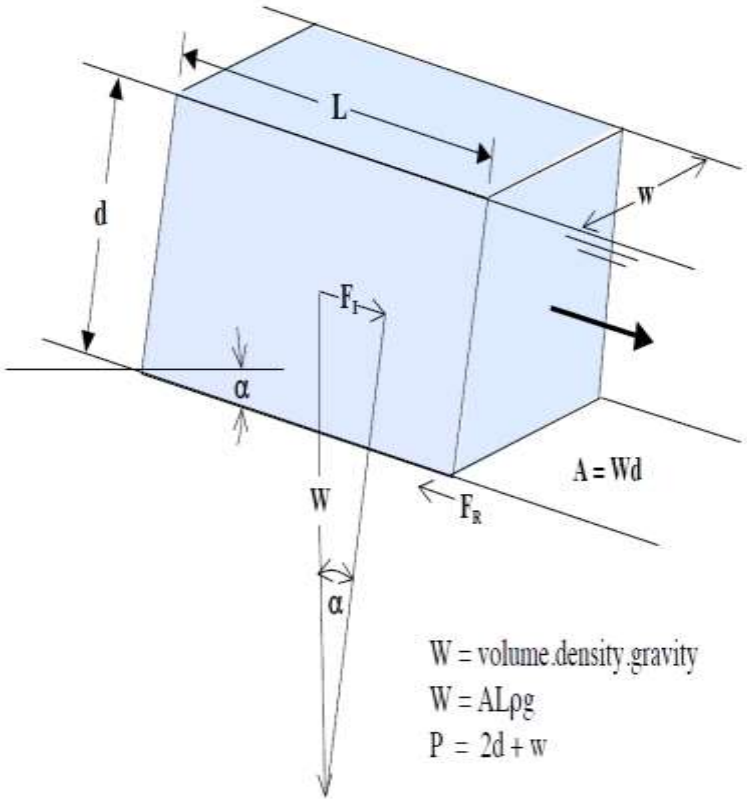
Forces Acting in a River

Shear Stress: τ_c

The balance of forces applied to river determines how it will move. Water moves through a channel because the impelling force (Fg) of gravity is able to overcome the frictional resisting force (FR) opposing to flow. The Fg , is the force of gravity acting over unit area on the bed. This force is simply:

$$Fg = \sin\alpha ALpg$$

The resisting force, FR, is taken to be the force of friction being exerted by the bed And vertical banks on the flow moving in Our rectangular channel. This frictional force acts in the plane of the channel boundary so it is a *shear stress* (τ_0). Stated mathematically: $FR = \tau_0 (2d+w)L \cos\alpha$



Forces Acting in a River

Since Fg = FR:

$\sin\alpha ALpg = \tau_0 (2d+w)L \cos\alpha$

$\tau_0 = \sin\alpha ALpg / (2d+w)L \cos\alpha = pgR \tan\alpha$ since $(2d+w) = P$

For $\tan\alpha = s$ and

For wide channel $R \approx d$ (denoted as h)

So, $\boxed{\tau_0 = pghs}$

Stream Power: Ω

Stream power is a measure of the main driving forces acting in a channel and determines a river's capacity to transport sediment and perform geomorphic work. It is defined as:

Stream power is the rate of energy dissipation against the bed and banks of a river per unit downstream length. It is given by the equation:

$\Omega = pgQs = pgwhVs = Vw\tau_0$; Hence, stream power/width = $\boxed{\Omega/w = V\tau_0}$

where Ω is the **stream power**, ρ is the density of water, g is acceleration due to gravity, Q is discharge (m^3/sec), and S is the channel slope.

Forces Acting in a River

Flow Resistance

$v = C\sqrt{hs}$ (Chezy's equation)

Since $\tau = \rho ghs$, we can say that

$V = C\sqrt{hs} = C\sqrt{(\tau/\rho g)}$

The shear stress has been shown to be proportional to the velocity squared: Or $v \propto \sqrt{\tau}$

$\tau \propto V^2$

Again: $\Omega = \rho g Q s = \rho g w h V s \approx w V^2 V$; Hence, stream power/width;

$\Omega \propto V^3$ Since $\Omega = V \tau_0$

Strickler's equation for sediment roughness:

$V = k R^{2/3} S^{1/2}$ where $k = 21.1 / d^{1/6}$ and d is the d_{50}

Considering Manning's n with Strickler's k: $1/n = k$

or $1/n = 21.1 / d_{50}{}^{1/6}$

$n = 0.047\, d_{50}{}^{1/6}$ (relating Manning's n with sediment diameter)

Sediment Movement

Competence (*also called* ***calibre)*** refers to the largest size (diameter) of sediment particle or grain that the flow is capable of moving; it is a hydraulic limitation. So a river may be competent or incompetent with respect to a given grain size.

Capacity refers to the maximum amount of sediment of a given size that a stream can transport in traction as bed load. Given a supply of sediment, capacity depends on channel gradient, discharge and the **calibre** of the load (the presence of fines may increase fluid density and increase capacity; the presence of large particles may obstruct the flow and reduce capacity).

Fall Velocity: The fall velocity of sediment particle in quiescent column of water is the average velocity that a particle would attain if falling alone in water of infinite extent at temperature of 24^0 C. The fundamental property governing the motion of a sediment in a fluid is its fall velocity and is function of its volume, shape, density, surface roughness and the viscosity and density of the fluid. Fall velocity of particle can be calculated from a balance of buoyant weight and the resisting force resulting from fluid drag.

Basic Mechanism of Sediment Transport

The ability of the channel to erode and transport sediment depends on the balance between:

(1) gravitational forces acting to settle particles on the bed and
(2) drag and lift forces that act to either suspend them in flow or move them along downstream.

