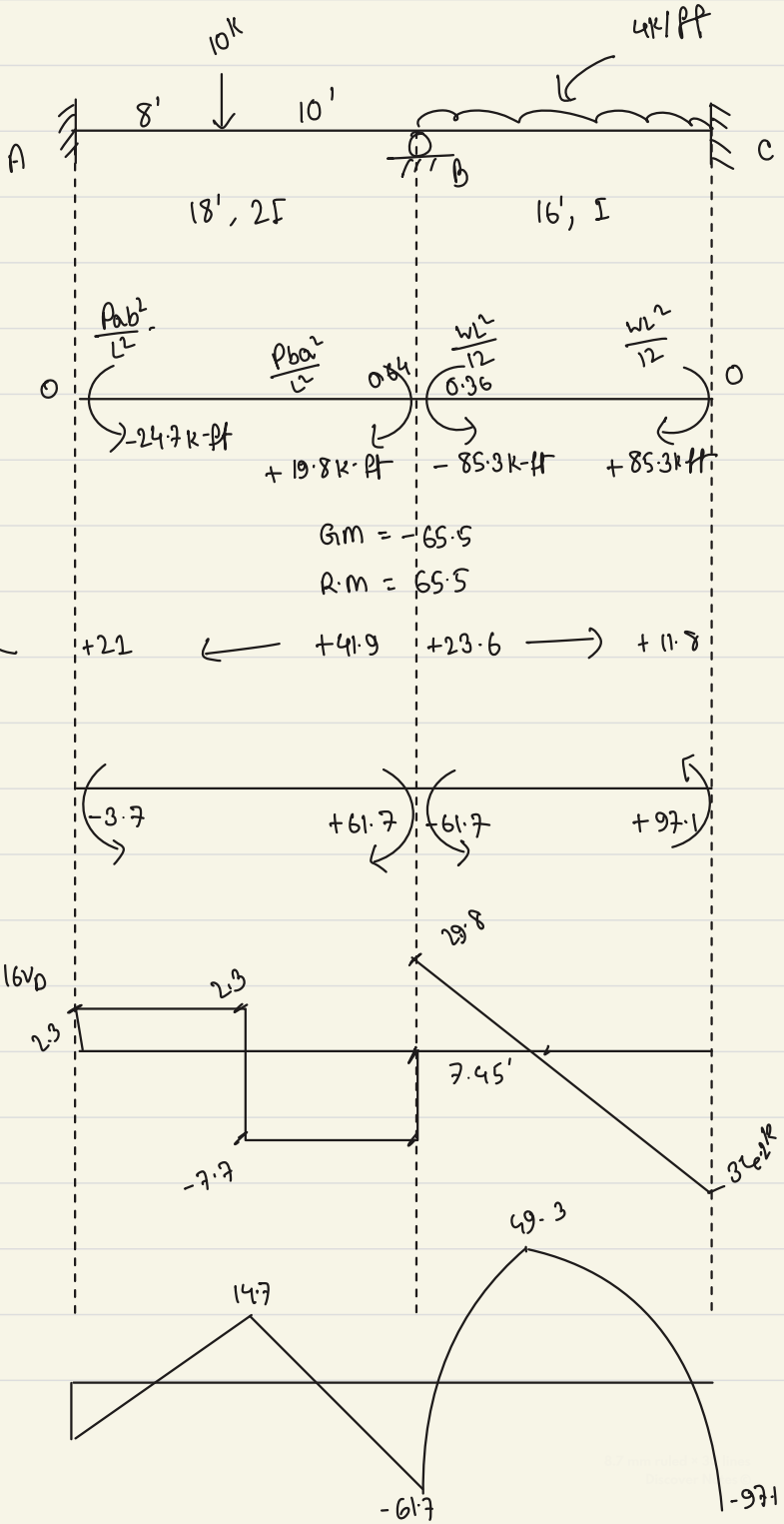


$\overset{+}{\circlearrowleft} \sum M_A = 0 \Rightarrow -61.7 + 4\times 16\times \frac{16}{2} + 97.1 - 16V_D$

$\Rightarrow V_D = 34.2\ K$

$\overset{+}{\uparrow} \sum F_y = 0 \Rightarrow V_{BD} - 4\times 16 + 34.2$

$\Rightarrow V_{BD} = 29.8\ K$



Problem 2:

A rotates clockwise 0.03 rad

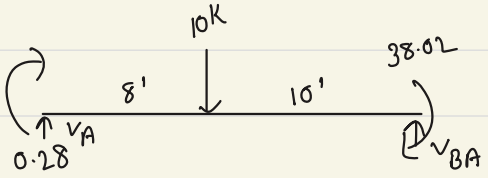
A moves upward 0.15'

B settles down 0.2'

$EI = 4500\ k\text{-}ft^2$

$$DF_{BA} = \frac{\frac{4 \times 2EI}{18}}{\frac{4 \times 2EI}{18} + \frac{4 \times EI}{16}} = 0.64$$

$$DF_{BC} = \frac{\frac{4 \times EI}{16}}{\frac{4 \times 2EI}{18} + \frac{4 \times EI}{16}} = 0.36$$

