Derived hydraulic relations

Alluvial channels are free to adjust their shape in response to changes in flow because they are competent to move the material that forms their boundaries.

The detailed hydrodynamics of how these channels establish their preferred dimensions and shape are complex and still not fully understood. However, the similarities in the behavior of these channels worldwide have resulted in development of empirical characteristics to predict channel behavior.

Hydraulic geometry describes the basic channel and flow properties of a river. The basic hydraulic geometry can be used to deduce how derived parameters such as shear stress and Manning's n vary downstream.

For a typical river, downstream hydraulic geometry is described by the following equations:

$$w = aQ^{0.5} \qquad d = cQ^{0.4} \qquad v = kQ^{0.1} \qquad s = tQ^{-1.0}$$

The Manning's roughness factor:

$$n = \frac{d^{2/3} s^{1/2}}{v}$$

The hydraulic geometry implies that the roughness factor must decline as bankfull discharge increases downstream.

$$n = \frac{(cQ^{0.4})^{2/3}(tQ^{-1.0})^{1/2}}{(kQ^{0.1})}$$

$$n = \left(c^{2/3} t^{1/2} k^{-1}\right) Q^{0.27-0.5-0.1}$$

$$n = C_3 Q^{-0.33} = \frac{C_3}{Q^{0.33}}$$

Similarly, we can repeat the exercise for the Darcy-Weisbach resistance coefficient and say that:

$$ff = \frac{8gds}{v^2}$$

The hydraulic geometry implies that friction factor must decline as bankfull discharge increases downstream.

$$ff = \frac{8g(cQ^{0.4})(tQ^{-1.0})}{(kQ^{0.1})^2} = \left(\frac{8gct}{k^2}\right) Q^{-1.0+0.4-0.2}$$

$$ff = C_2 Q^{-0.8} = \frac{C_2}{Q^{0.8}}$$

Manning-Strickler relation and the hydraulic geometry relations are used to derive a relationship between D_{50} and Q. The equation shows that bed-material grain-size (D_{50}) must decline as bankfull discharge increases downstream.

$$n = 0.0478 D_{50}^{1/6} = \frac{d^{2/3} s^{1/2}}{v}$$

$$0.0478 D_{50}^{1/6} = \frac{(cQ^{0.4})^{2/3} (tQ^{-1.0})^{1/2}}{kQ^{0.1}}$$

$$D_{50}^{1/6} = \left[\frac{c^{2/3} t^{1/2} k^{-1.0}}{0.0478}\right] Q^{-0.33}$$

$$D_{50} = \left[\frac{c^{2/3} t^{1/2} k^{-1.0}}{0.0478}\right]^6 \left(Q^{-0.33}\right)^6$$

$$D_{50} = C_4 Q^{-2.0} = \frac{C_4}{Q^2}$$

Chezy's coefficient

$V = C (RS)^{1/2} = C (dS)^{1/2}$ …… (1)

$V = (1/n) R^{1/6} (RS)^{1/2} = (1/n) d^{1/6} (dS)^{1/2}$ …….. (2)

From 1nd 2:

$C = (1/n) d^{1/6} = (C_1 Q^{0.33}) (C_2 Q^{0.06}) = C_3 Q^{0.39}$

The hydraulic geometry implies that the Chezy's coefficient must increase as bankfull discharge increases downstream.

Hydraulic Geometry Characteristics: change in velocity

Manning's Equation

$$V = \frac{R^{2/3} S^{1/2}}{n}$$

V is average velocity (m/s)
R = hydraulic radius (m)
S = energy slope (m/m)
n = Manning's roughness coefficient

As the river moves downstream:
S ⟶ decrease
Sediment size ⟶ decrease
Hence n ⟶ decrease

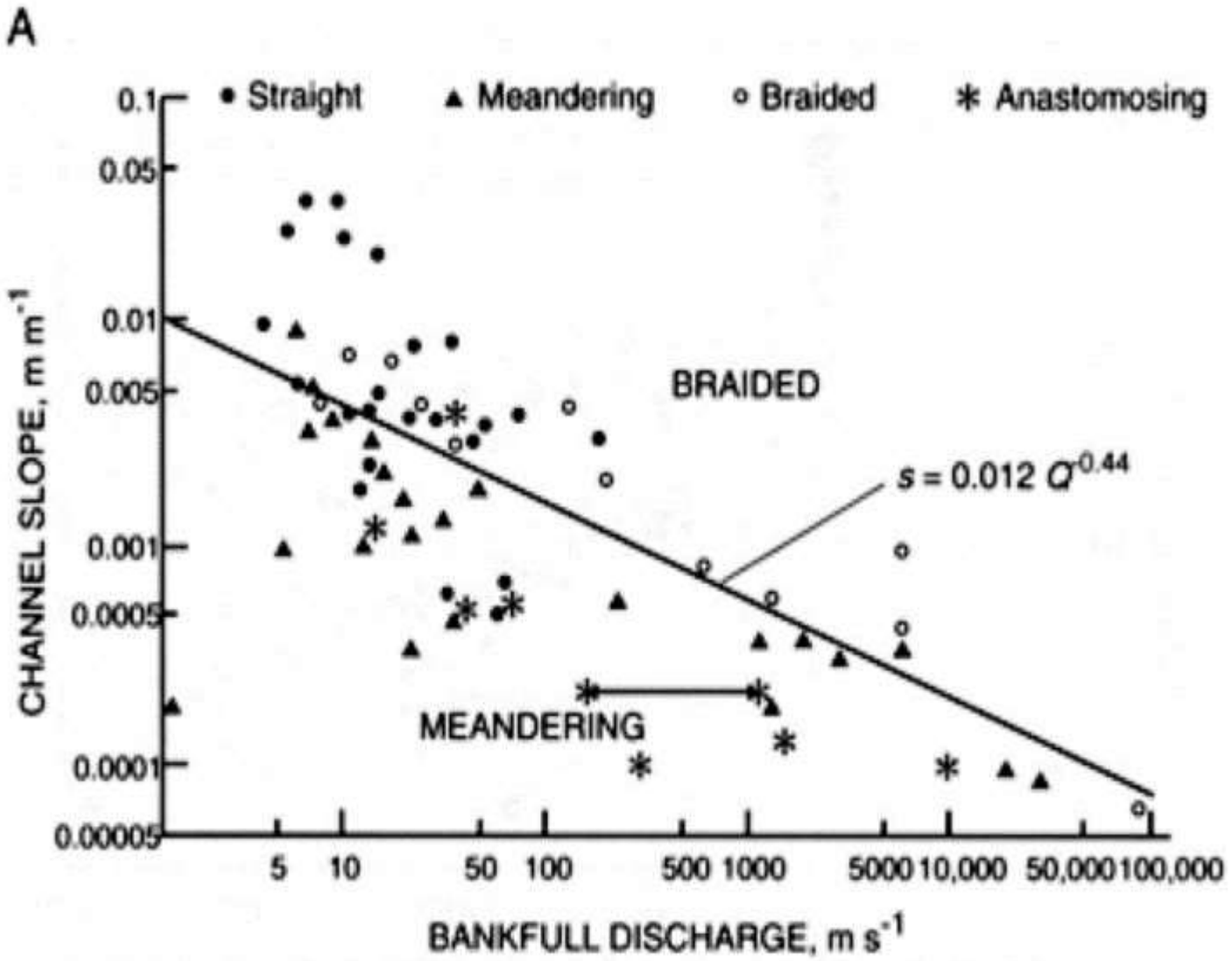
So, $V \alpha R^{2/3} \alpha d^{2/3}$
As the river moves downstream
d ⟶ increases
Hence, V ⟶ increase down-stream (not at a big change)

Flow/Channel Property	Downstream
Slope	Declines downstream so $s = gQ^{-z}$
D_{50}	Declines downstream so $D_{50} = gQ^{-z}$
Channel width	Remarkably constant for typical gravel-bed rivers so that $w = aQ^{0.5}$ where $a \cong 5$ In noncohesive sediments exponent can be high: $w = aQ^{0.9}$
Flow velocity	Declining slope and grain size often tend to cancel each other so velocity changes little or declines so Cohesive channels: $v = kQ^{0-0.1}$ Typical gravel-bed: $v = kQ^{0-0.1}$ Noncohesive channels: $v = kQ^{0-0.1}$
Channel depth	Typically, for Cohesive channels: $d = cQ^{0.3-0.4}$ Typical gravel-bed: $d = cQ^{0.3-0.4}$ Noncohesive channels: $d = cQ^{0.3-0.4}$
Flow resistance	Tends to decline or be constant; as slope and grain size declines tend to cancel so with increasing discharge, $ff = hQ^{0-0.1}$