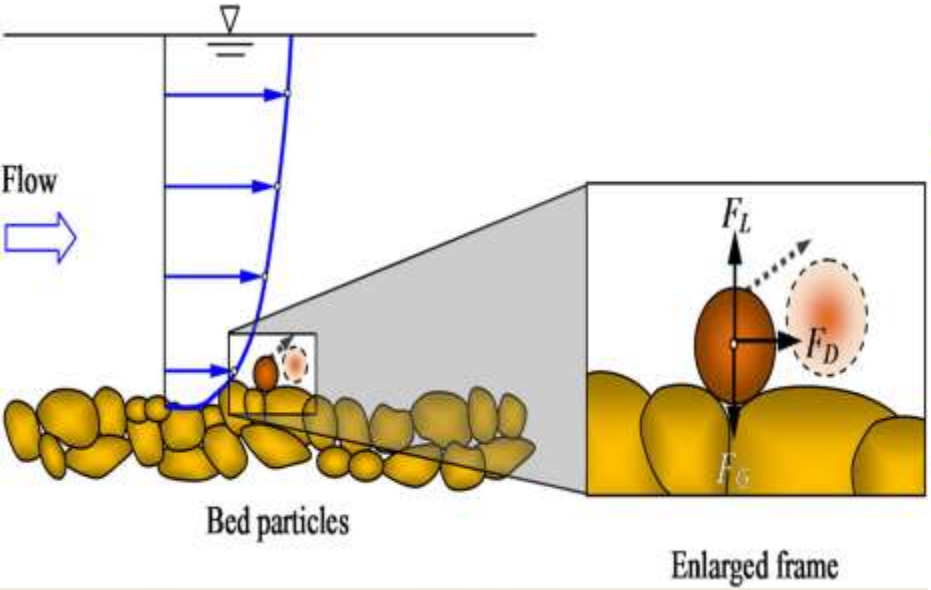
Assuming that the sediment is non-cohesive, opposing forces act on the grain:
- the submerged weight of the grain acts to hold it on the bed;
- the fluid lift and drag forces act to lift, roll, and slide the grain along the bed.

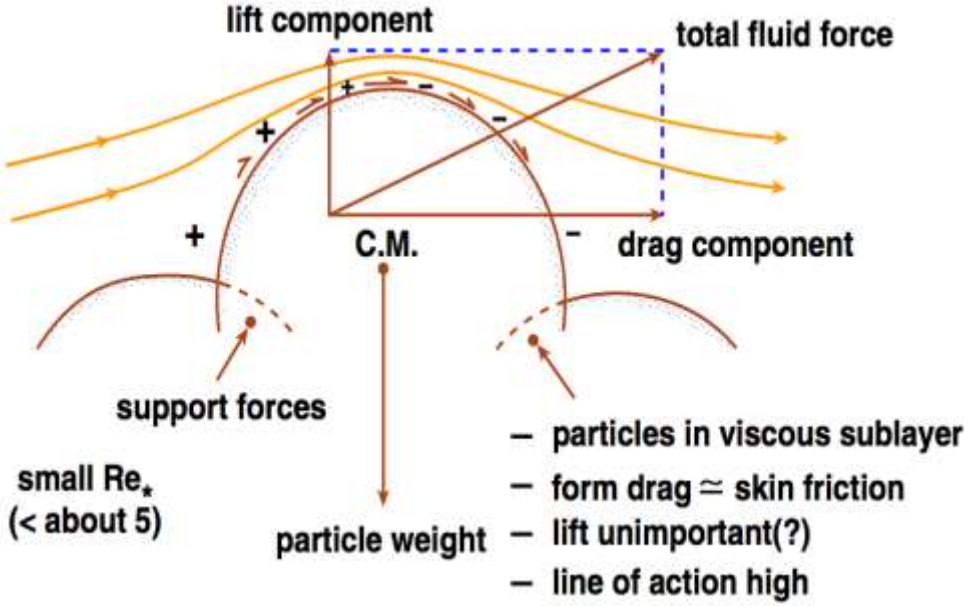
All of these forces are highly variable in nature and the general problem of sediment transport is an incredibly difficult, complex and still not clearly understood. Hence, many simplifications, assumptions and empirical derivations are used in sediment transport.

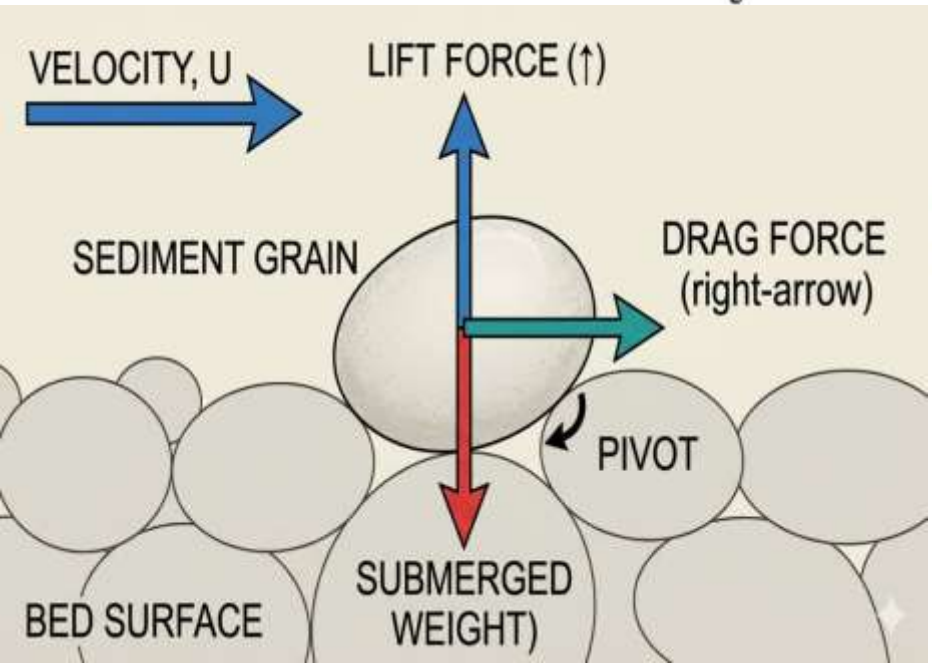
There are three ways that grains can move:

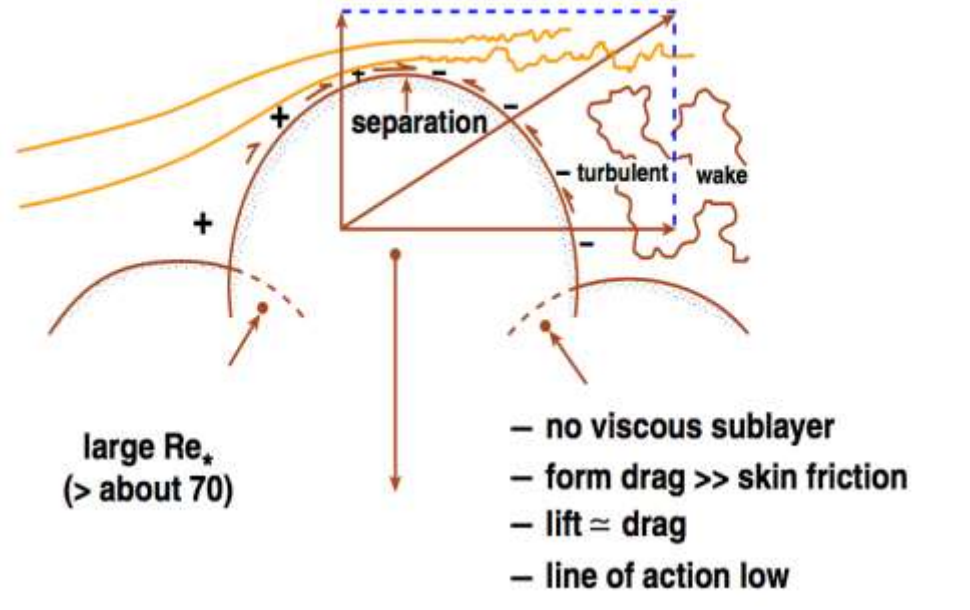
- lifting of the grain off the grains beneath it
- sliding of the grain up and out of its position on the bed
- rotation of the grain about a pivot point formed by neighboring grains

Basic Mechanism of Bed Load Sediment Transport

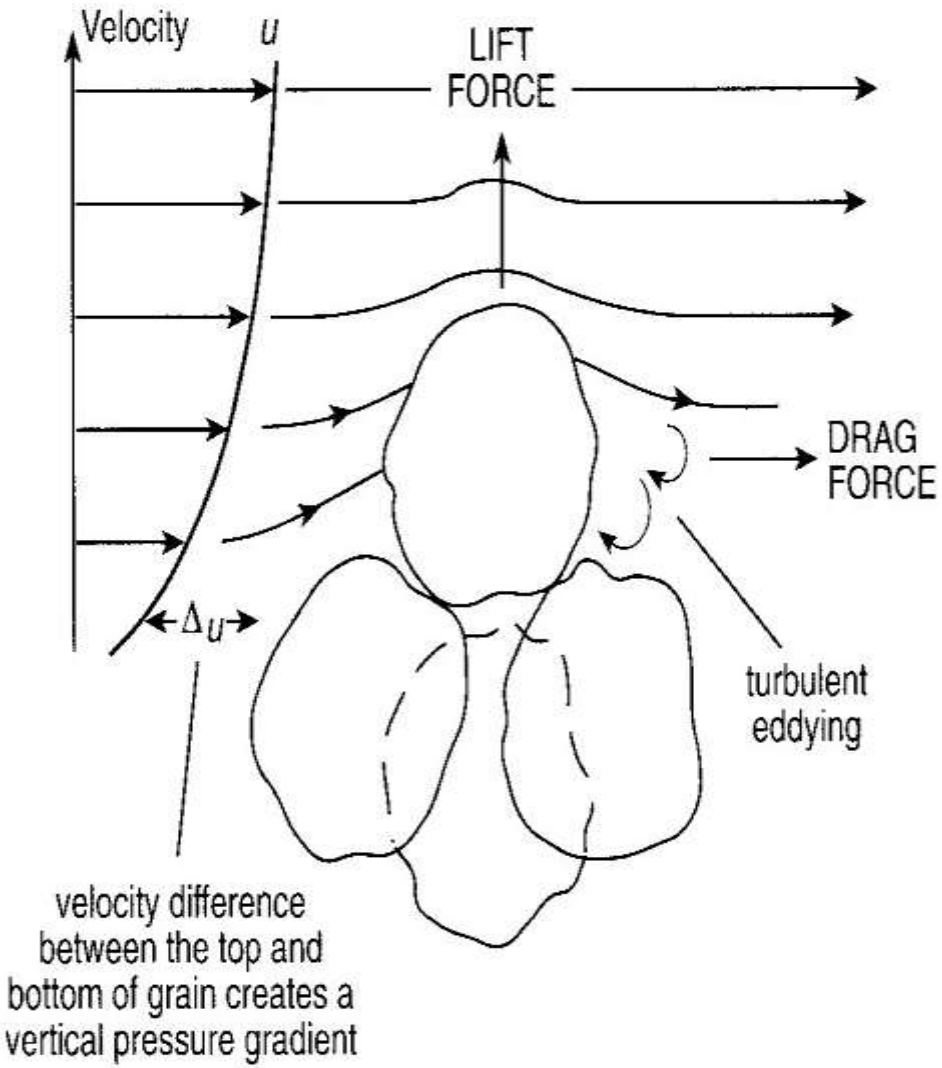


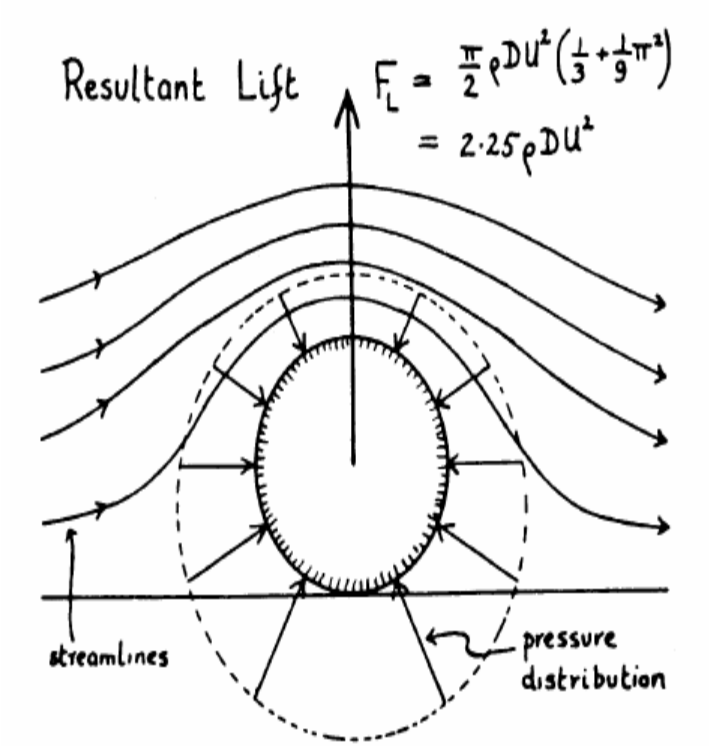






B


Velocity
u
LIFT
FORCE
DRAG
FORCE
Δu
turbulent
eddying
velocity difference
between the top and
bottom of grain creates a
vertical pressure gradient