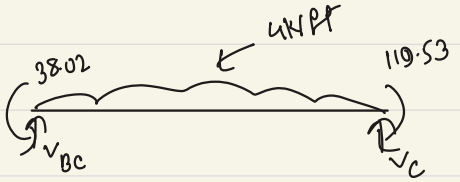


$\Sigma M_A = 0 \Rightarrow 0.28 + 10 \times 8 + 38.02 - 18 V_{BA} = 0$

$\Rightarrow V_{BA} = 6.57^K$

$\Sigma F_y = 0 \Rightarrow V_A - 10 + 6.57 = 0$

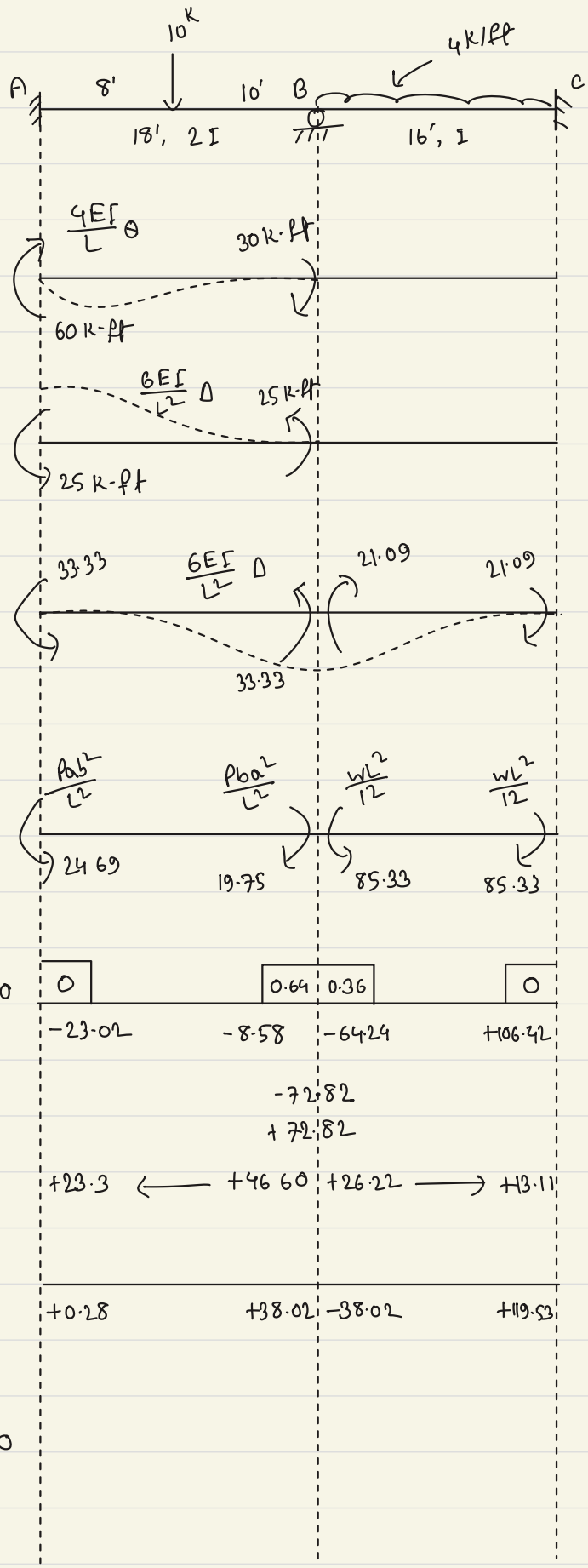
$\Rightarrow V_A = 3.43^K$

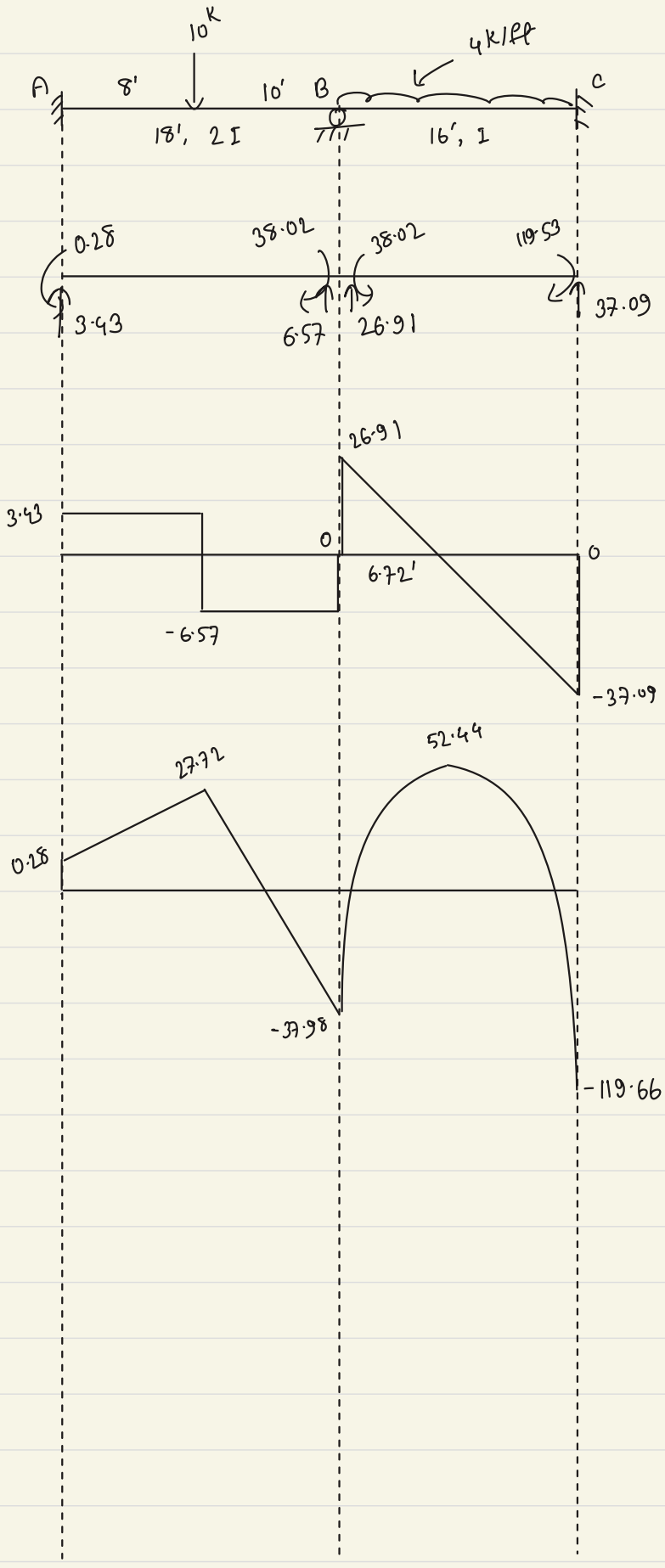


$\Sigma M_B = 0 \Rightarrow -38.02 + 4 \times 16 \times 8 + 119.53 - 16 V_C = 0$

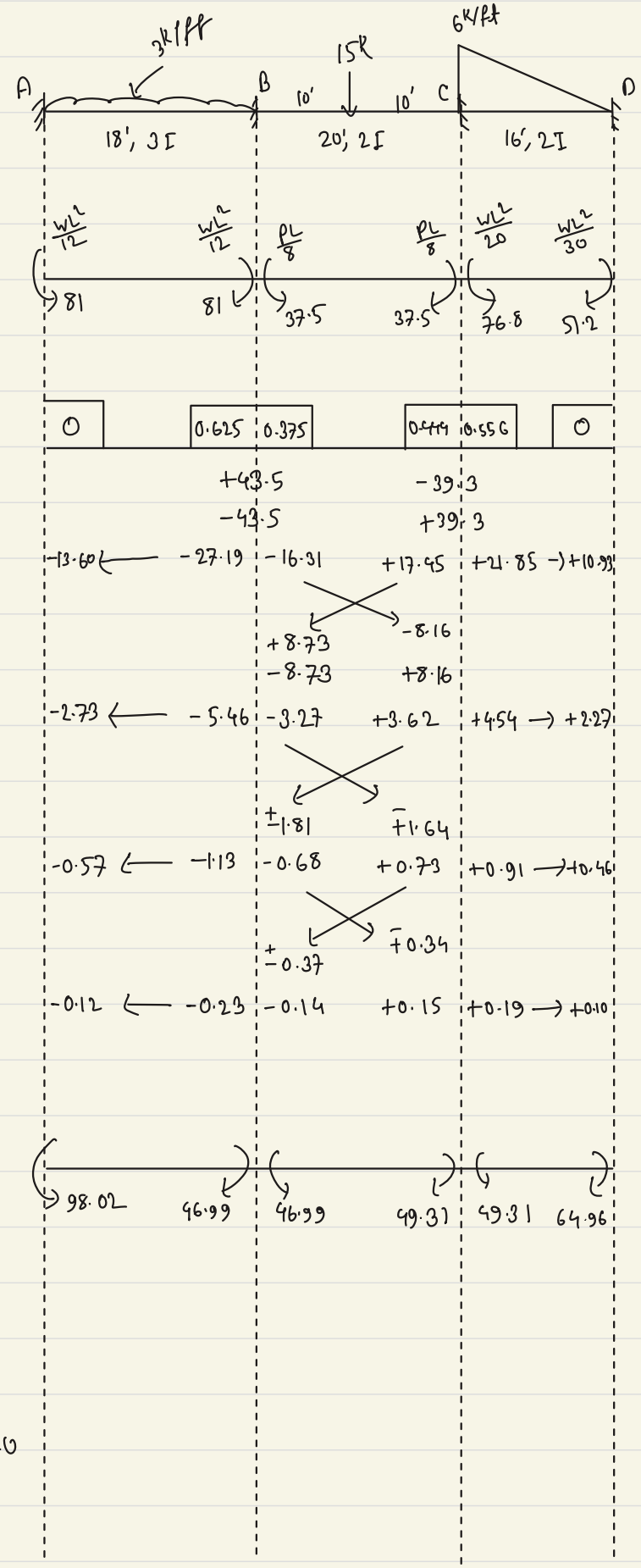
$\Rightarrow V_C = 37.09^K$

$\therefore \quad V_{BC} = 26.91^K$




10^K
4K/ft
A
8'
10'
B
C
18', 2I
16', I
0.28
38.02
38.02
119.53
3.43
6.57
26.91
37.09
26.91
3.43
0
0
6.72'
-6.57
-37.09
27.72
52.44
0.28
-37.98
-119.66

Problem 3:

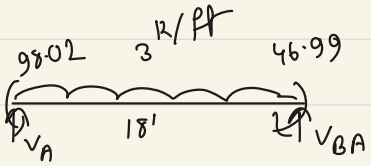


$DF_{BA} = 0.625$

$DF_{BC} = 0.375$

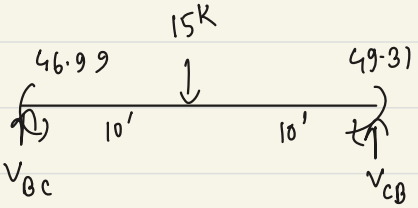
$DF_{CB} = 0.444$

$DF_{CD} = 0.556$



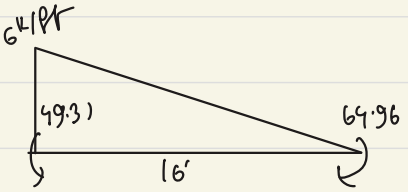
$\Sigma M_A = 0 \Rightarrow V_{BA} = 24.17^K$

$V_A = 29.83^K$



$V_{BC} = 7.62^K$

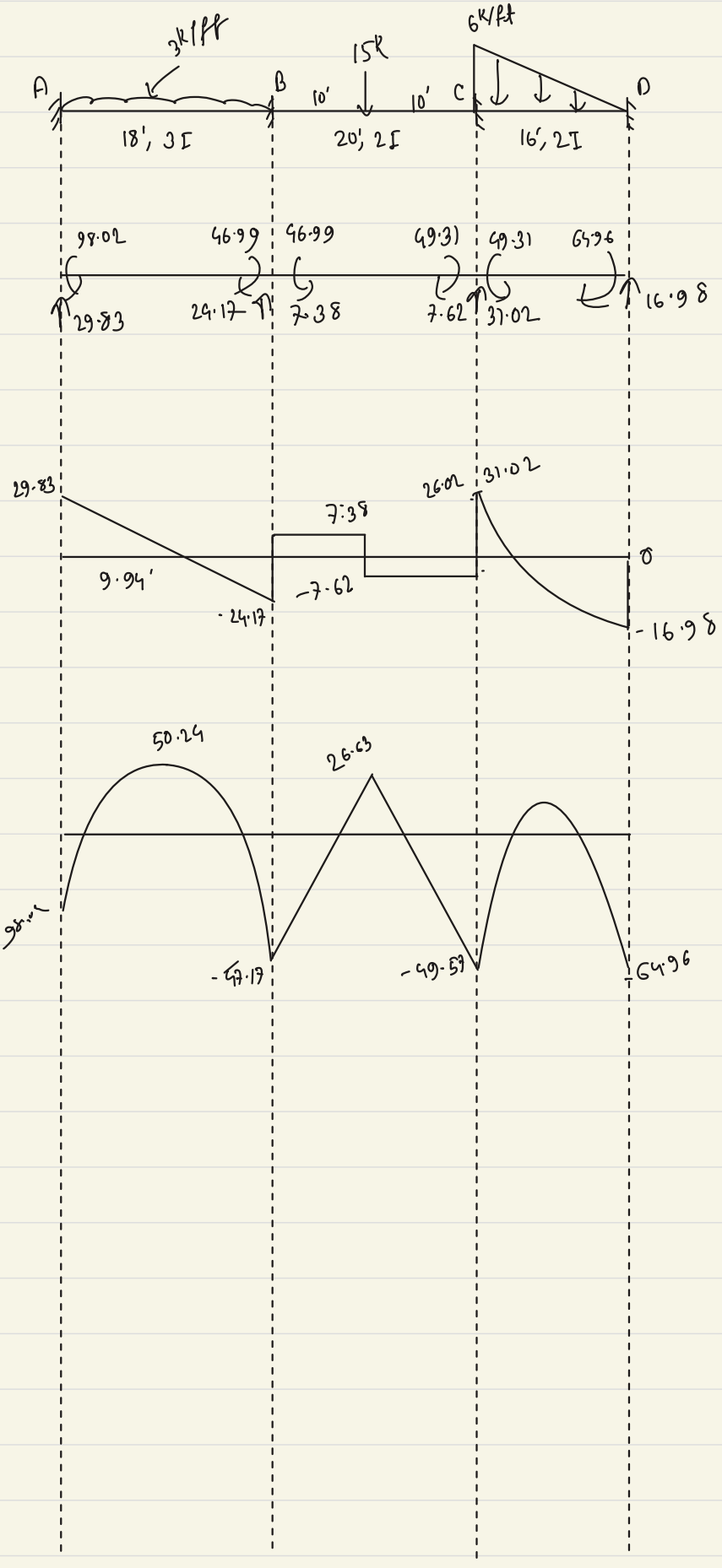
$V_{CB} = 7.38^K$

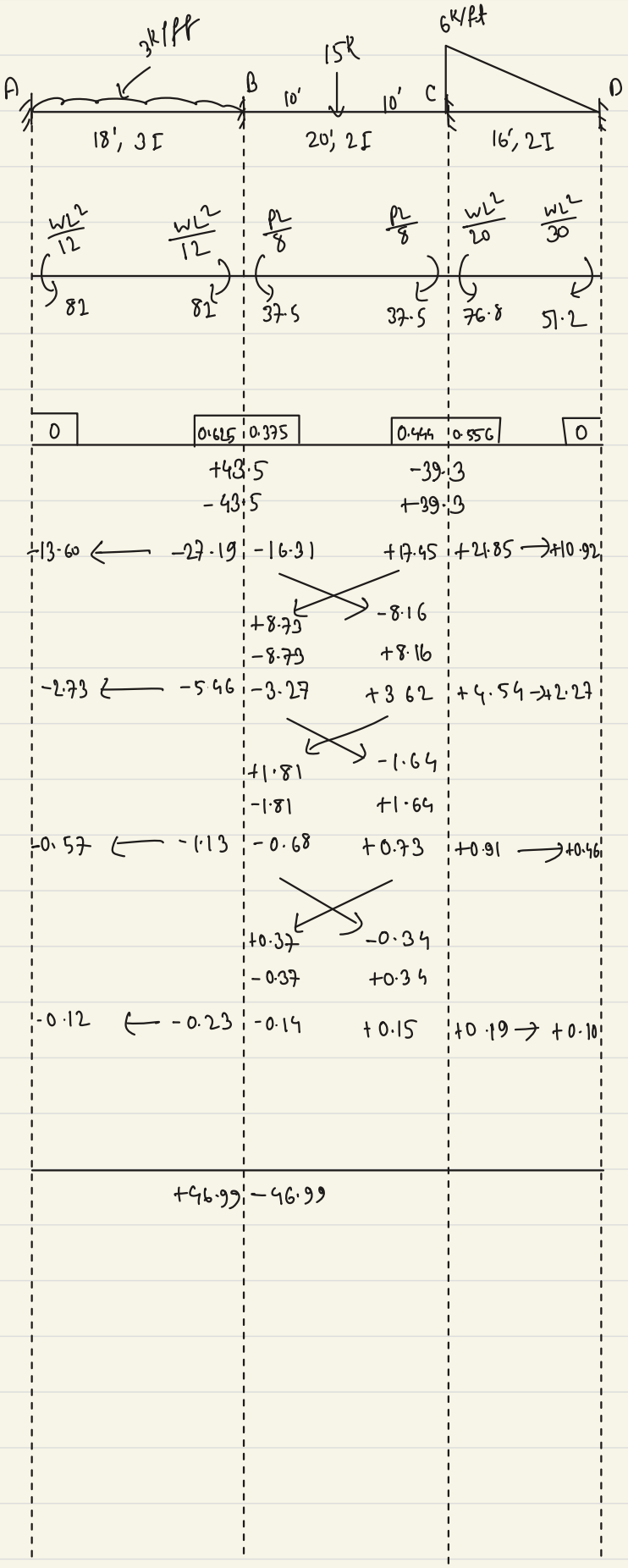


$(\overset{+}{\circlearrowleft}) \Sigma M_A = 0 \Rightarrow -49.31 + \frac{1}{2} \times 6 \times 16 \times \frac{16}{3} + 64.96 - V_D \times 16 = 0$