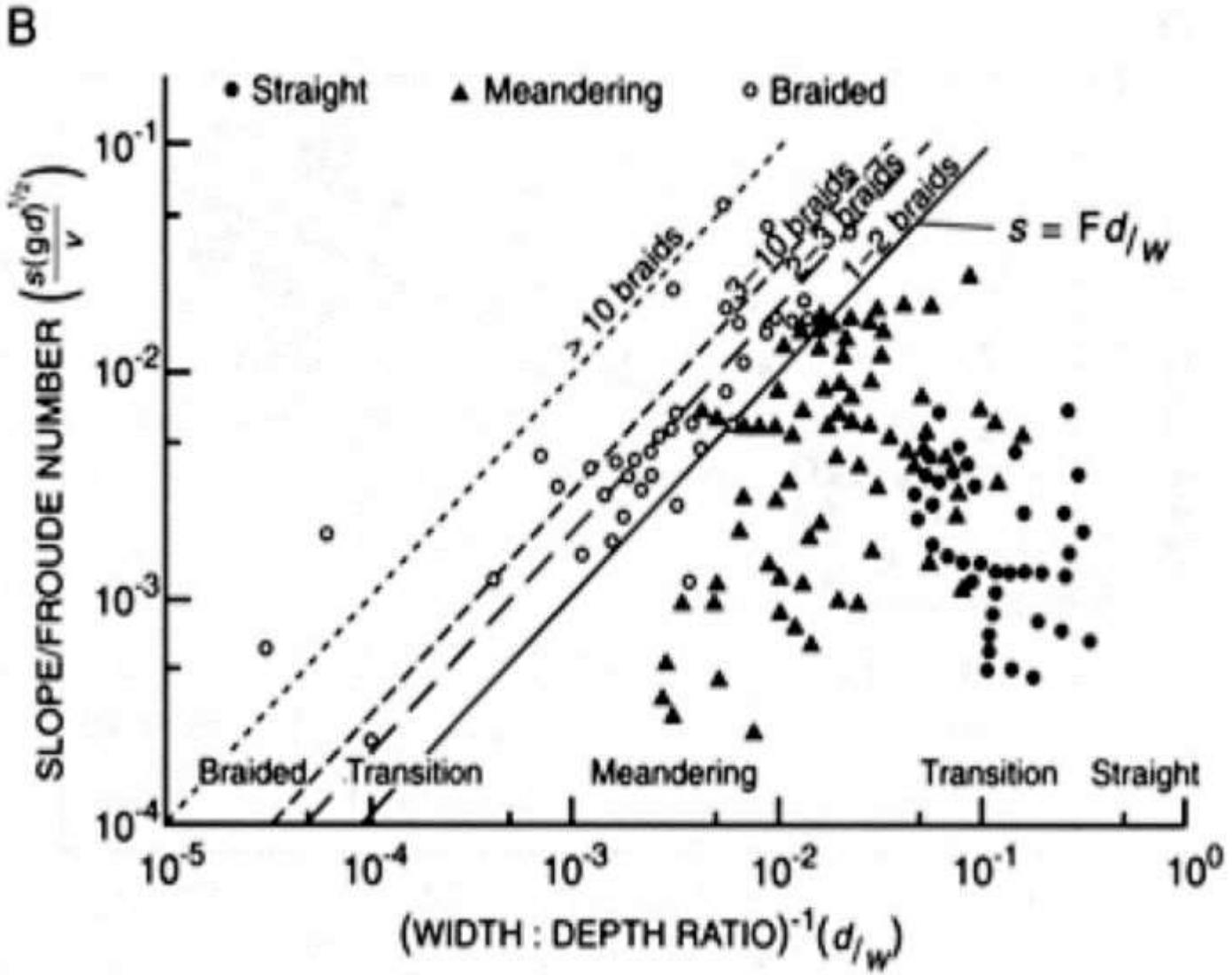
Effect of S, w/d Ratio and Sediment type on River Pattern

Meandering channels could be distinguished from braided channels on a graph of slope and bankfull discharge (Figure A). At a given bankfull discharge, meanders occur at lower slopes than braided channels and critical slope for planform transition ($S = 0.012\ Q^{-0.44}$) declines as bankfull discharge increases.

Figure B shows the plot of (s/F) and (d/w) in which single-thread channels (straight and meandering) are separated from braided channels and braided channels are subdivided by the number of active channels (2-10 braids). Here F is the Froude number.
The form ratio (w/d) can be taken as a measure of the strength of the boundary materials (wide shallow channels have high w/d values and weak banks while narrow deep channels have low w/d values and strong banks). The inverse form ratio (*d/w)* therefore indicates increasing bank-strength from left to right *on* the abscissa of the graph. The transition from single to multiple channels occurs at a given value of s/F on the ordinate as bank-strength decreases.

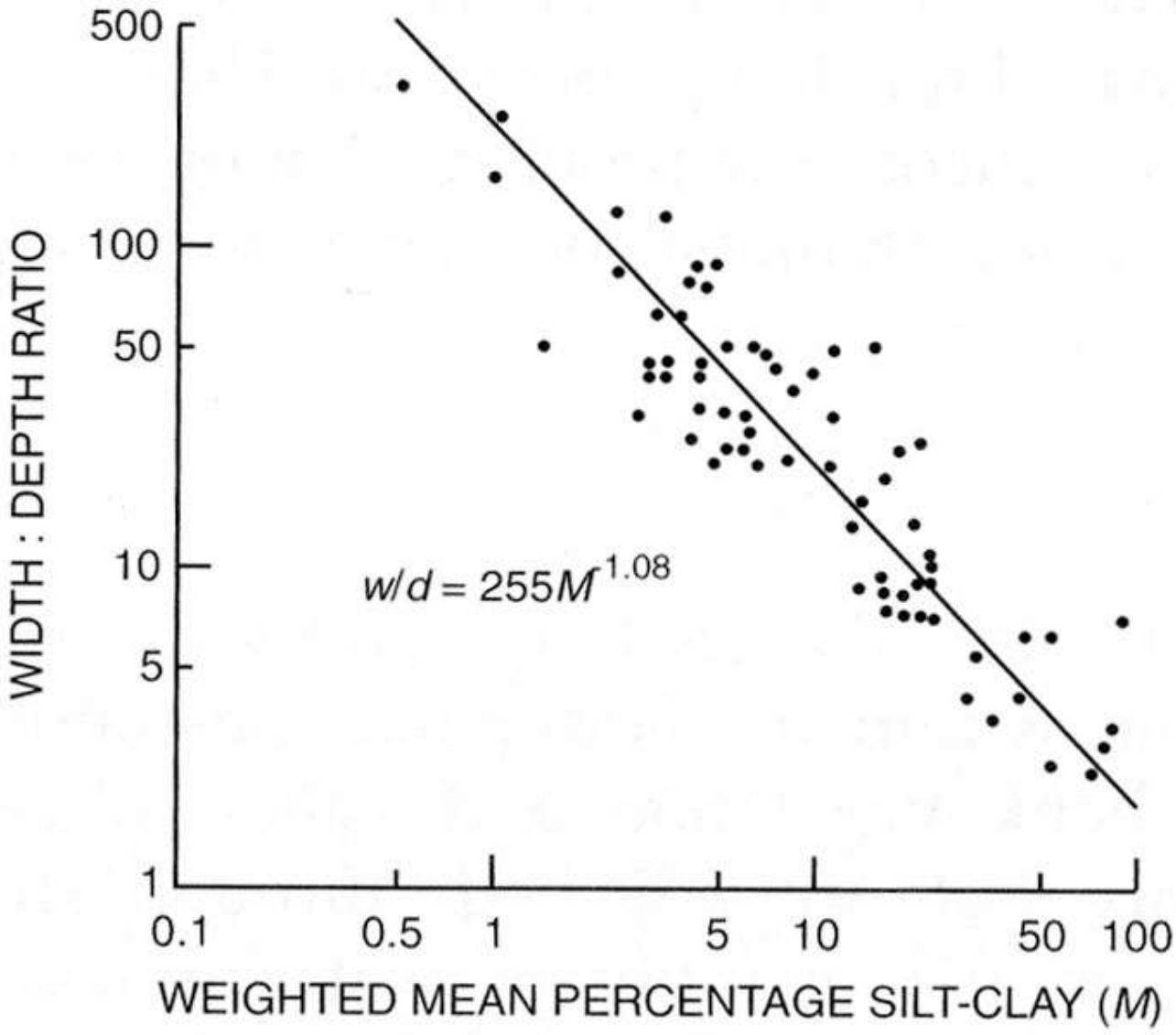


Meandering channels are formed at a lower slope.
The critical slope (between braided and meander) decreases with the increase in discharge. Higher discharge is associated with lower slope.