Resultant Lift
$F_L = \frac{\pi}{2}\rho D U^2\left(\frac{1}{3}+\frac{1}{9}\pi^2\right)$
$= 2{\cdot}25\,\rho D U^2$
streamlines
pressure
distribution

Basic Mechanism of Bed Load Sediment Transport

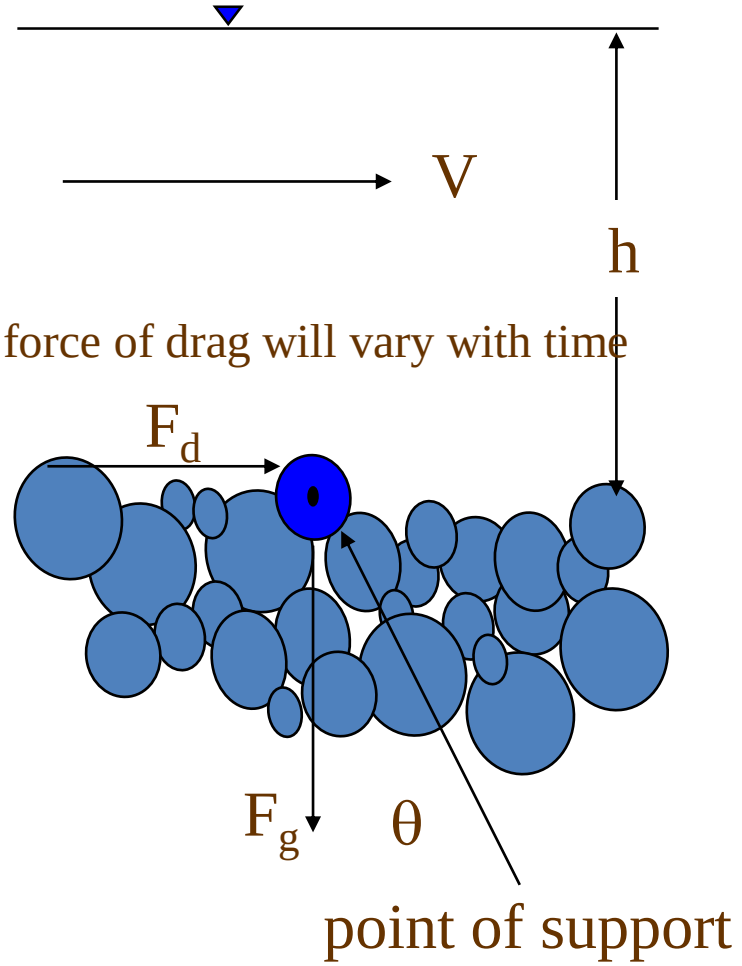
Threshold of movement

Forces exerted by the fluid vary from grain to grain due to variation of grain size, relative exposure to the flow and due to turbulent velocity fluctuations. For a particle/sediment to move (**incipient motion or threshold of movement**), the drag and lift forces in the direction of easiest movement must exceed the combined frictional and gravitation force in the opposite direction.

For case 1, lift forces must exceed the gravity force.

For case 2, drag forces in the direction of easiest movement must exceed the combined frictional and gravitation force in the opposite direction.

For case 3, the moment of the fluid forces must exceed the moment of the gravitational force.



Gravitational Force

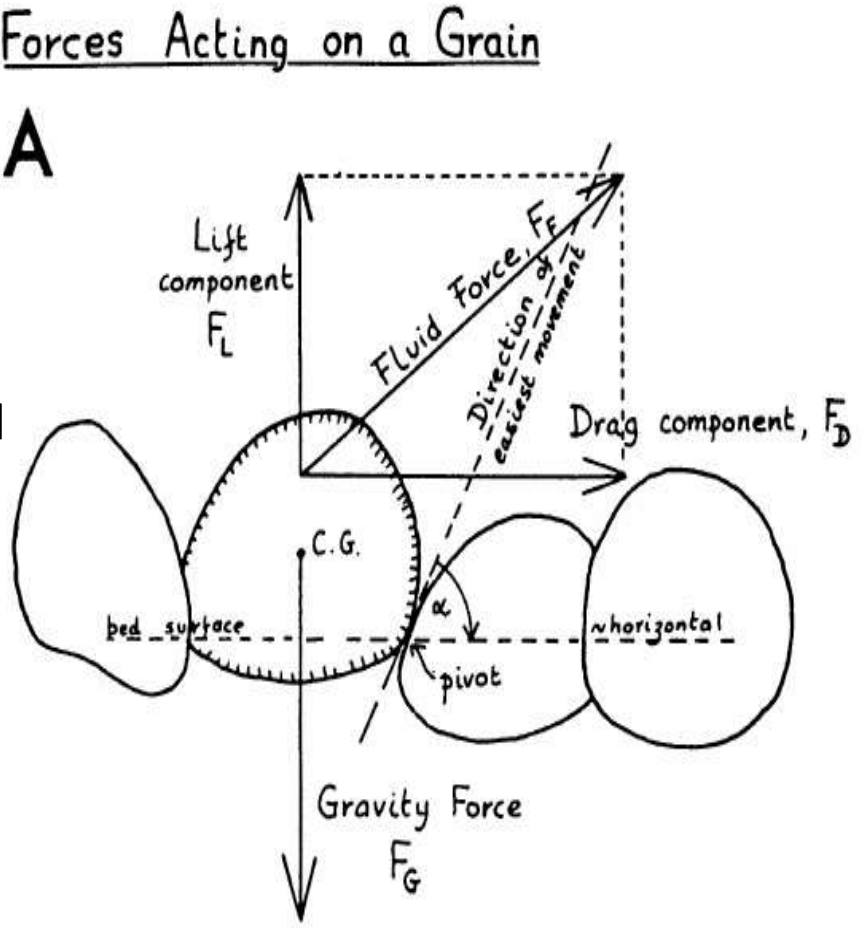
The submerged weight of a sphere (Fg) is:
Fg = mass * gravity = volume * density * g

$$Fg = (4/3)\,\pi\,(d/2)^3\,(\rho s - \rho)\,g$$

By rearranging terms this can be expressed as:

$$Fg = (\pi/6)\,(d)^3\,(\rho s - \rho)\,g$$

The part of this force that opposes sliding of the grain in direction of easiest movement is equal to:
Fg sinα



where α is the friction angle of the material on the bed, which takes into account both the geometry and the frictional properties of the bed material.