$\Rightarrow V_D = 16.98^K$

$V_{CD} = 31.02^K$


3K/ft
15K
6K/ft
A
B
C
D
10'
10'
18', 3I
20', 2I
16', 2I
98.02
46.99
46.99
49.31
49.31
64.96
29.83
24.17
7.38
7.62
31.02
16.98
29.83
9.94'
-24.17
7.38
-7.62
26.02
31.02
0
-16.98
50.24
26.63
-47.17
-49.59
-64.96


3K/ft
15K
6K/ft
A
B
C
D
10'
10'
18', 3I
20', 2I
16', 2I
$\frac{wL^2}{12}$
$\frac{wL^2}{12}$
$\frac{PL}{8}$
$\frac{PL}{8}$
$\frac{wL^2}{20}$
$\frac{wL^2}{30}$
82
82
37.5
37.5
76.8
51.2
0
0.625
0.375
0.444
0.556
0
+43.5
-43.5
-39.3
+39.3
-13.60
-27.19
-16.91
+17.45
+24.85
+10.92
+8.79
-8.79
-8.16
+8.16
-2.73
-5.46
-3.27
+3.62
+4.54
+2.27
+1.81
-1.81
-1.64
+1.64
-0.57
-1.13
-0.68
+0.73
+0.91
+0.46
+0.37
-0.37
-0.34
+0.34
-0.12
-0.23
-0.14
+0.15
+0.19
+0.10
+46.99
-46.99

LOS 23/07

Exterior end roller on hinge support

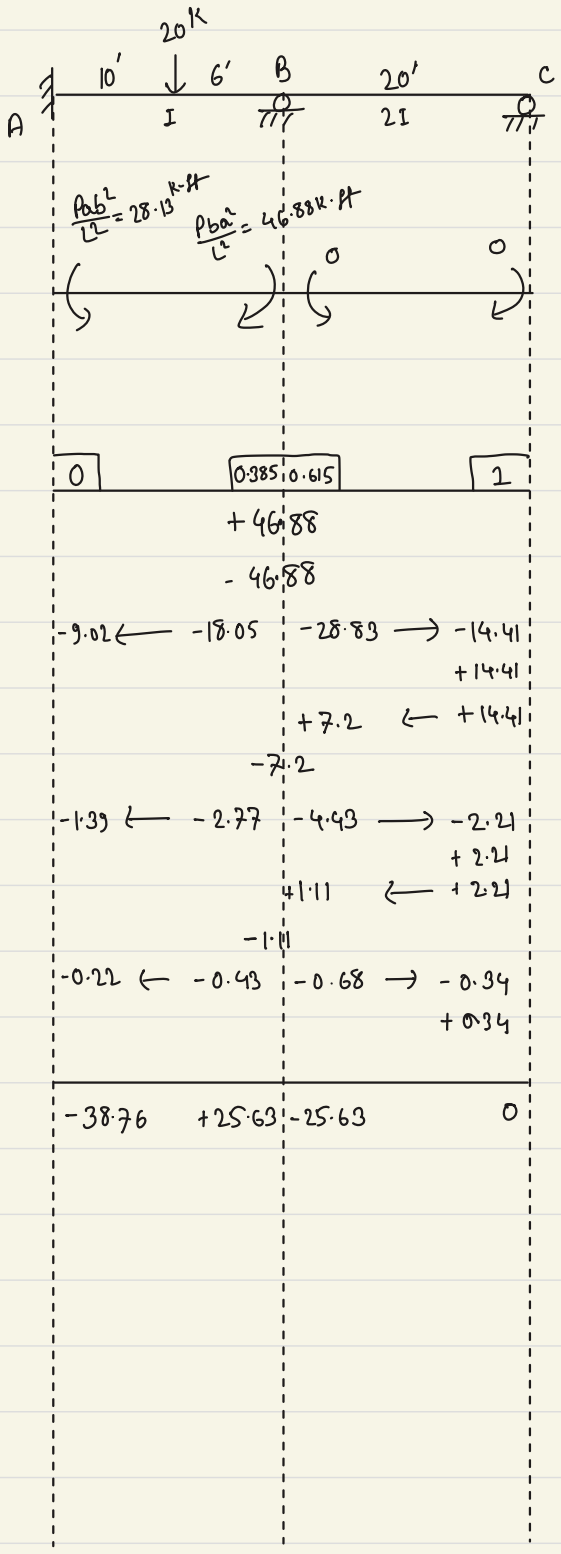
DF at exterior roller on hinge support = 1

Example:

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= \text{---} = 0.385$$

$$DF_{BC} = \text{---} = 0.615$$

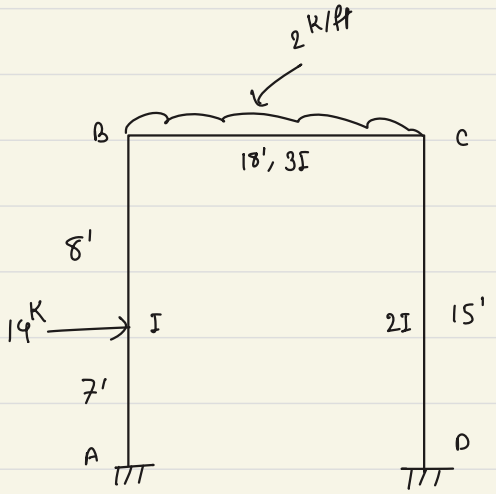


Modified stiffness method

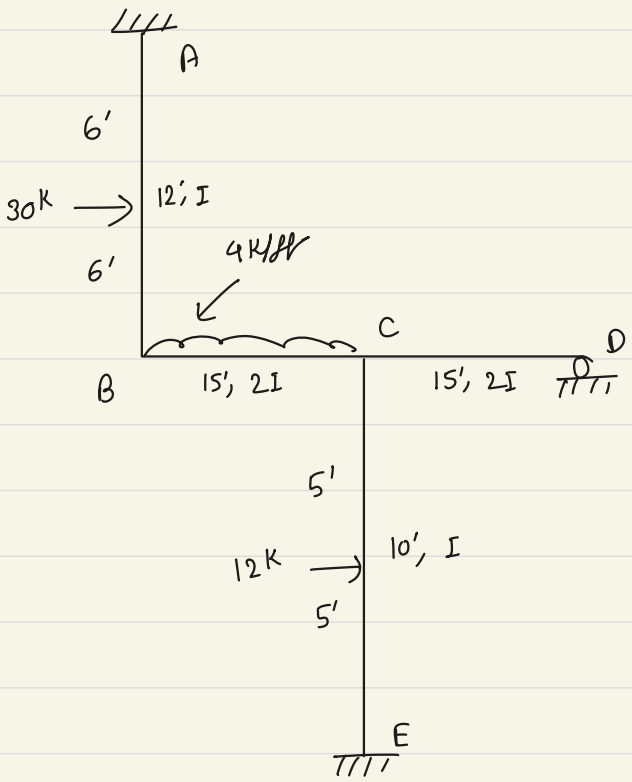
Regular stiffness: $K = \frac{4EI}{L}$

Modified stiffness: $K = \frac{3EI}{L}$; Pin / Hinge / Roller [not fixed]
no carry-over

Frame: P2



$$D_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CE}}$$



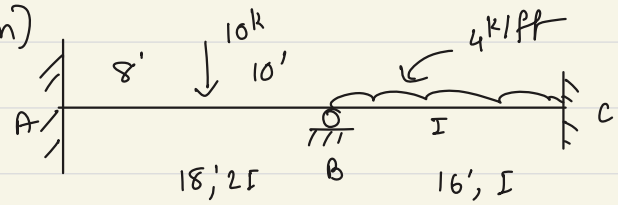
Assignment: 2 frame problems (PDF MOM)

Page 25 & 27

Incl. SFD & BMD Deadline: Last day

Slope Deflection Method

P10/ [very simple, not significant for xam]

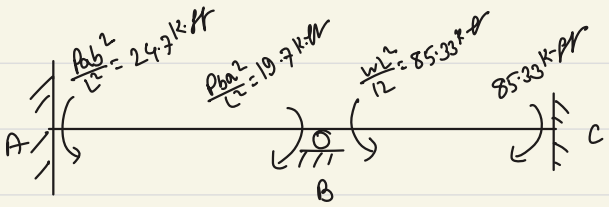


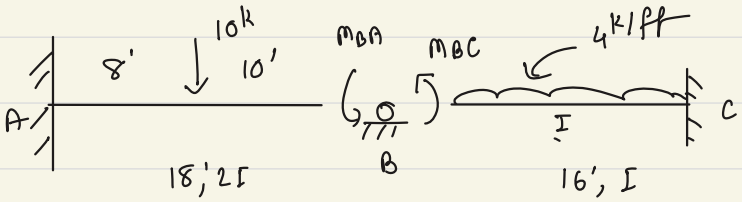
$$M_{AB} = \frac{4EI}{L}(2\theta_A + \theta_B - 3\psi_{AB}) + FEM_{AB} \quad (FEM_{AB} = -24.7)$$

$$M_{BA} = \frac{4EI}{L}(2\theta_B + \theta_A - 3\psi_{BA}) + FEM_{BA} \quad (FEM_{BA} = +19.7)$$

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\psi_{BC}) + FEM_{BC} \quad (FEM_{BC} = -85.33)$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B - 3\psi_{CB}) + FEM_{CB} \quad (FEM_{CB} = +85.33)$$





$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow EI\theta_B = +94.51$$

After substitution:

$$M_{AB} = +3.7 \text{ k-ft}$$

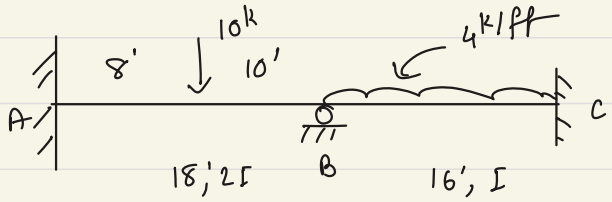
$$M_{BA} = +61.7 \text{ k-ft}$$

$$M_{BC} = -61.7 \text{ k-ft}$$

$$M_{CB} = +97.14 \text{ k-ft}$$

Log 10/09

P01/

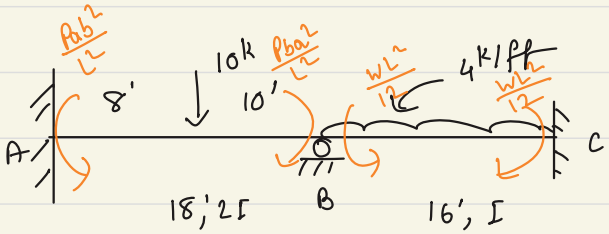


P02/ Support yields

A ↻ 0.03 rad

A ↑ 0.15'

B ↓ 0.2'



$$\psi_{AB} = \frac{|\Delta_A - \Delta_B|}{L} = \frac{|0.15+0.2|}{18} = +0.0194$$

$$\psi_{BC} = \frac{|-2+0|}{16} = -0.011$$

$\theta_A = +0.03$ rad

$\theta_B = ??$

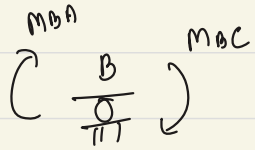
$\theta_C = 0$

$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - 3\psi_{AB}) + FEM_{BA}$$

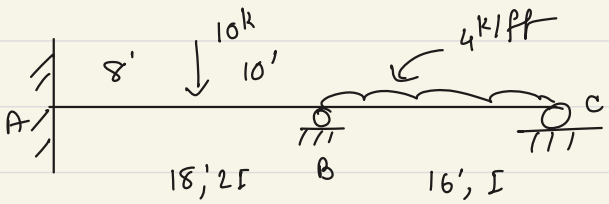
$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\psi_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B - 3\psi_{BC}) + FEM_{CB}$$



$$\Rightarrow M_{BA} + M_{BC} = 0$$

Problem 3/ Unyielded two rollen



$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi_{AB}) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - 3\psi_{AB}) + FEM_{BA}$$