Braids ⟶ Active channels in a section

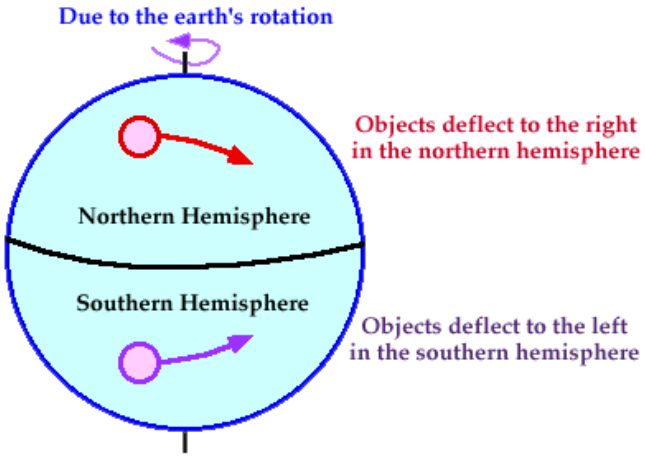
Meandering channels would be more in cohesive soil with high contents of silt-clay and hence would have a low width/depth ratio (< 10).
High width/depth ratio is associated with bed load (low % silt-clay) and braided channel.



Meandering River

Several theories /hypotheses have been proposed to understand the process of meandering:

1. Earth's rotation theory – any object moving over earth's surface experiences a force normal to its path (known as Coriolis force). This force (to right in northern hemisphere and to left in southern hemisphere) deflects the river and results in meanders.



2. Disturbance theory – the disturbance caused by various reasons (deposition of sediment load, obstacle etc.) in a straight channel may cause an asymmetry in flow which may result in meanders.

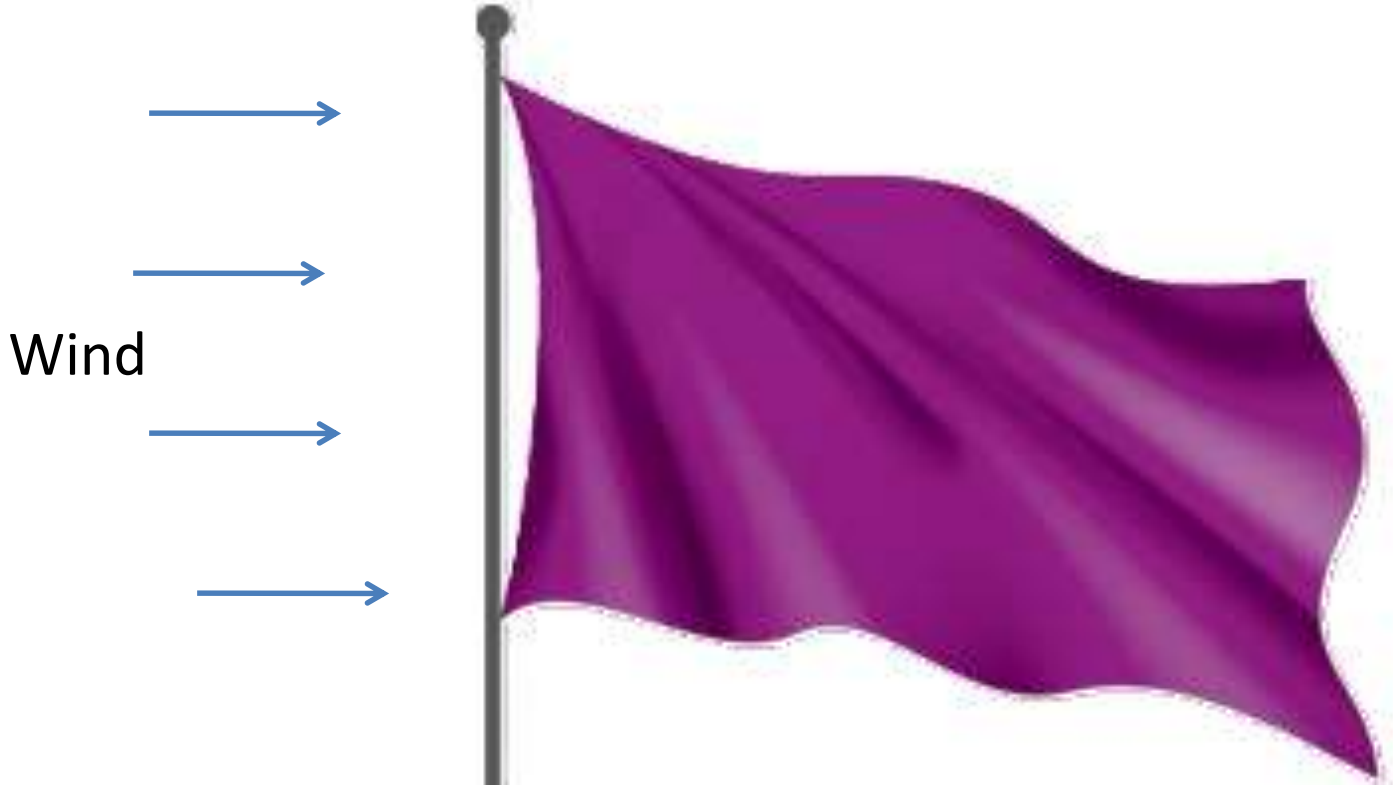
The primary cause of disturbance is excess of total sediment load during floods. A river tends to build a steeper slope by depositing the sediment on the bed when the sediment load is in excess of that required for equilibrium. This increase in slope reduces the depth and increases the width of the river channel if the banks do not resist erosion. Only a slight deviation from uniform axial flow is then required to cause more flow towards one bank than the other. Additional flow is immediately attracted towards the former bank, leading to shoaling along the latter ascending the curvature of the flow and finally producing meanders.

3. Excess energy theory – any channel having excess energy tries to increase its length by meandering (increase in sinuosity) thereby decreasing its slope. Thus, the minimum energy (equilibrium) is associated with the formation of meanders.