Incipient motion is often calculated for the median bed surface grain size (d_{50}) to characterize general bed mobility.

Fluid Forces

A flowing fluid exerts two kind of forces forces on the bed – drag and lift. The viscous drag on the upper exposed surface of the grain acts parallel to the bed.

Drag Force - Drag is the friction due to the flow moving past the exposed grain surface. It therefore is a function of:

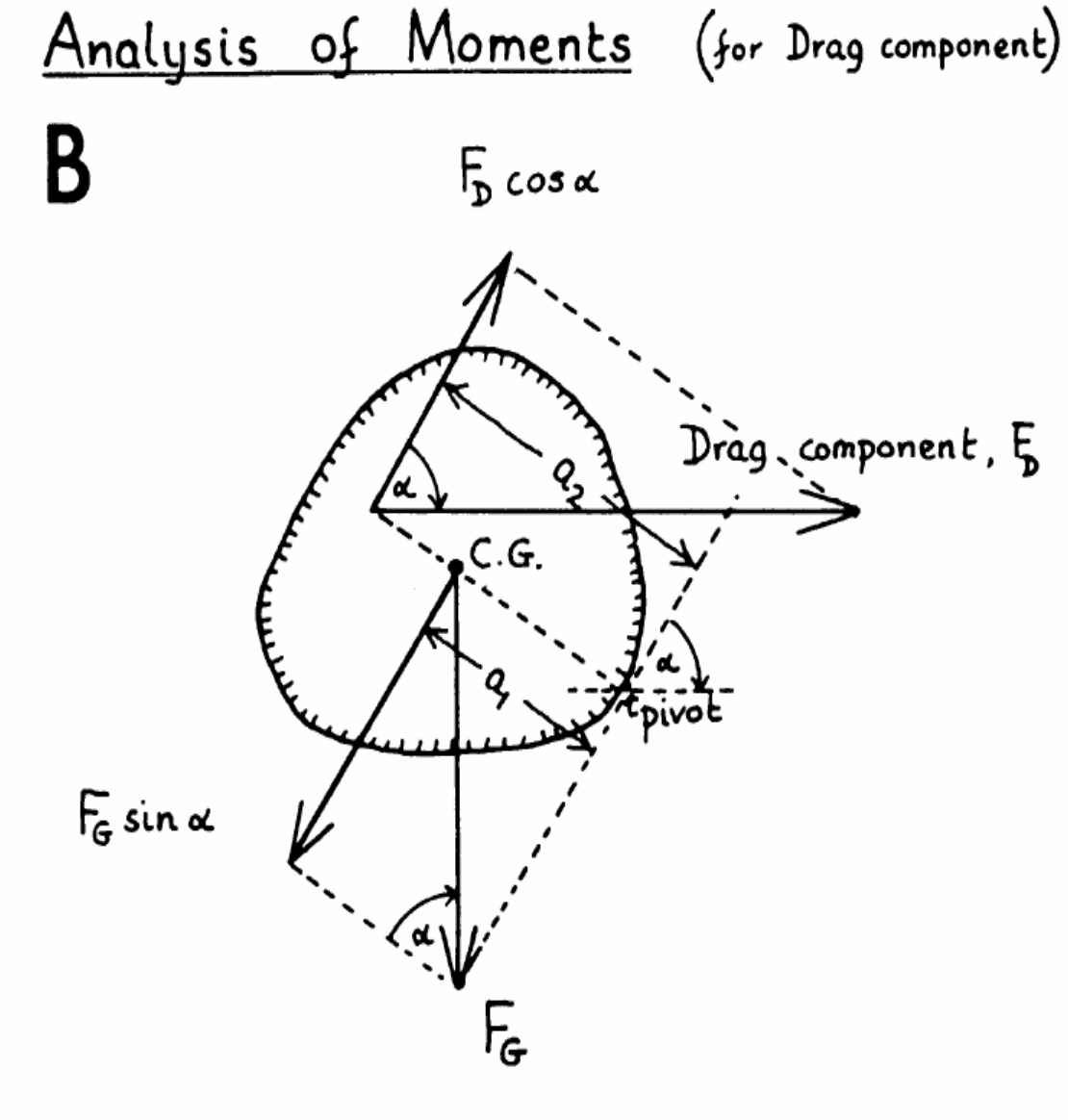
1) surface roughness of the grain; 2) flow velocity; and 3) the exposed cross-sectional area of the grain that the flow hits.

The standard drag equation has the form: $F_d = C_d (\rho u^2 / 2) A = C\tau A$

Where: Fd is the drag force, Cd is the drag coefficient, u is flow velocity and A is the cross sectional area of the grain exposed to flow. C is the combined coefficient.

Lift Force – Lift force due to unequal distribution of dynamic pressure on the grain surface act
normal to the bed. Higher fluid velocities develop over the top of the grain than underneath it, which gives rise to higher pressures on the lower surface of the grain than on the upper surface, which results in a net upward force. Experiments have shown that static lift forces can be as much as 80% of the drag force at the bed surface, but decrease rapidly with height and are negligible at a few grain heights above the bed.

Basic Mechanism of Bed Load Sediment Transport



Initiation of Bed Movement (incipient motion/threshold of movement)

An individual grain begins to move when:

$F_D + F_L > F_g$

It is impossible to define this balance for all grains on a stream bed:
- some grains lie in positions from which they are more easily lifted, slid or rolled
- some grains are more exposed to the flow and subjected to larger fluid forces
- fluid forces fluctuate with time because of turbulence in the flow

Simplifying and considering only drag force:
Total moment = product of the total force and distance to the pivot point. The condition for the onset of motion is given by:
$a_1 (F_g \sin \alpha) = a_2 (F_d \cos \alpha)$

The gravity force may be written as
$F_g = c_1 d^3 g (\rho s - \rho)$ where c_1 is a coefficient that accounts for non-uniform grain shape.
The fluid drag force may be assumed to be equal to the product of the average boundary shear stress (τ_b) and the area of the grain, which may be written as:
$F_d = c_2 d^2 \tau_b$
c_2 is a coefficient that accounts for the geometry and packing of the grains (through the exposed area) and for variation in the drag coefficient.