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\psi_{BC}) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B - 3\psi_{BC}) + FEM_{CB}$$

$\theta_A = 0$

$\theta_B = ?$

$\theta_C = ?$

$\psi_{AB} = 0$

$\psi_{BC} = 0$

$M_{BC} + M_{BA} = 0$

$M_{CB} = 0$

Problem 4/

Rough / Assignment

16/09

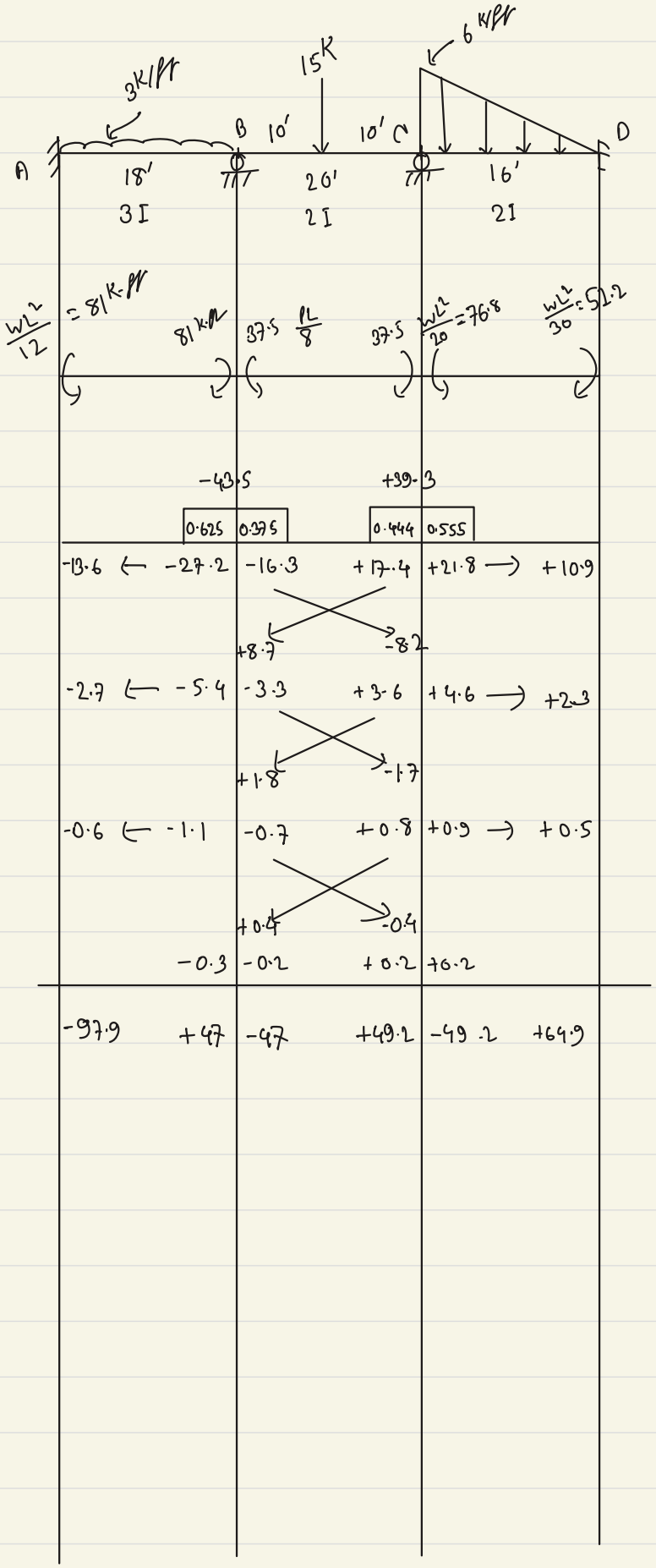
Prob 3

$$DF_{BA} = \frac{\frac{12}{18}}{\frac{12}{18} + \frac{8}{20}} = 0.625$$

$$D_{BC} = 0.375$$

$$D_{CD} = \frac{\frac{8}{16}}{\frac{8}{16} + \frac{8}{20}} = 0.555$$

$$D_{CB} = 0.444$$

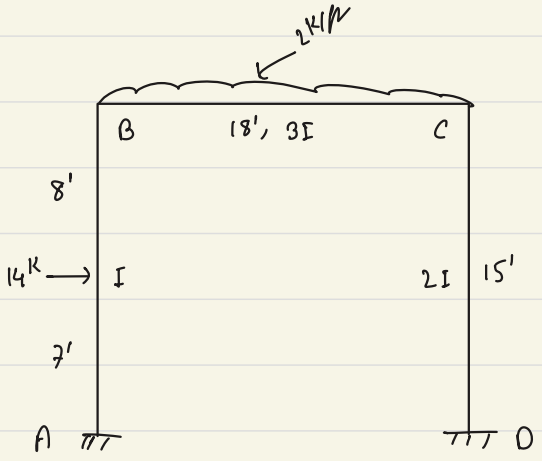


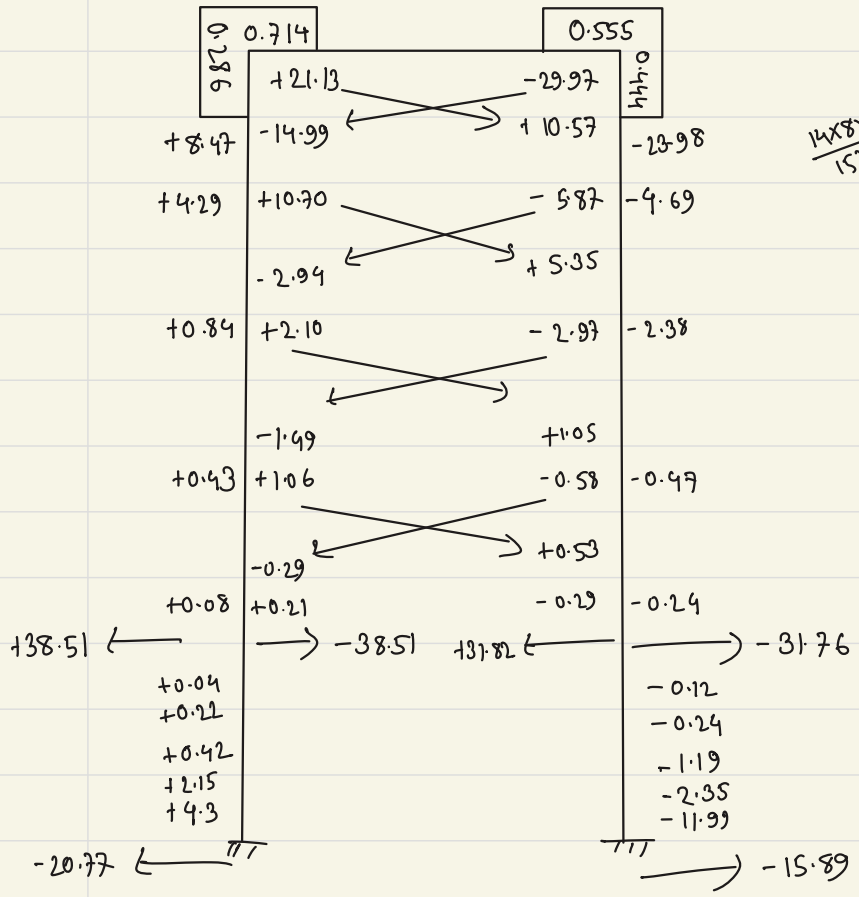
P25/

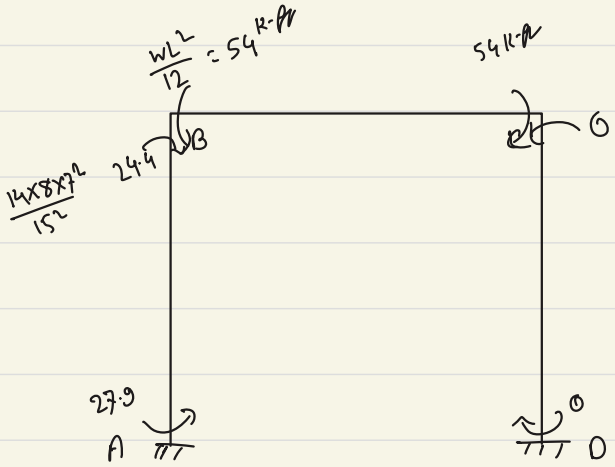
$$DF_{BA} = \frac{9/15}{4/15 + 12/18} = 0.286$$

$$DF_{BC} = 0.714$$

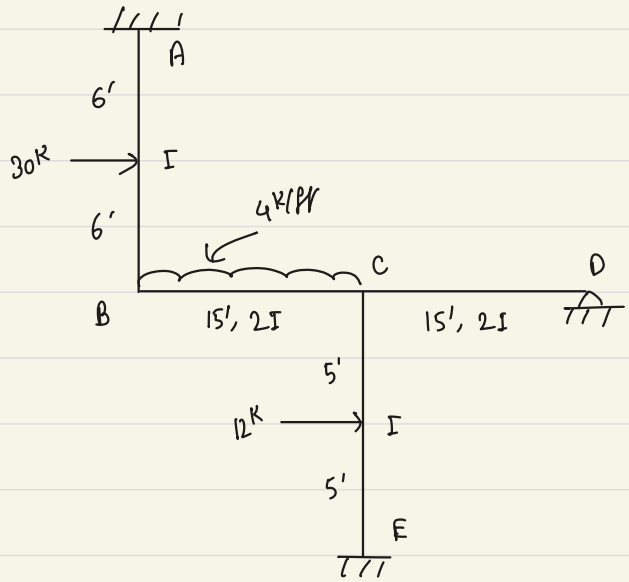
$$DF_{CD} = \frac{8/15}{8/15 + 12/18} = 0.44 \;/\; DF_{CB} = 0.555$$



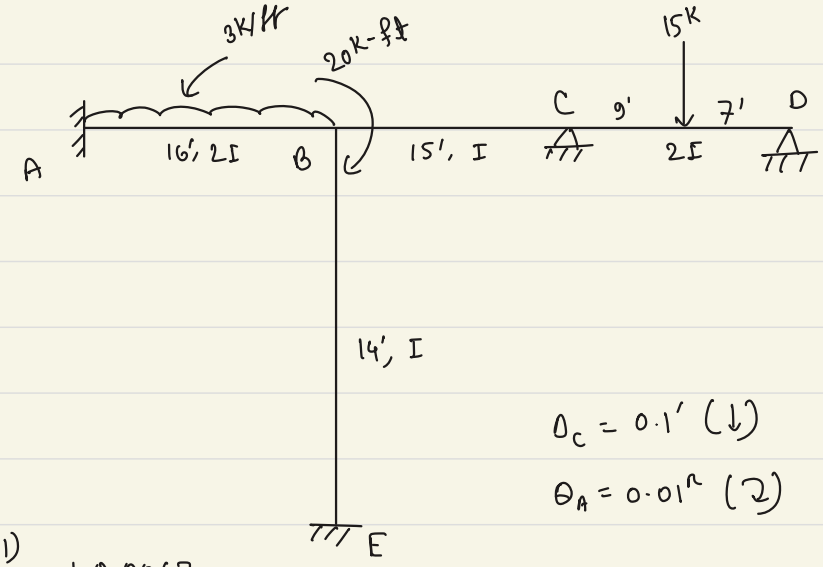




P27/



P8/



$\theta_A = 0$

$\theta_B \neq 0$

$\theta_C \neq 0$ } ?