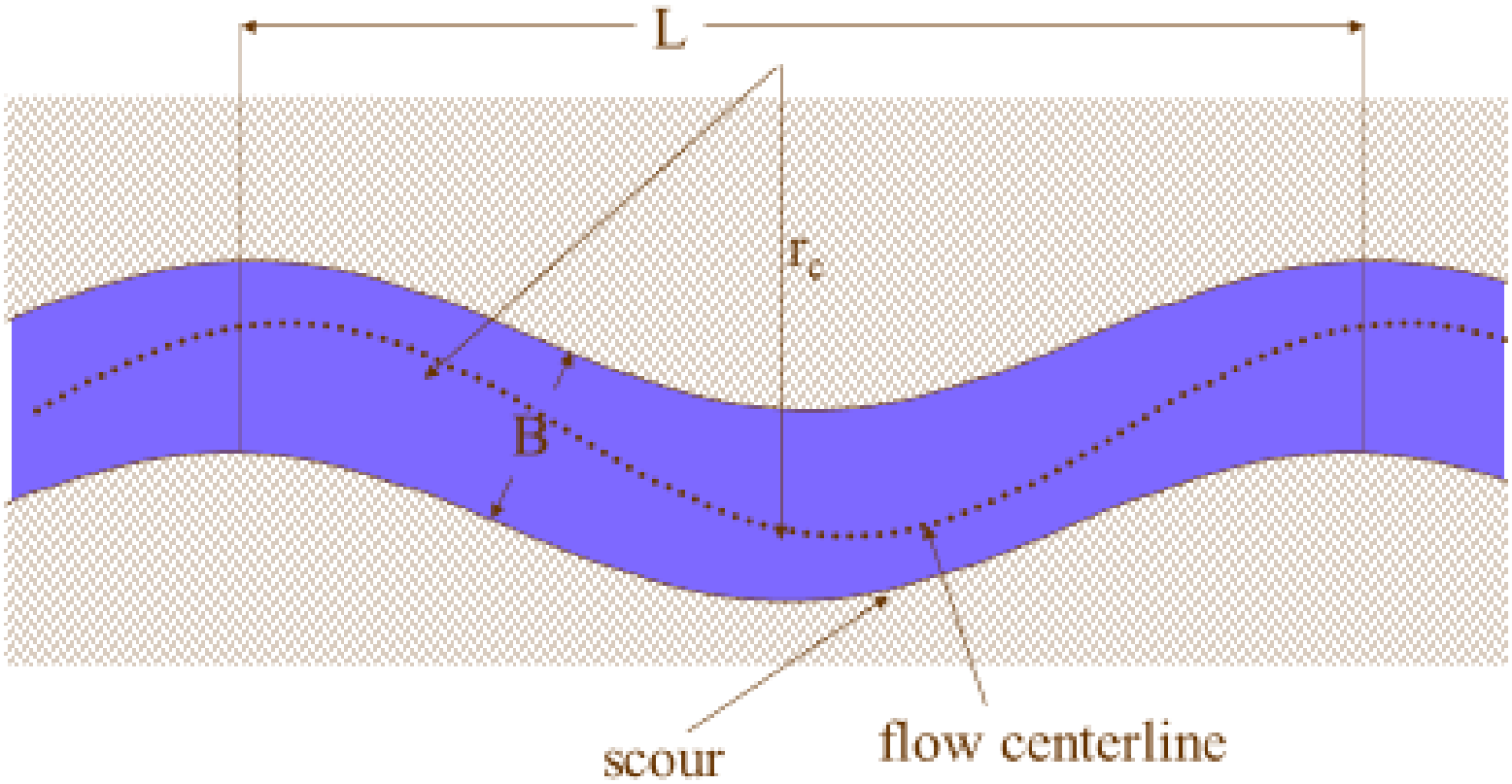
4. Secondary flow theories – Any disturbance in the river may cause secondary currents leading to asymmetry and producing meanders.

5. General wave motion arguments (Flag fluttering)

Meandering is an oscillatory motion in space and that the resulting waveform is similar to many other naturally oscillating flow systems and likely part of the same general phenomenon. When a fluid flows so that it exerts a shearing force on its boundaries it has been shown that instability in these turbulent shearing flows can develop and cause the fluid to oscillate. The precise reasoning is complex and demonstrable only in terms of some rather complicated mathematics.

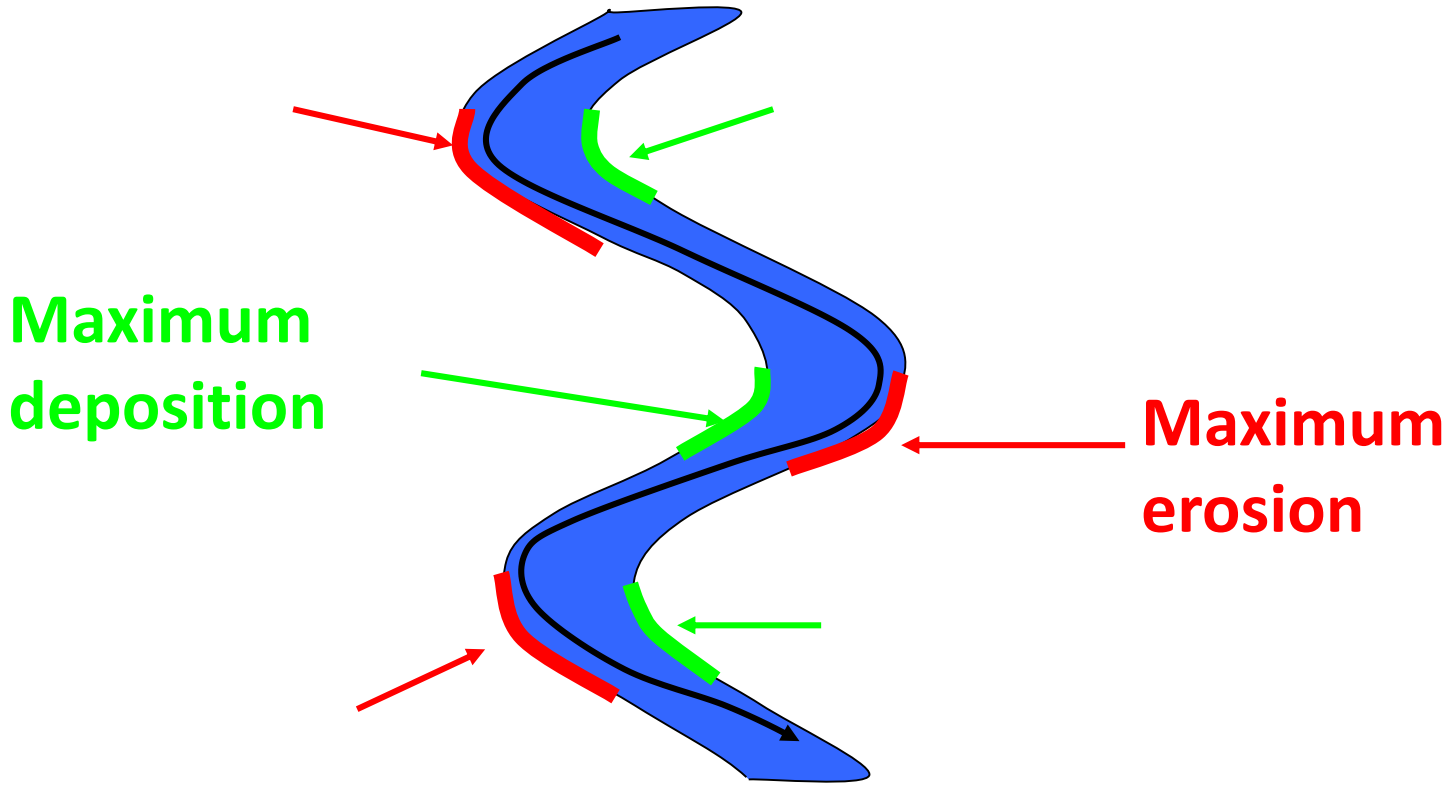


Dimensional Relationships and Stability in Meandering Channel



$\frac{L}{B} \approx 7 \text{ to } 10$ $\frac{r_c}{B} \approx 2 \text{ to } 3$ surprisingly small variation!

Erosion and Deposition in a Meandering River



The regions of erosion and deposition are predictable (known) in a meandering river (pool) unlike a braided river (which is both unstable and unpredictable).

With data collected from almost 200 rivers world-wide, a number of consistent relationships emerge for meandering rivers. These can be used to evaluate the relative stability or degree of disturbance in a river under consideration.
They can also be used as design guidelines for constructed channels, assuming that a meandering river is the most stable channel pattern for the conditions of discharge, sediment load, and slope.