Initiation of Bed Movement (incipient motion/threshold of movement)

For initial motion, τ_b = the "critical" shear stress, τ_c and so with substitution:

$a_1 c_1 (d)^3 g (\rho s - \rho) \sin \alpha = a_2 c_2 (d)^2 \tau_c \cos \alpha$

Rearranging in terms of τ_c yields an expression for the critical shear stress required to
initiate motion:

$$\tau_c = (a_1 c_1 / a_2 c_2) d g (\rho s - \rho) \tan \alpha$$

Collecting all of the constants together into a single constant called τ^*, the dimensionless critical shear stress, or **Shields parameter**, this simplifies to:

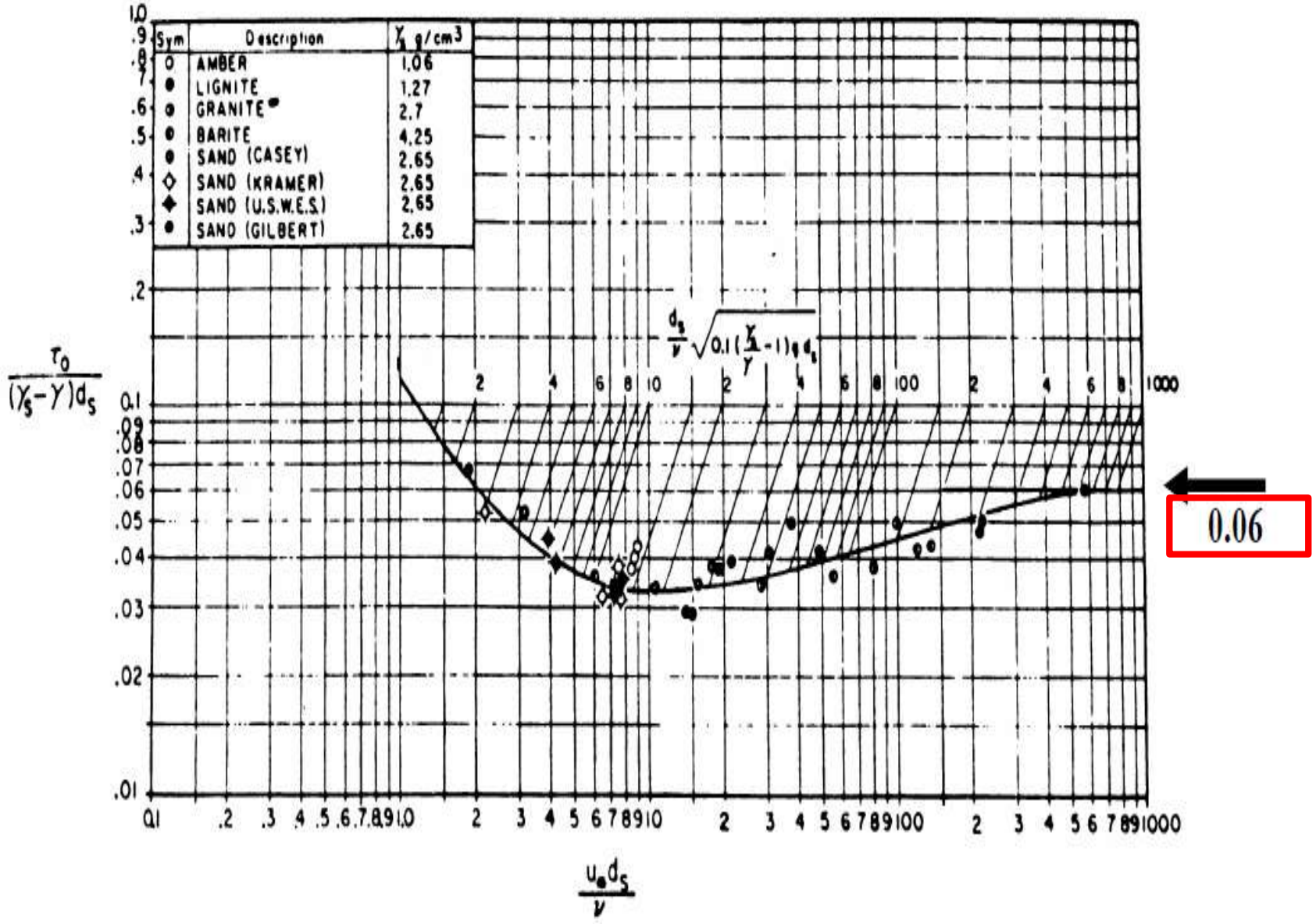
$$\tau_c = \tau^* d g (\rho s - \rho)$$

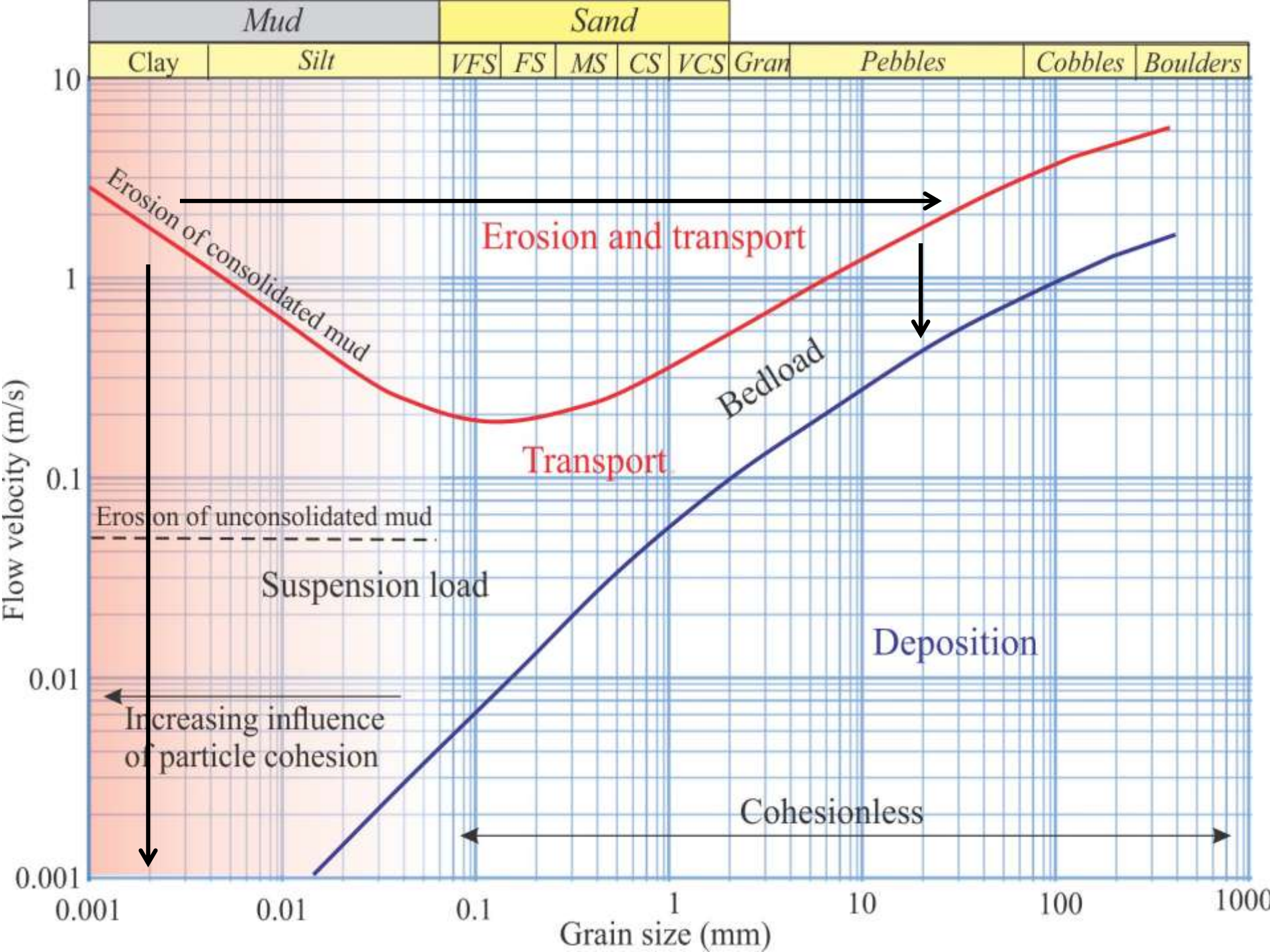
Rearranging in terms of the constant yields:

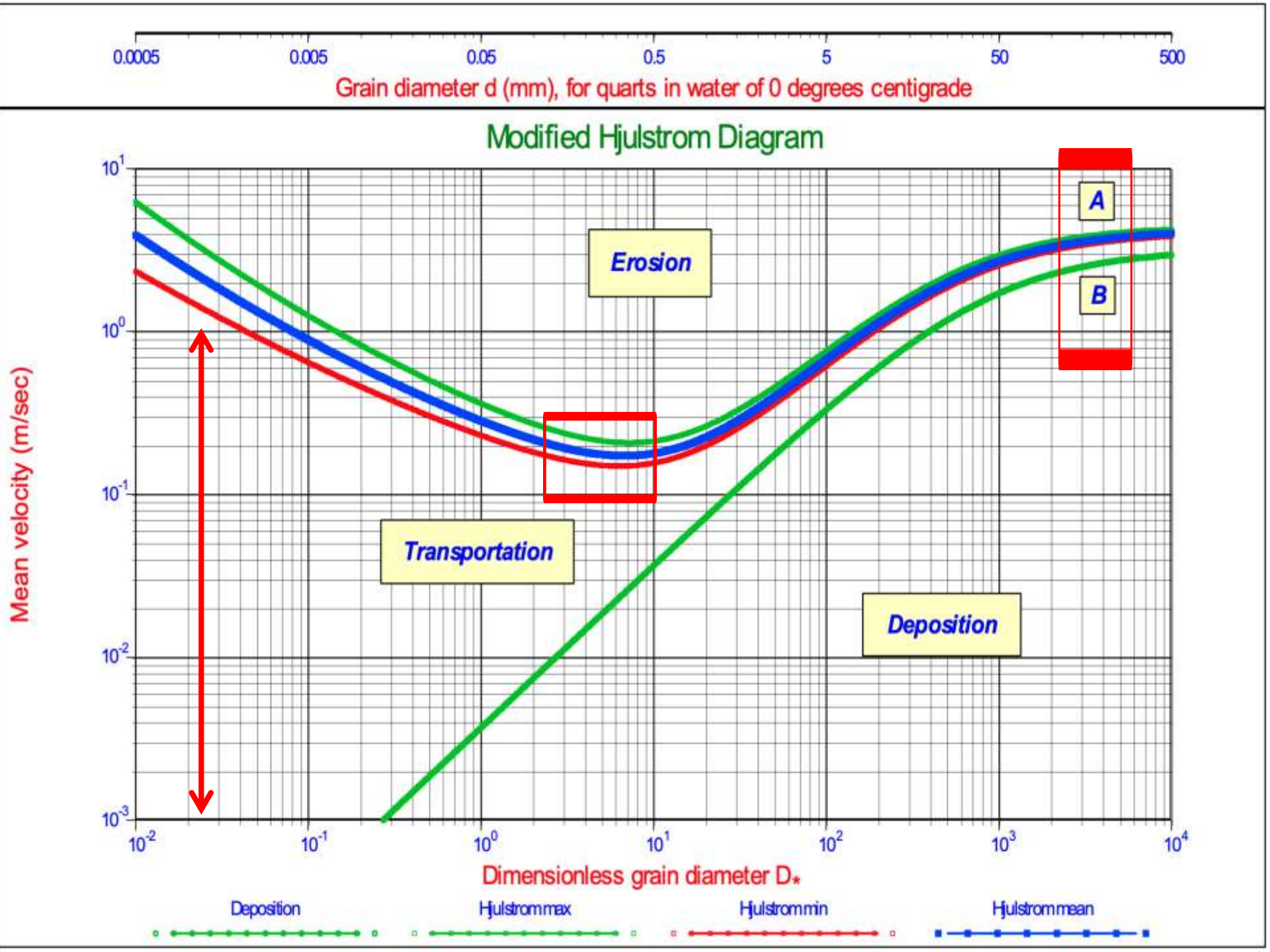
$$\tau^* = \tau_c / [d g (\rho s - \rho)]$$

Note that τ^* is the critical force divided by the submerged weight of the grain.