$\theta_D \neq 0$

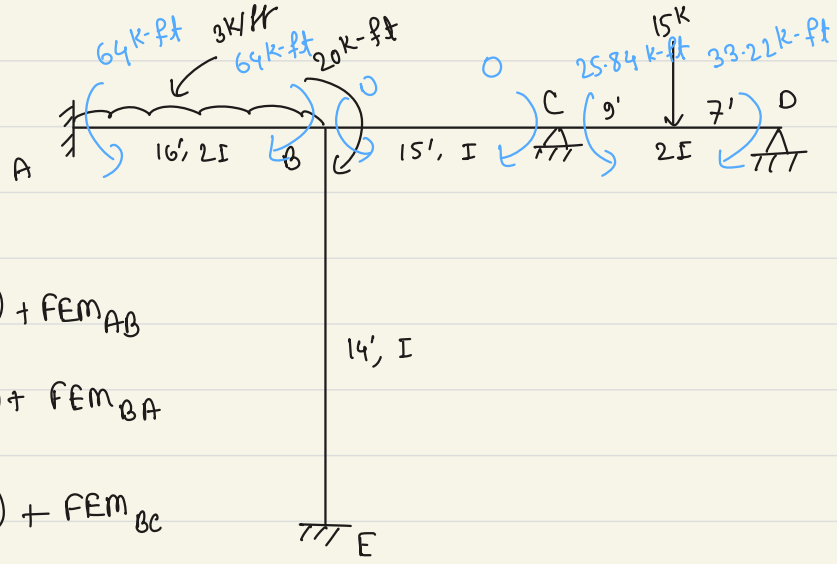
$\theta_E = +0.01$ rad

$\Delta_C = 0.1'$ (↓)

$\theta_A = 0.01^R$ (↻)

$$\psi_{BC} = \frac{\Delta_B - \Delta_C}{L} = \frac{0-(-0.1)}{15} = +0.0067$$

$$\psi_{BE} = \frac{\Delta_C - \Delta_D}{L} = \frac{-0.1-0}{16} = -0.0063$$



$$m_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi_{AB}) + FEM_{AB}$$

$$m_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - 3\psi_{BA}) + FEM_{BA}$$

$$m_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\psi_{BC}) + FEM_{BC}$$

$$m_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B - 3\psi_{CB}) + FEM_{CB}$$

$$m_{CD} = \frac{2EI}{L}(2\theta_C + \theta_D - 3\psi_{CD}) + FEM_{CD}$$

$$m_{DC} = \frac{2EI}{L}(2\theta_D + \theta_C - 3\psi_{DC}) + FEM_{DC}$$

$$m_{BE} = \frac{2EI}{L}(2\theta_B + \theta_E - 3\psi_{BE}) + FEM_{BE}$$

$$m_{EB} = \frac{2EI}{L}(2\theta_E + \theta_B - 3\psi_{EB}) + FEM_{EB}$$

At equilibrium conditions:

Joint B: $M_{BA} + M_{BC} + M_{BE} = 20$ — (i)

Joint C: $M_{CB} + M_{CD} = 0$ — (ii)

Joint D: $M_{DC} = 0$ — (iii)

(i), (ii), (iii) → $\theta_B = —$

$\theta_C = —$

$\theta_D = —$

SEM = ?? for all 4 segments (altogether 8 segment moment)
Followed by calculating reactive forces as well as eventually plotting SFD & BMD with max bending mmnt values

Compulsory → Slope deflection or moment distribution

Ques 1 — 15+15

2 — 15+15

- - -

Rough / Final

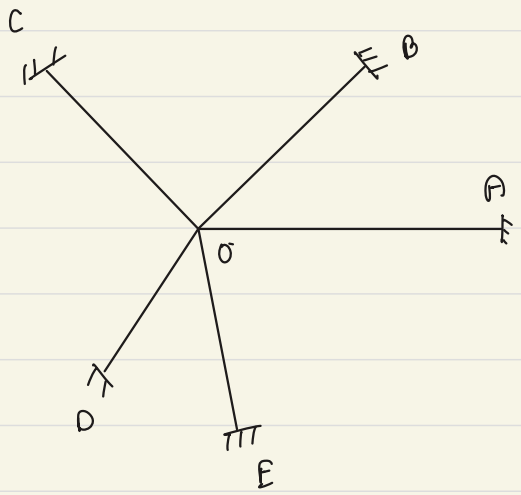
13/10

Moment Distribution method

$$m = m_{OA} + m_{OB} + m_{OC} + m_{OD} + m_{OE}$$

$$m = \theta \Sigma K ; \quad m_{OA} = \theta K_{OA}$$

$$\frac{m_{OA}}{m} = \frac{\theta K_{OA}}{\theta \Sigma K} \Rightarrow \boxed{m_{OA} = \frac{K_{OA}}{\Sigma K} m}$$

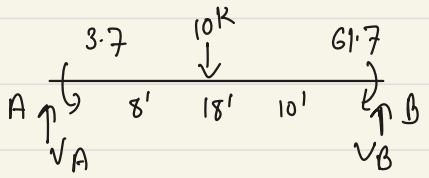


Ex. 1 / P07

$$DF_{BA} = \frac{8/18}{8/18 + 4/16} \times \frac{EI}{EI} = 0.64$$

$$DF_{BC} = \frac{4/16}{8/18 + 4/16} = 0.36$$

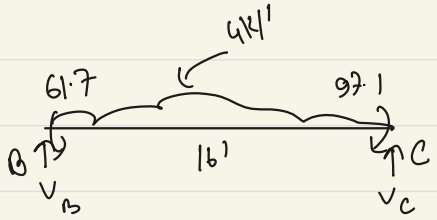
Support reactions:



$\Sigma M_A = 0 \Rightarrow -3.7 + 10 \times 8 + 61.7 - 18 V_B = 0$

$\Rightarrow V_B = 7.7^K \ (\uparrow)$

$\uparrow \Sigma F_y = 0 \Rightarrow 2.3^K \ (\downarrow)$

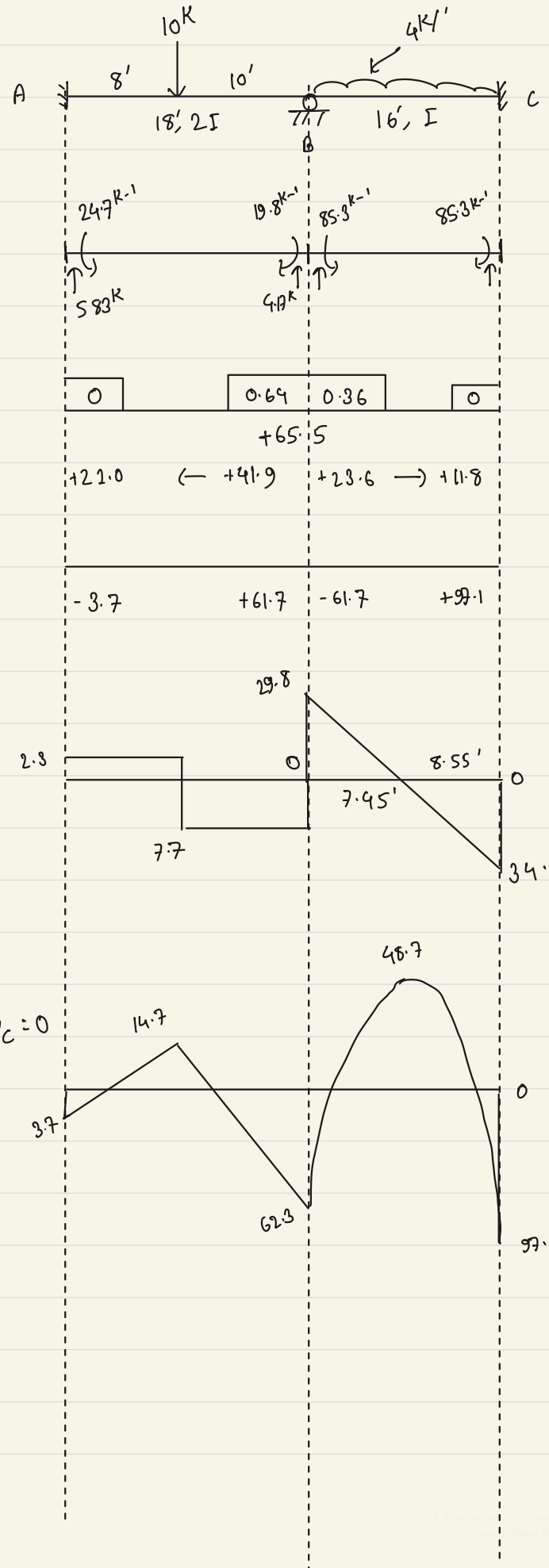


$\overset{+}{\circlearrowleft} \Sigma M_B = 0 \Rightarrow -61.7 + 4 \times 16 \times 8 + 97.1 - 16 V_C = 0$

$\Rightarrow V_C = 34.2^K \ (\uparrow)$

$\uparrow \Sigma F_y = 0 \Rightarrow V_B - 4 \times 16 + 34.2 = 0$

$\Rightarrow V_B = 29.8^K \ (\uparrow)$



Ex 2/

$EI = 4500\ k\text{-}ft^2$

$\theta_A = 0.03^R$ ↻

$\Delta_A = 0.15'$ ↑

$\Delta_B = 0.2'$ ↓

$$DF_{BA} = \frac{8/18}{8/18 + 4/16}\,\frac{EI}{EI} = 0.64$$

$$DF_{BC} = \frac{4/16}{8/18 + 4/16} = 0.36$$

Support reactions:

$\overset{\circlearrowleft}{\Sigma M_A = 0} \Rightarrow 0.3 + 10\times 8 + 38 - 18V_{B_1} = 0$

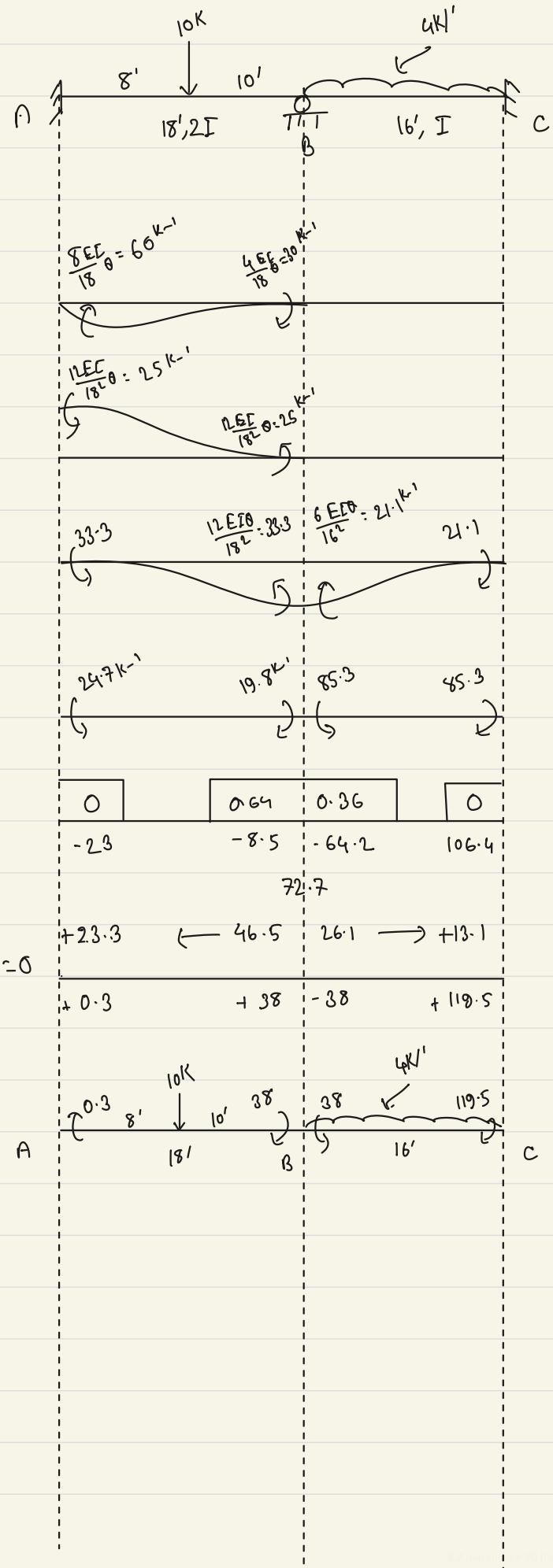
$\Rightarrow V_{B1} = 6.6^K$

$\overset{+}{\uparrow}\Sigma F_y = 0 \Rightarrow V_A = 3.4^K$

$\overset{\circlearrowleft +}{\Sigma M_B = 0} \Rightarrow -38 + 4\times 16\times 8 + 119.5 - 16V_C = 0$