By defining the following variables:

r_c = the radius of curvature of the channel centerline;
Λ = the down valley distance of one complete meander bend (i.e. the wave length or meander length);
W_{bf} and d_{bf} = bankfull width and depth;
P = the sinuosity (channel length/wave length)

The most useful relationships, with all measurements in meters, are then as follows:

- $1.5 < P < 2.0$, for most "true" meandering channels (27)

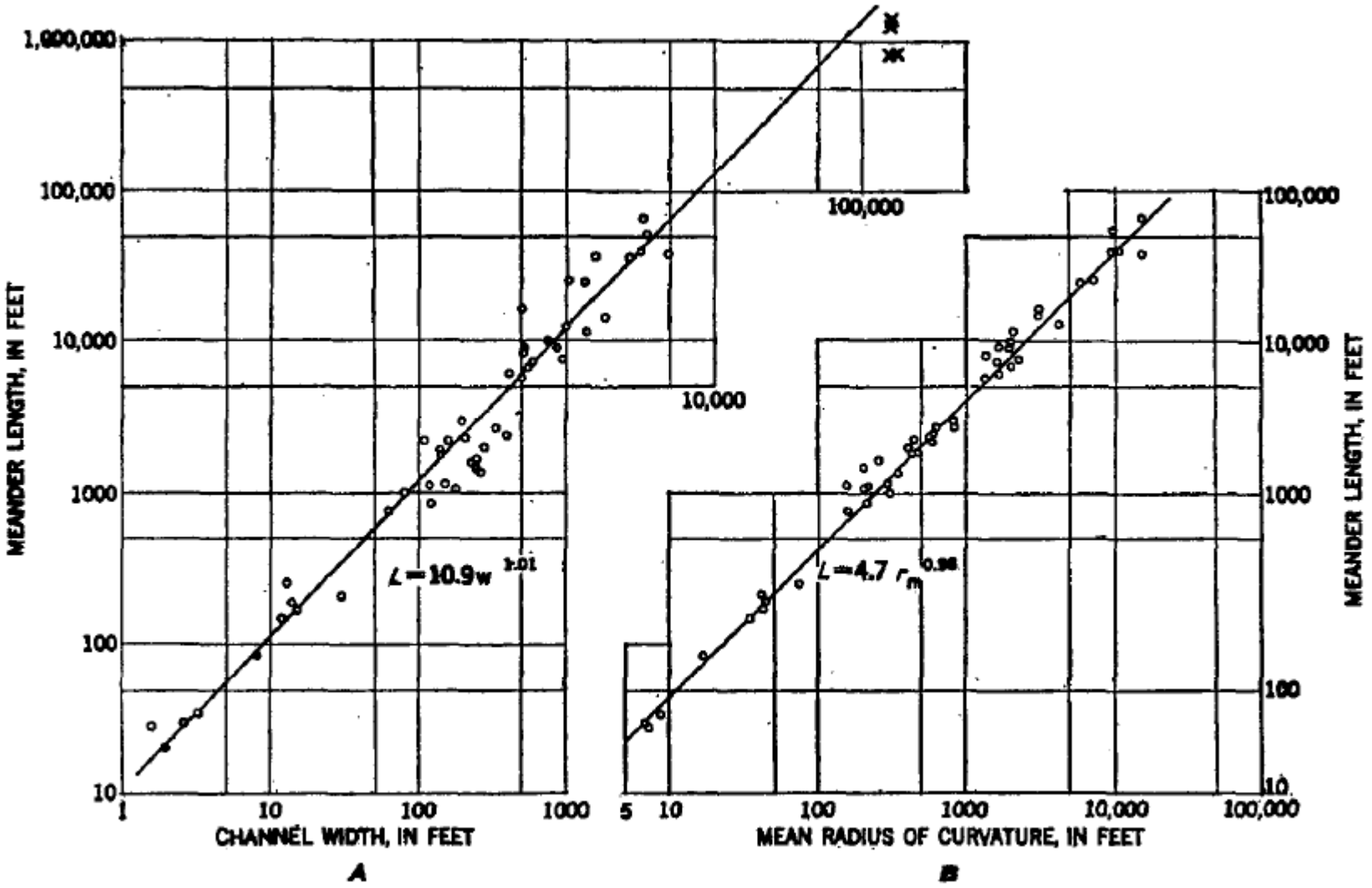
- $r_c = 1.5\, w_{bf}^{1.12}$ (28)

- $\lambda = 4.53\, r_c\, (= 6.8\, w_{bf}^{1.12})$ (29)

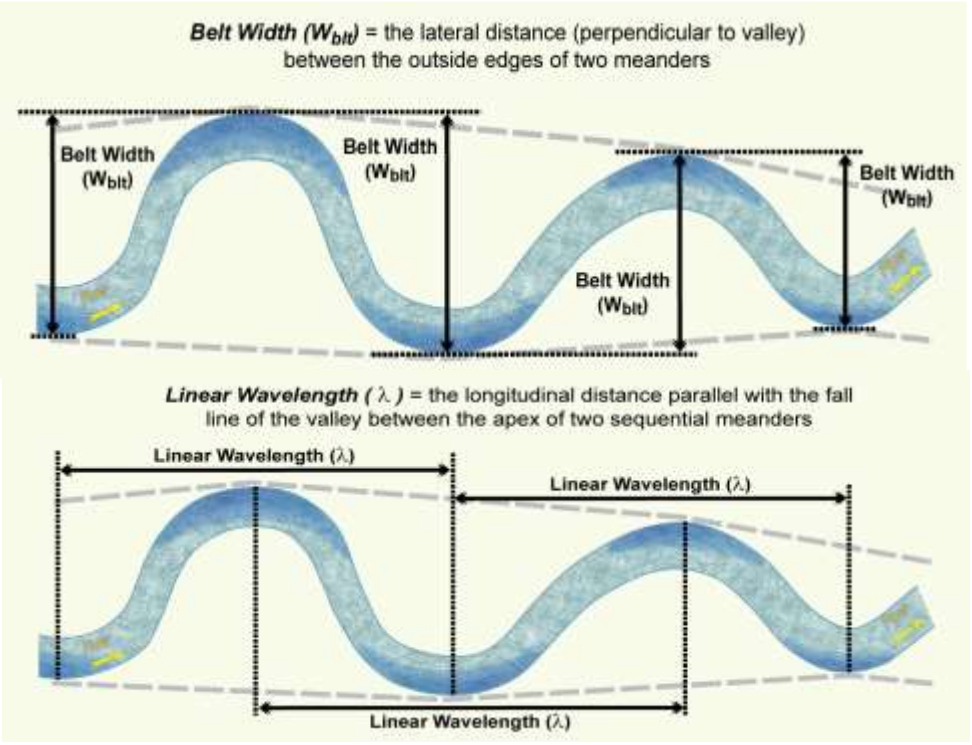
  $\Lambda / 4.53 = r_c$ or $\Lambda = 6.8\, w^{1.12}$

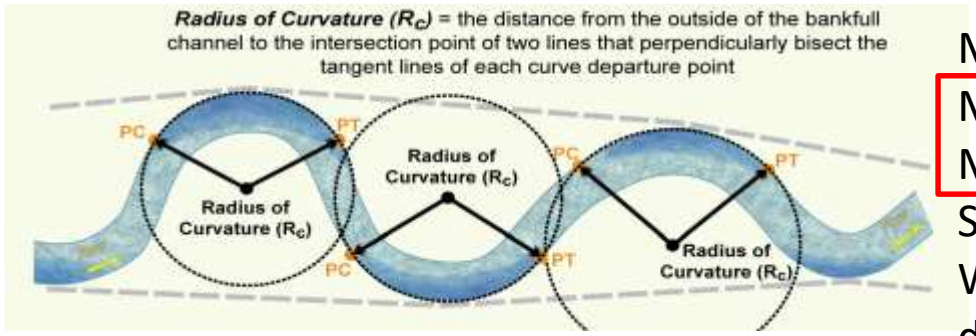
- $\lambda = 29.3\, (w_{bf} d_{bf})^{0.65}$ (30)

Λ (meander length) is a direct function of bankfull width and radius of curvature but not bankfull depth.



RELATIONS BETWEEN MEANDER LENGTH AND CHANNEL WIDTH (LEFT DIAGRAM) AND MEAN RADIUS OF CURVATURE (RIGHT DIAGRAM)





THE VARIABLES

Q = discharge of water;
Q_s = discharge of sediment,
υ = kinematic viscosity of water
η = dynamic viscosity and density respectively;
D = mean size of sand plain material (50 per cent of material on the cumulative curve is finer than the size D),
M_L = meander length
M_B = meander width;
R_M = bend radius;
B = surface width :
H = depth of flow
S = valley slope