$\Rightarrow V_C = 37.1\ K$

$V_{B2} = 26.9\ K$



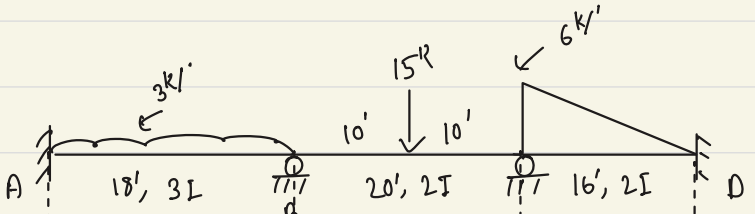
P15/

$$DF_{BA} = \frac{12/18}{12/18 + 8/20} = 0.625$$

$$DF_{BC} = \frac{8/20}{12/18 + 8/20} = 0.375$$

$$DF_{CB} = \frac{8/20}{8/20 + 8/16} = 0.444$$

$$DF_{CD} = \frac{8/16}{8/20 + 8/16} = 0.555$$



	A	B		C		D
DF	0	0.625	0.375	0.444	0.555	0
	-81	+81	-37.5	+37.5	-76.8	+51.2
	+13.6	← -27.2	-16.3	+17.4	+21.8	→ +10.9
			+8.7	-8.2		
	-2.7	← -5.4	-3.3	+3.6	+4.6	→ 2.3
			+1.8	-1.7		
	-0.6	← -1.1	-0.7	+0.8	+0.9	→ +0.5
			+0.4	-0.4		
		-0.3	-0.2	+0.2	+0.2	
	-98.3	+47	-47.1	+49.2	-49.3	+64.9

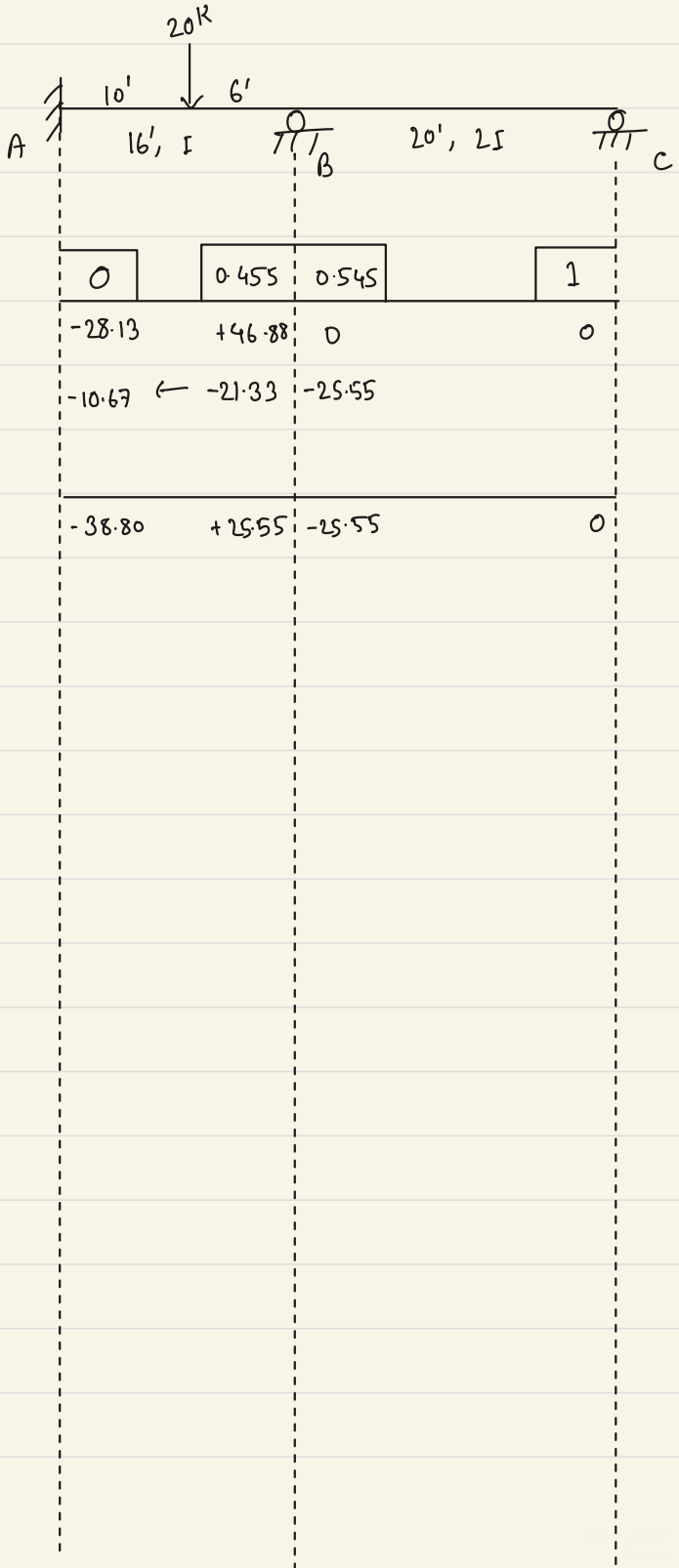
mom / modified stiffness :

If exterior end is roller / hinge

use → $K = \frac{3EI}{L}$ and perform no carryover to there.

$$DF_{BA} = \frac{4/16}{4/16 + 6/20} = 0.455$$

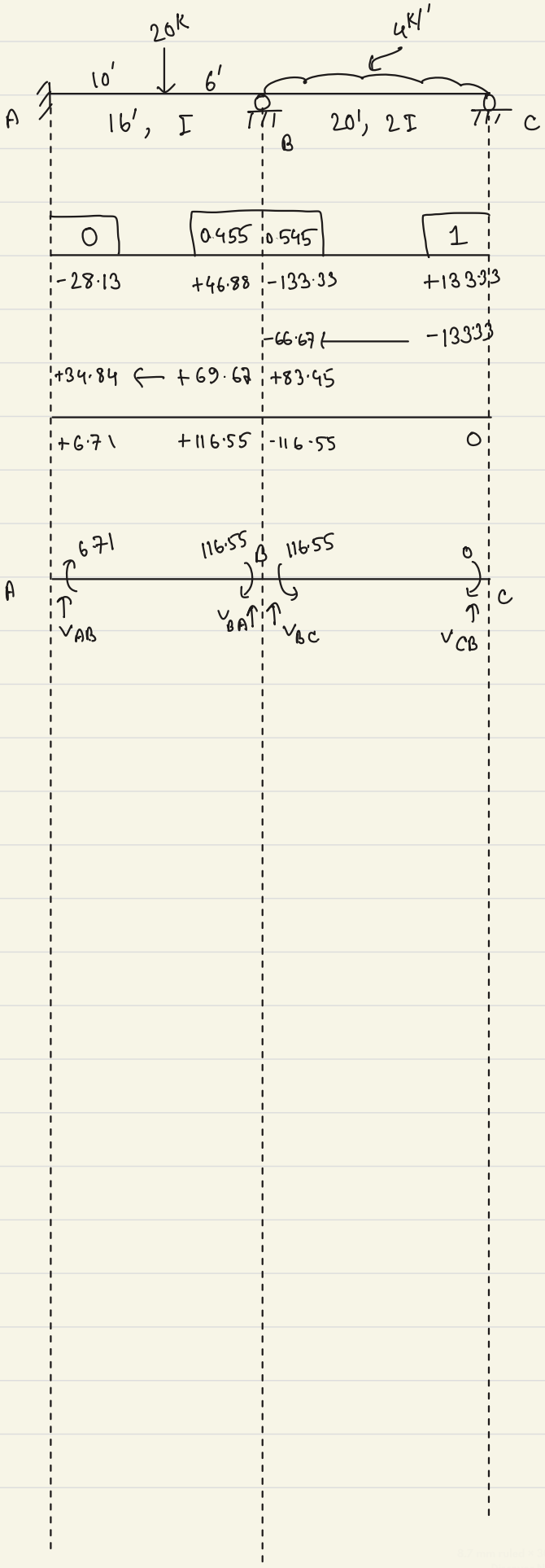
$$DF_{BC} = \frac{6/20}{4/16 + 6/20} = 0.545$$



P23/

$$DF_{BA} = \frac{4/16}{4/16 + 6/20} = 0.455$$

$$DF_{BC} = \frac{6/20}{4/16 + 6/20} = 0.545$$

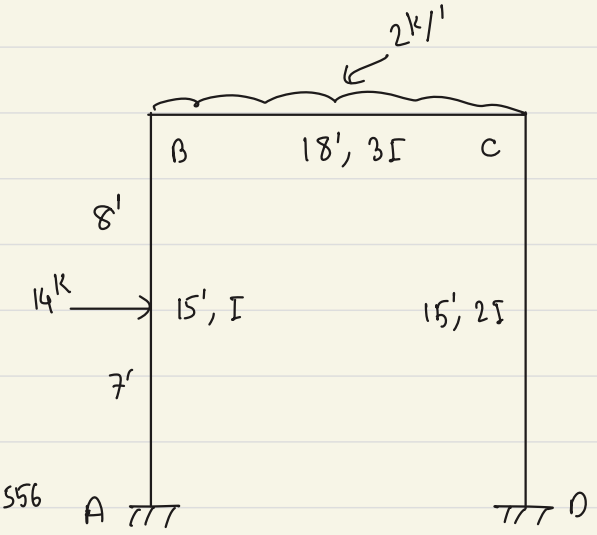


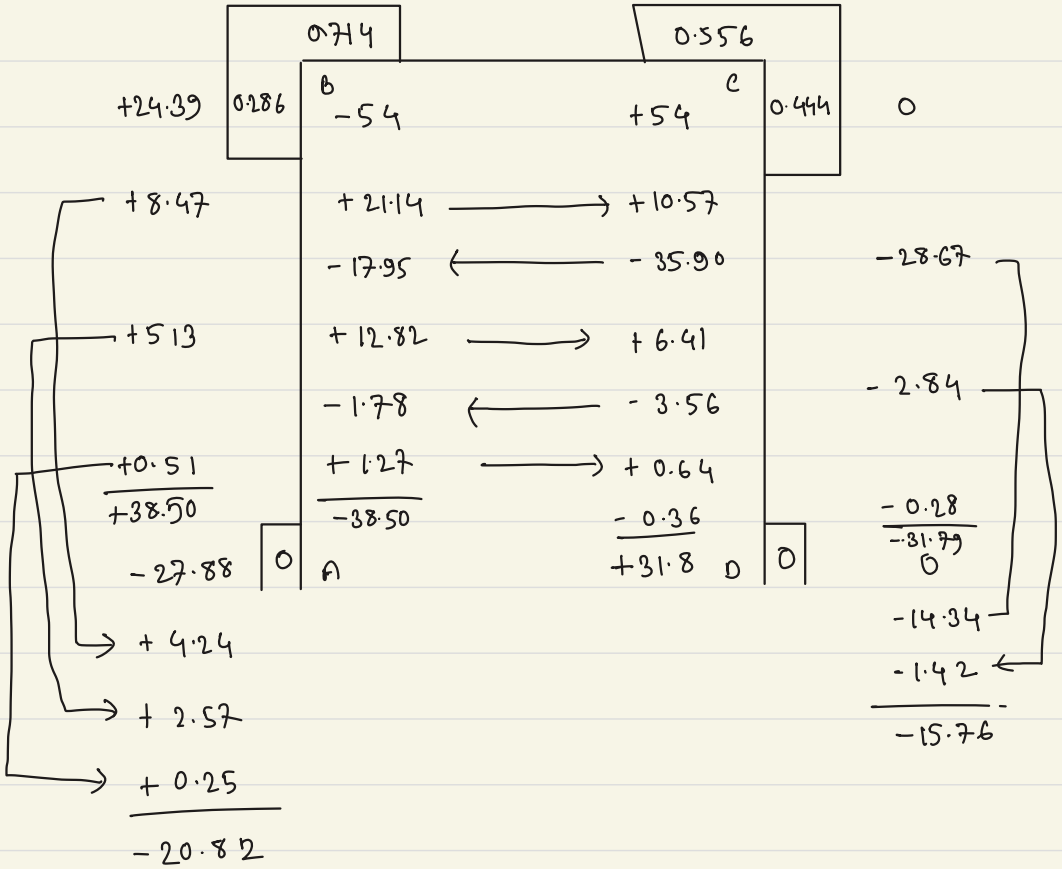
P24/

$$DF_{BA} = \frac{4/15}{4/15 + 12/18} = 0.286$$

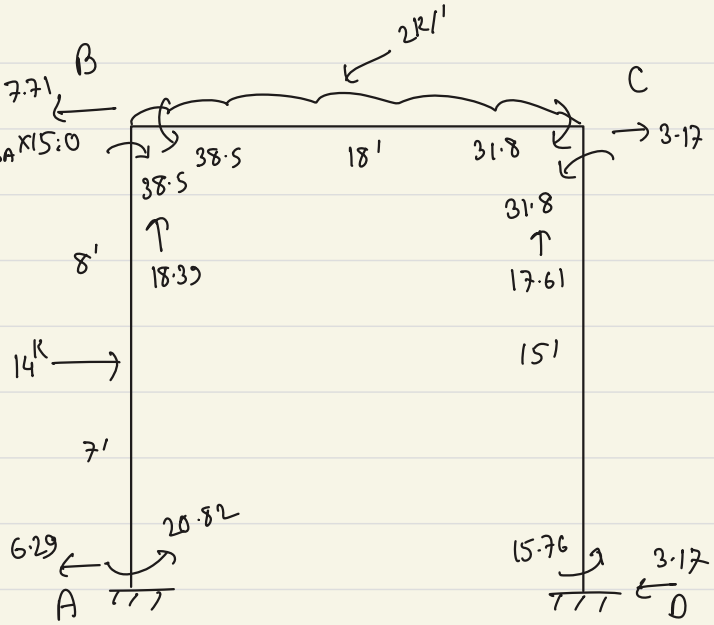
$$DF_{BC} = \frac{12/18}{4/15 + 12/18} = 0.714$$

$$DF_{CD} = \frac{8/15}{8/16 + 12/18} = 0.444 \; ; \quad DF_{CB} = 0.556$$





Support reactions:



AB: $\Sigma M_A = 0 \Rightarrow -20.82 + 7 \times 14 + 38.5 + H_{BA} \times 15 = 0$

$\Rightarrow H_{BA} = 7.71^K \ (\leftarrow)$

$\Sigma F_x = 0 \Rightarrow 14 - 7.71 + H_{AB} = 0$

$\Rightarrow H_{AB} = 6.29^K \ (\leftarrow)$

BC: $\Sigma M_B = 0 \Rightarrow 2 \times 18 \times 9 - 38.5 + 31.5 - 18 V_{CB} = 0$

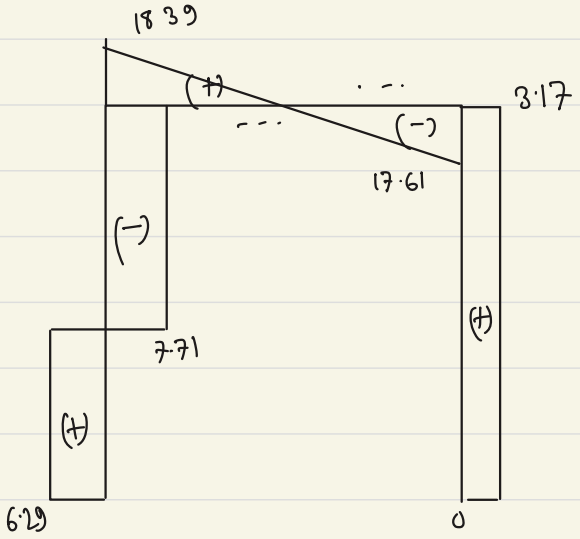
$\Rightarrow V_{CB} = 17.61^K \ (\uparrow)$

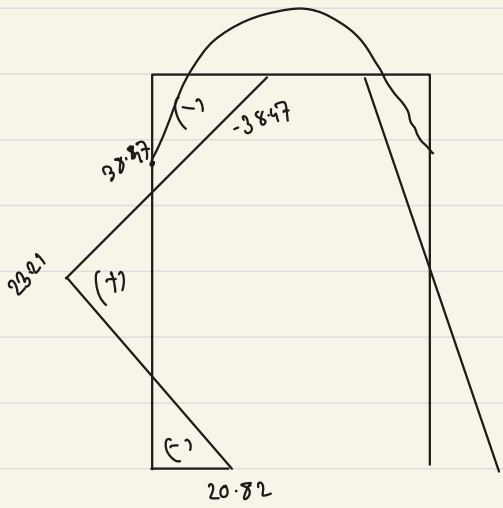
$\Sigma F_y = 0 \Rightarrow V_{BC} - 2 \times 18 + 17.61 \Rightarrow V_{BC} = 18.39^K \ (\uparrow)$

CD: $\Sigma M_C = 0 \Rightarrow -31.8 - 15.76 - 15 H_{DC} = 0$

$\Rightarrow H_{DC} = 3.17^K \ (\leftarrow)$

$H_{CD} = 3.17^K \ (\rightarrow)$



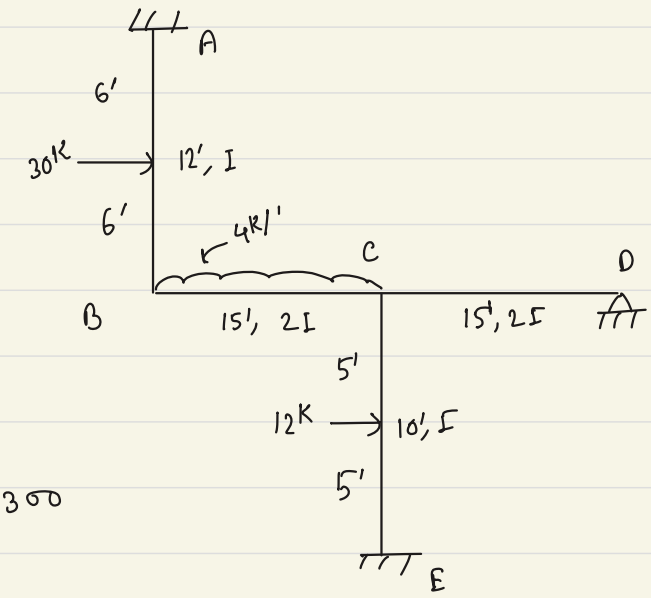


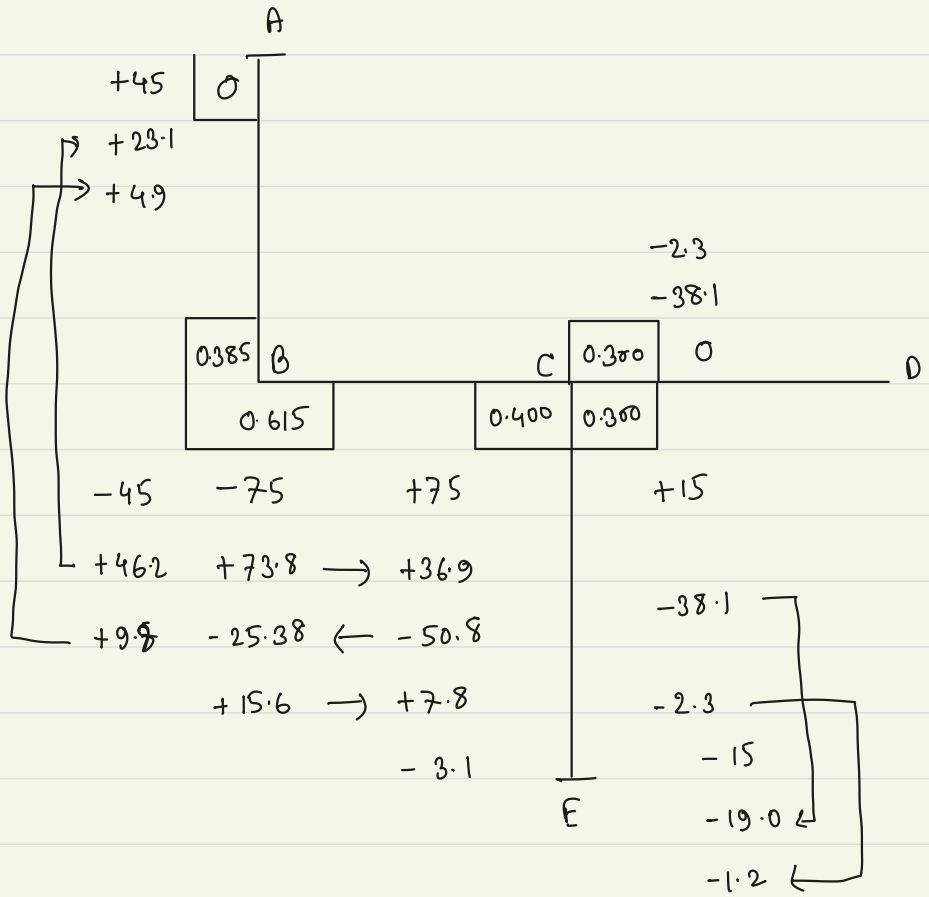
$$DF_{BA} = \frac{4/12}{4/12 + 8/15} = 0.385$$

$$DF_{BC} = 0.615$$

$$DF_{CB} = \frac{8/15}{8/15 + 4/10 + 6/15} = 0.400$$

$DF_{CD} = 0.300$ $\quad DF_{CE} = 0.300$





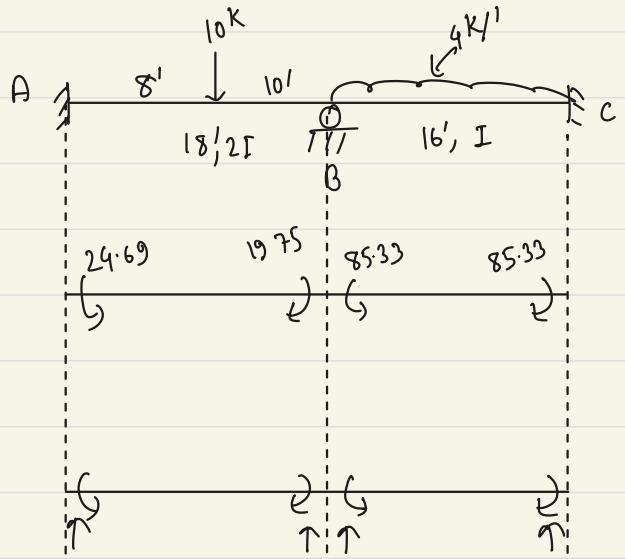
Slope Deflection method

$$M_{AB} = FEM_{AB} + \frac{4EI}{L}\theta_A - \frac{6EI}{L^2}\Delta_1 + \frac{2EI}{L}\theta_B + \frac{6EI}{L^2}\Delta_2$$