$M_L = 36\ Q^{0.5}$

$M_L = 10.9\ B^{1.01}$

$M_L = 4.7\ R_M^{0.98}$

$S = 0.06\ Q_b^{-0.44}$

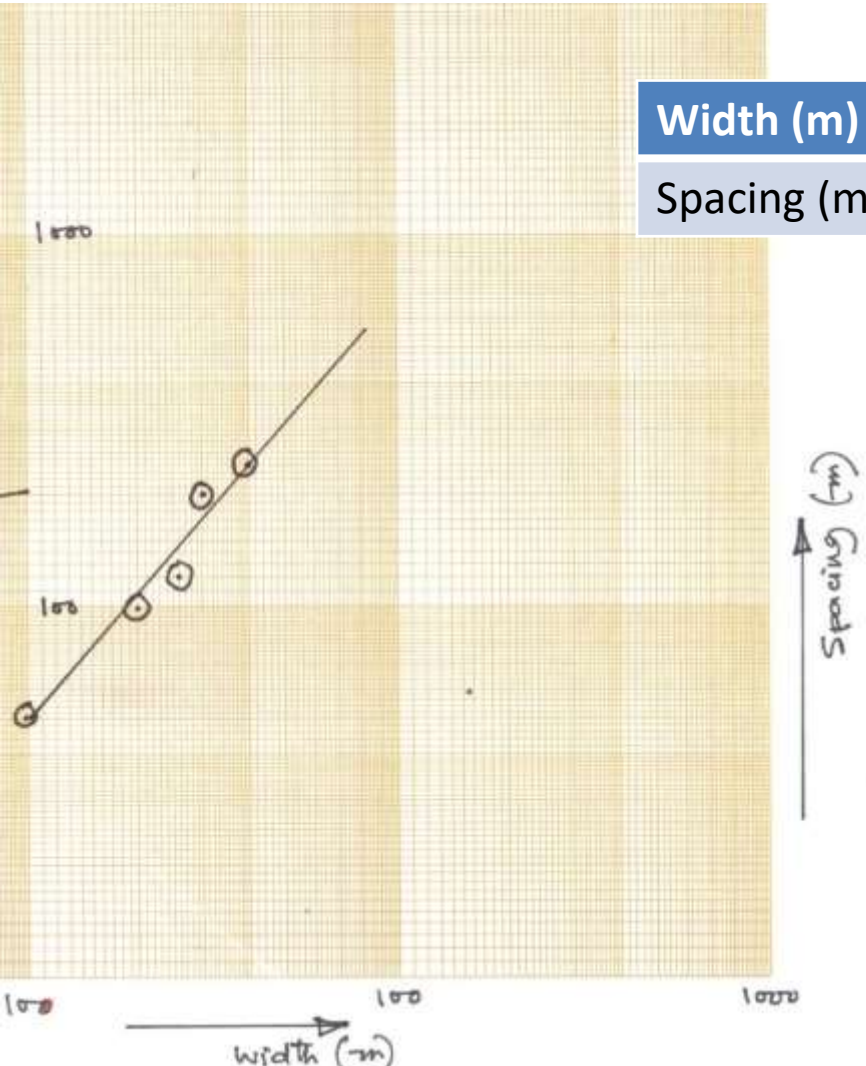
Where Q is in cfs and Q_b is the bankfull discharge

Slope is inversely related to discharge – meanders with high discharge occur at low slopes.

Problem

The pool to pool spacing of some meander channels and the corresponding river widths are shown in the following table. Develop the meander length-width relationship.

Width (m)	10	20	25	30	40
Spacing (m)	50	100	120	250	300



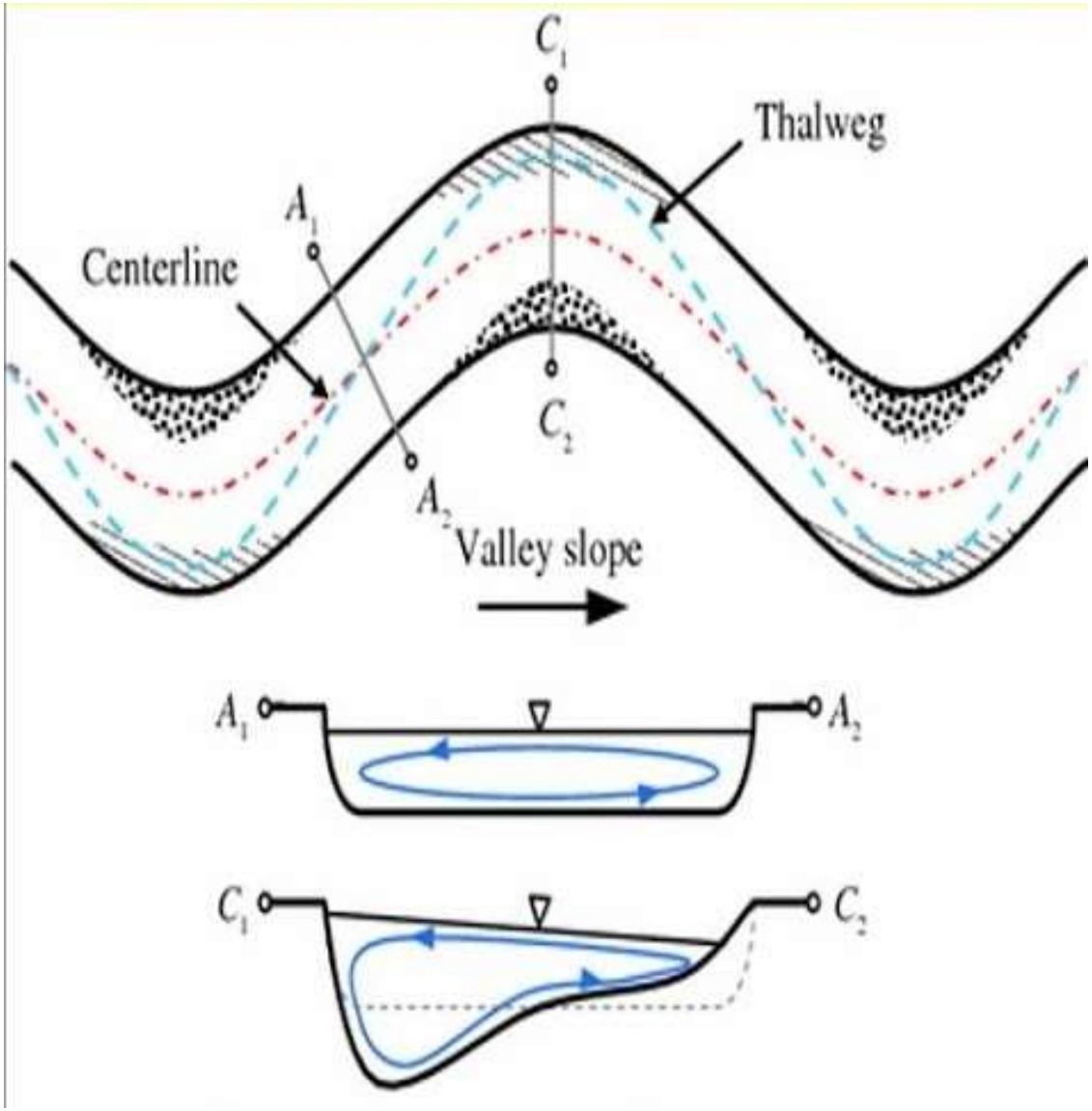
The relationship from the log-log Plot is:

$SP = 2.78\ W^{1.16}$

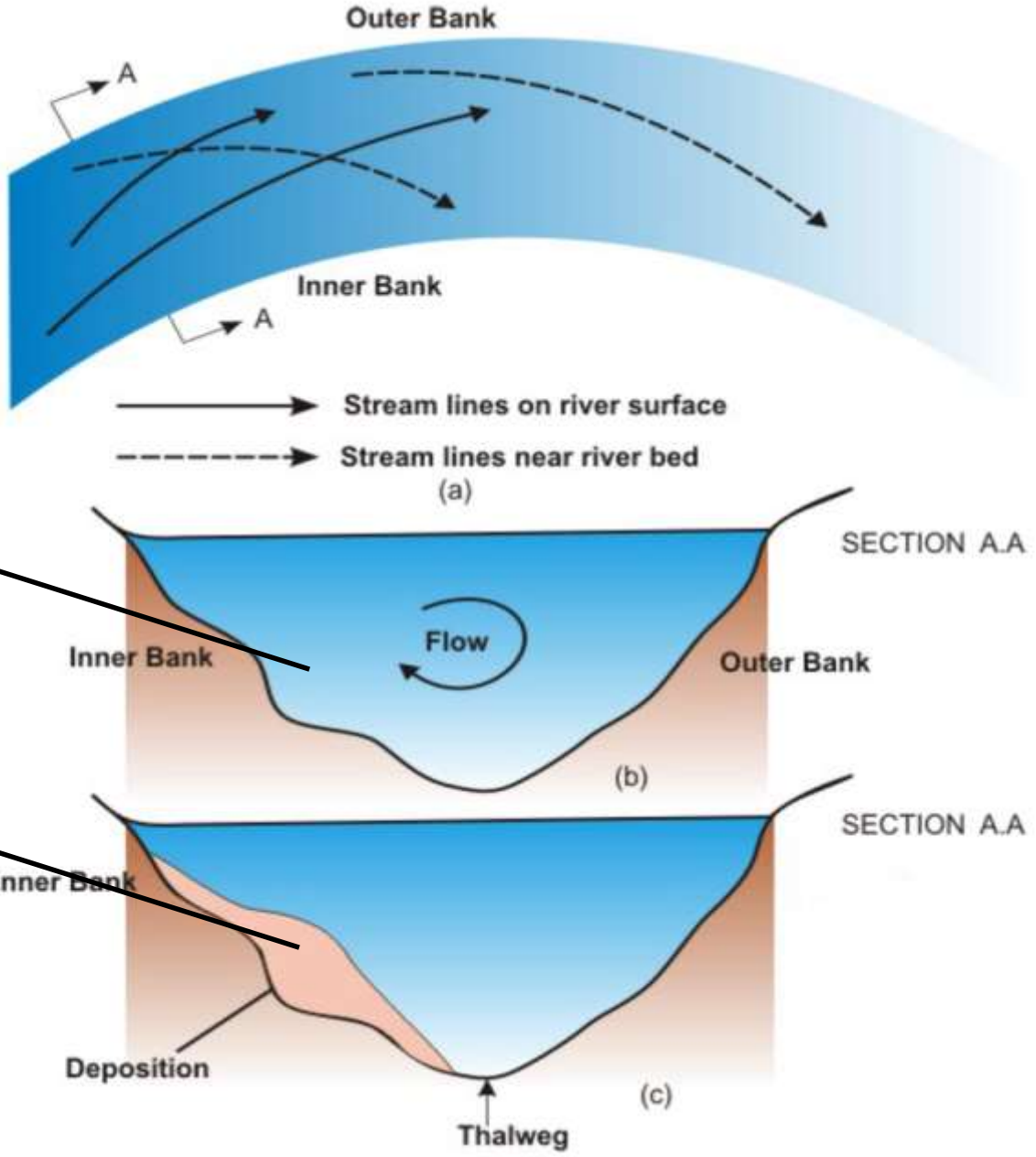
Hence, the meander length would be:

M_L = ??

Pool Formation in Meandering River and Helical Current



Pool Formation in Meandering River and Helical Current



Secondary current/ Helical flow

Point bar

Point bar – bar formed in the inner bend of a meandering river due to deposition of sediment from outer bend by secondary current.

(Pools and point bars)

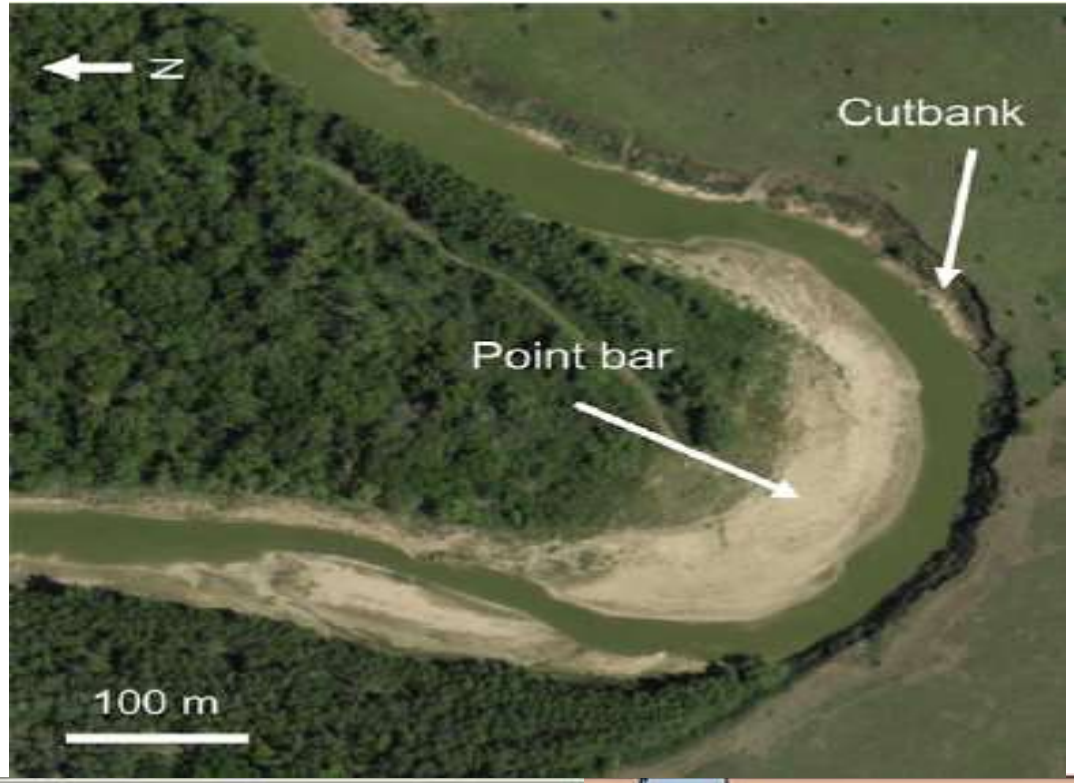


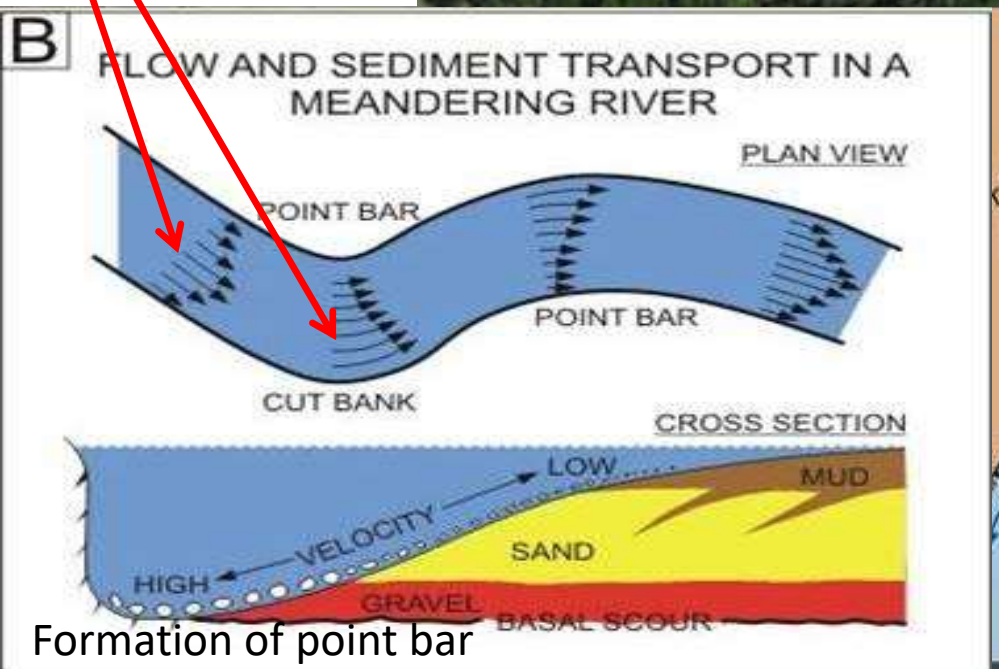
(b)