$$= FEM_{AB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3}{L}\underbrace{(\theta_1 - \theta_2)}_{L\psi_{AB}}\right]$$

$$\psi_{AB} = \frac{|\Delta_A - \Delta_B|}{L}$$

P09/



$\theta_A = 0$

$\theta_B = ?$

$\theta_C = 0$

$\psi_{AB} = 0$

$\psi_{BC} = 0$

$$M_{AB} = \frac{2 \times 2EI}{L}[2\theta_A + \theta_B - 3\psi_{AB}] + FEM_{AB}$$

$$= \frac{4EI}{18}\theta_B - 24.69$$

$$M_{BA} = \frac{2 \times 2EI}{L}[2\theta_B + \theta_A - 3\psi_{AB}] + FEM_{BA}$$

$$= \frac{8EI}{18}\theta_B + 19.75$$

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\psi_{BC}) + FEM_{BC}$$

$$= \frac{2EI}{16}[2\theta_B] - 85.33$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B - 3\psi_{BC}) + FEM_{CB}$$

$$= \frac{2EI}{16}[\theta_B] + 85.33$$

$$\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$$

$$\Rightarrow \frac{8EI}{18}\theta_B + 19.75 + \frac{2EI}{16} \times 2\theta_B - 85.33 = 0$$

$$\Rightarrow EI\theta_B = +94.44$$

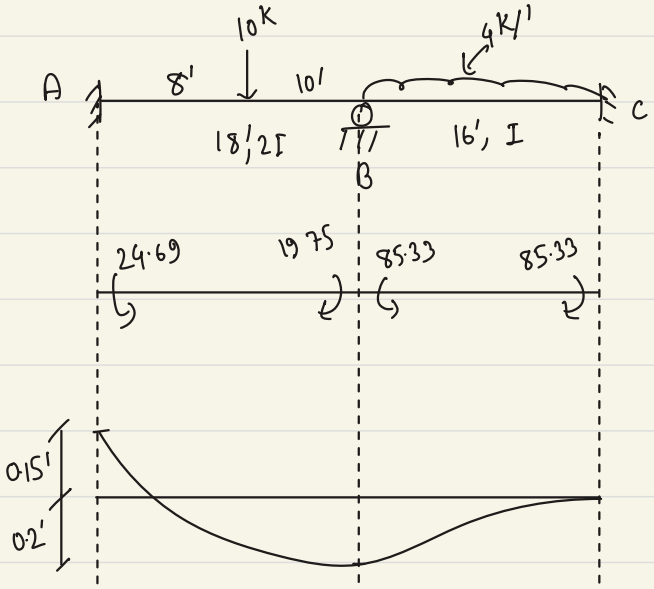
$\therefore M_{AB} = -3.70$ k-ft

$M_{BA} = +61.72$ k-ft

$M_{BC} = -61.72$ k-ft

$M_{CB} = 97.13$ k-ft

P15/



$\theta_A = 0.03^R$ (↻)

$\theta_B = ?$

$\theta_C = 0$

$\psi_{AB} = + \frac{|0.15+0.2|}{18} = 0.0194$

$\psi_{BC} = - \frac{|0.2|}{16} = -0.0125$

$M_{AB} = \frac{4EI}{L}[2\theta_A + \theta_B - 3\psi_{AB}] + FEM_{AB}$

$= \frac{4EI}{18}[0.06 + \theta_B - 3\times0.0194] - 24.69$

$M_{BA} = \frac{4EI}{18}[2\theta_B + 0.03 - 3\times0.0194] + 19.75$

$M_{BC} = \frac{2EI}{16}[2\theta_B + 3\times0.0125) - 85.33$

$M_{CB} = \frac{2EI}{16}[\theta_B + 3\times0.0125] + 85.33$

$\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$

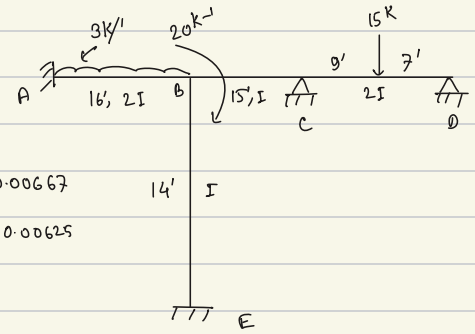
$\Rightarrow \frac{18000}{18}(2\theta_B + 0.03 - 3\times0.0194] + 19.75$

$+ \frac{9000}{16}[2\theta_B + 3\times0.0125) - 85.33 = 0$

$\Rightarrow \theta_B = 0.023$

$\therefore M_{AB} = 0.37^{K-'}$

P38/R08 $EI = 6000\ k\text{-}ft^2$



$\theta_A = 0$

$\theta_B = ?$

$\theta_C = ?$

$\theta_D = ?$

$\theta_E = 0.01$ (↷)

$\psi_{AB} = 0$

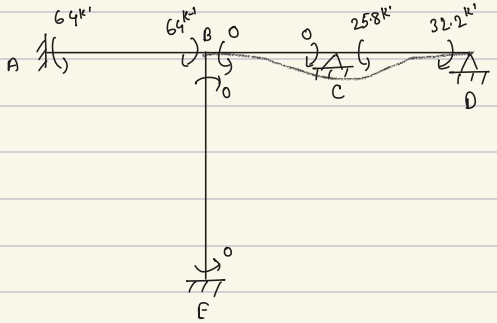
$\psi_{BC} = +\frac{0.1}{15} = +0.00667$

$\psi_{CD} = -\frac{0.1}{16} = -0.00625$

$\psi_{BE} = 0$

$m_{AB} = \frac{2EI}{L}[2\theta_A + \theta_B - 3\psi_{AB}] + FEM_{AB}$

$= 1250[\theta_B] - 64$



$m_{BA} = 1250[2\theta_B] + 64$

$m_{BC} = 666.7[2\theta_B + \theta_C - 3\times 0.00667]$

$m_{CB} = 666.7[2\theta_C + \theta_B - 3\times 0.00667]$

$m_{CD} = 1250[2\theta_C + \theta_D + 3\times 0.00625] - 25.8$

$m_{DC} = 1250[2\theta_D + \theta_C + 3\times 0.00625] + 32.2$

$m_{BE} = 714.3[2\theta_B + 0.01]$

$m_{EB} = 714.3[2\times 0.01 + \theta_B]$

At, Joint B: $m_{BA} + m_{BC} + m_{BE} = 20$

C: $m_{CB} + m_{CD} = 0$

D: $m_{DC} = 0$

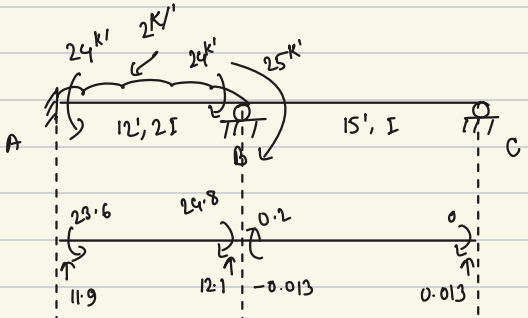
By solving: $\theta_B = -0.00914$

$\theta_C = 0.0155$

$\theta_D = -0.0300$

$m_{DC} =$

P5/



$\theta_A = 0$

$\theta_B = ?$

$\theta_C = ?$

$\psi_{AB} = 0$

$\psi_{BC} = 0$

$$M_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\psi_{AB}\right] + FEM_{AB}$$

$$= 0.333EI\left[\theta_B\right] - 24$$

$$M_{BA} = 0.333EI\left[2\theta_B\right] + 24$$

$$M_{BC} = 0.133EI\left[2\theta_B + \theta_C\right]$$

$$M_{CB} = 0.133EI\left[2\theta_C + \theta_B\right]$$

Joint B: $M_{BA} + M_{BC} = 25$

C: $M_{CB} = 0$

$EI\,\theta_B = 1.155$

$EI\,\theta_C = -0.578$

$M_{AB} = -23.6$

$M_{BA} = 24.8$

$M_{BC} = 0.2$

$M_{CB} = 0$

P07/

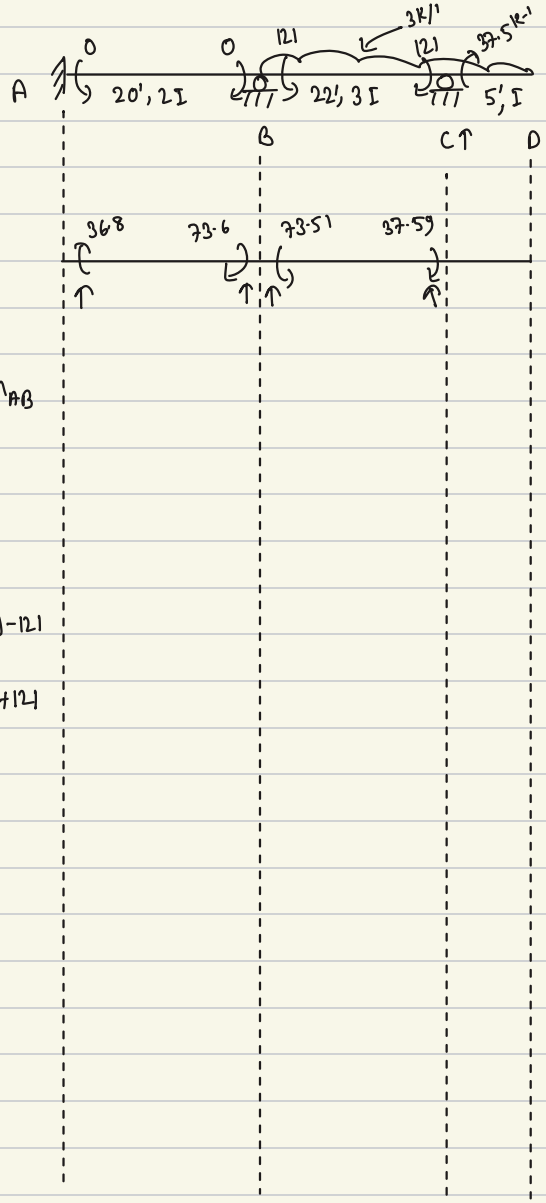
$\theta_A = 0$ $\quad EI = 5000\ k\text{-}ft^2$

$\theta_B = ?$

$\theta_C = ?$

$\psi_{AB} = 0$

$\psi_{BC} = -\frac{0.15}{22} = -0.00681$



$M_{AB} = \frac{2EI}{L}[2\theta_A + \theta_B - 3\psi_{AB}] + FEM_{AB}$

$= 1000[\theta_B]$

$M_{BA} = 1000[2\theta_B]$

$M_{BC} = 1363.6[2\theta_B + \theta_C + 3\times 0.00681] - 121$

$M_{CB} = 1363.6[2\theta_C + \theta_B + 3\times 0.00681] + 121$

Joint B: $M_{BA} + M_{BC} = 0$

C: $M_{CB} = 37.5$

$\therefore \theta_B = 0.0368$

$\theta_C = -0.0592$

$M_{AB} = 36.8$

$M_{BA} = 73.6$

$M_{BC} = -73.51$

$M_{CB} = 37.59$