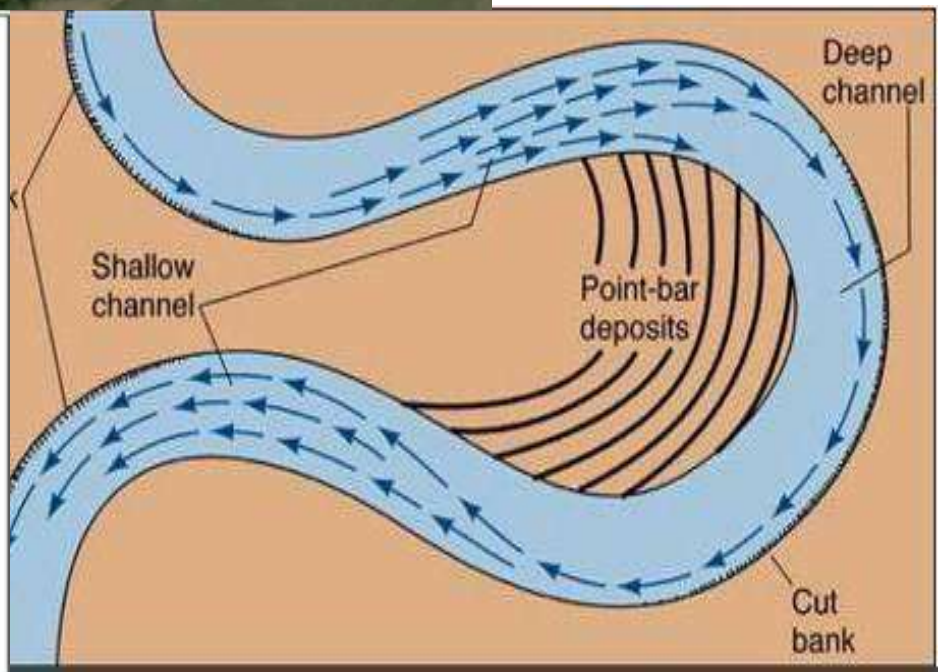
FIGURE 17.12

Earth: Portrait of a Planet, 2nd Edition
Copyright (c) W.W. Norton & Company

Symmetry and asymmetry



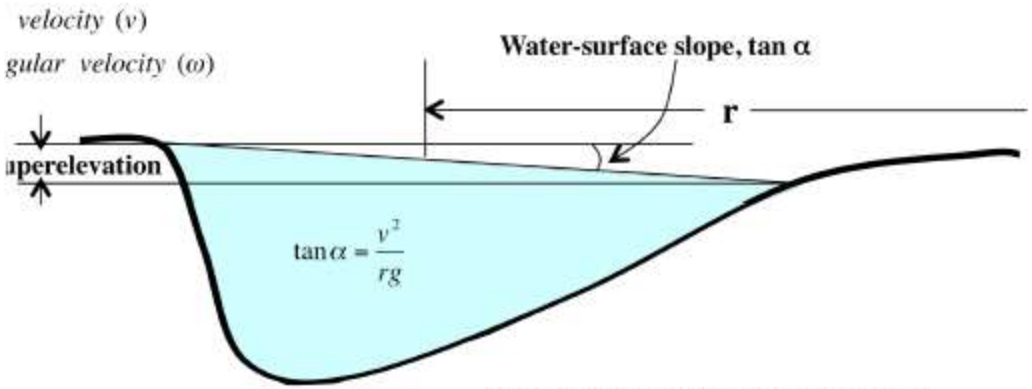




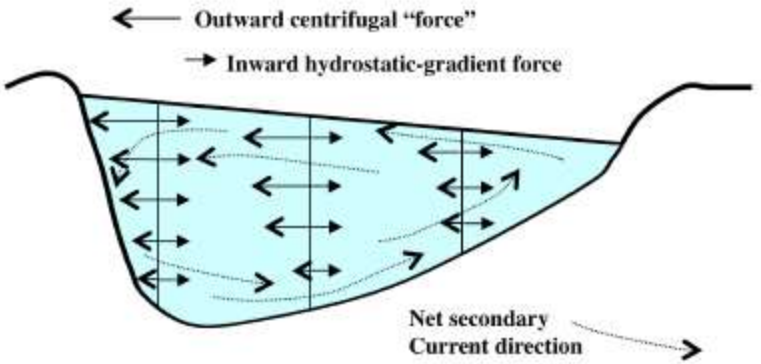
Formation of point bar

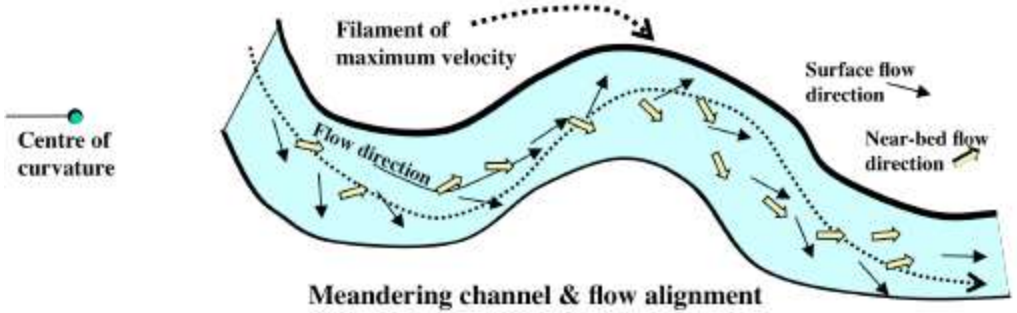
Friction at the boundary is important because it causes velocity to vary throughout the section (lower near the bed and higher in the upper flow) and in turn causes the centrifugal force to vary in the same way. The transverse water-surface gradient exerts a hydrostatic force towards the centre of bend curvature that causes the water to flow towards the inside bank. Although it is opposed by the centrifugal force this force varies with velocity and depth so that in a vertical the two forces are not balanced at all points.

Centrifugal force dominates at the surface and the hydrostatic force dominates near the bed. This causes a rotating secondary circulation in two dimensions and is called *helical flow*. Helical flow helps in the formation of the point bar.



Channel cross-section on a river bend





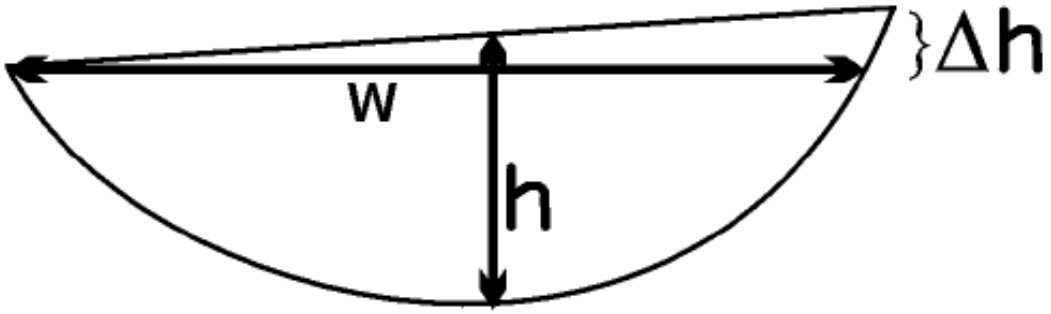
Meandering channel & flow alignment

Superelevation

The centrifugal force of the water accelerating around the bend must be balanced by the cross-stream pressure gradient imparted by the superelevation (Δh):
Hence, Centrifugal force = Hydrostatic force

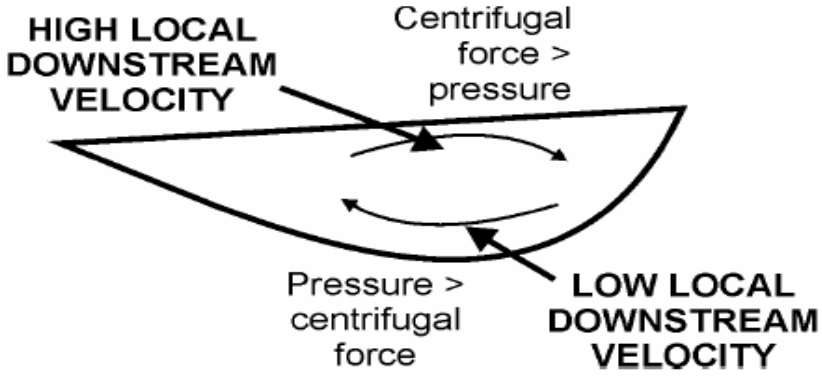
$$\frac{\rho u^2 h}{r_c} = \rho g h \frac{\Delta h}{w}$$

$$\Delta h = \frac{u^2 w}{r_c g}$$



Note the typical scale of the superelevation: rc / w is about 2 or 3; for a river with a flow of about 1 m/s, Δh is a few cm.

The pressure gradient balances the *average* flow velocity, but near the surface the local velocity exceeds the average velocity and so the centrifugal force exceeds the pressure; at depth near h, average velocity exceeds the local velocity and so the pressure exceeds the centrifugal force:



So bottom flow is towards the inside of the bend.

This is secondary flow (helical flow).