Because of all the uncertainties surrounding particle shape and the exact nature of grain-fluid interaction, Shields (1936) conducted a set of laboratory experiments to investigate how τ^* would vary with the roughness Reynolds number ($Re^* = u^* d/\nu$).

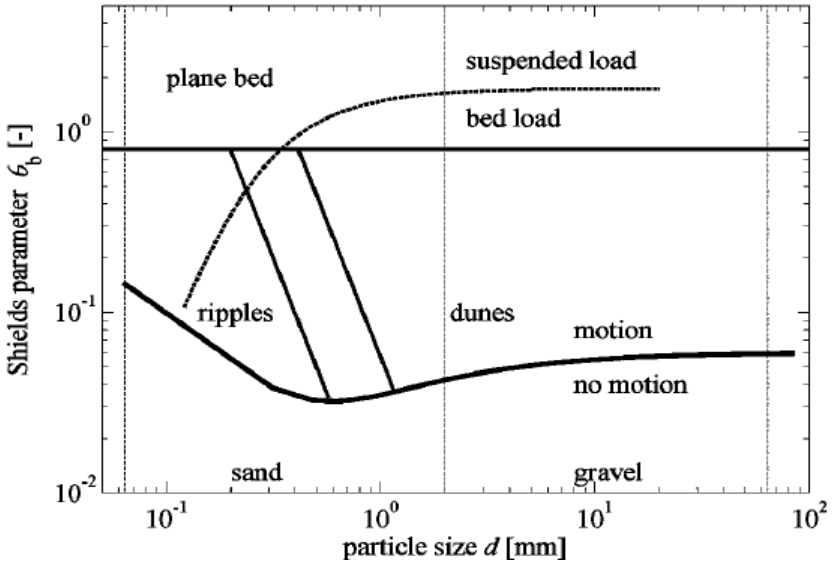

Sym | Description | γ_s g/cm3
AMBER 1.06
LIGNITE 1.27
GRANITE 2.7
BARITE 4.25
SAND (CASEY) 2.65
SAND (KRAMER) 2.65
SAND (U.S.W.E.S.) 2.65
SAND (GILBERT) 2.65
$\frac{\tau_0}{(\gamma_s-\gamma)d_s}$
$\frac{d_s}{\nu}\sqrt{0.1(\frac{\gamma_s}{\gamma}-1)g\,d_s}$
$\frac{u_* d_s}{\nu}$
0.06

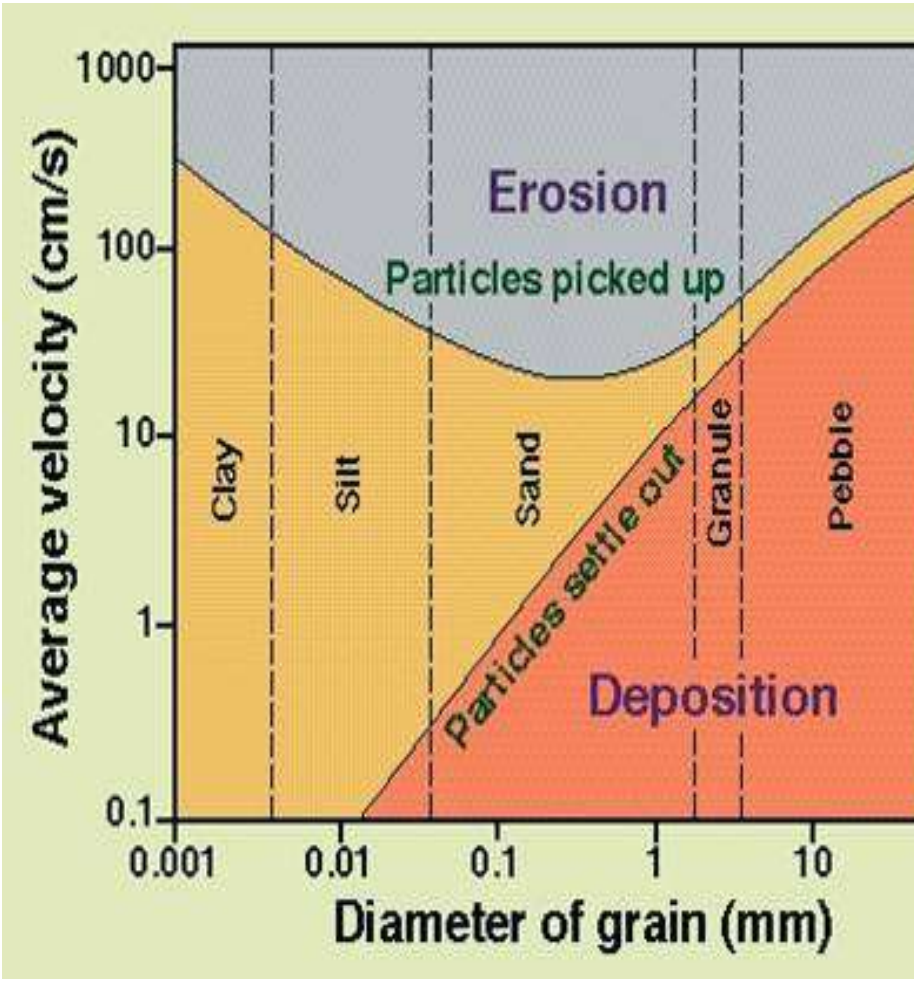

Mud
Sand
Clay
Silt
VFS
FS
MS
CS
VCS
Gran
Pebbles
Cobbles
Boulders
10
1
0.1
0.01
0.001
Flow velocity (m/s)
Erosion of consolidated mud
Erosion and transport
Bedload
Transport
Erosion of unconsolidated mud
Suspension load
Deposition
Increasing influence
of particle cohesion
Cohesionless
0.001
0.01
0.1
1
10
100
1000
Grain size (mm)


0.0005
0.005
0.05
0.5
5
50
500
Grain diameter d (mm), for quarts in water of 0 degrees centigrade
Modified Hjulstrom Diagram
Erosion
Transportation
Deposition
A
B
Mean velocity (m/sec)
Dimensionless grain diameter D*
Deposition
Hjulstrommax
Hjulstrommin
Hjulstrommean

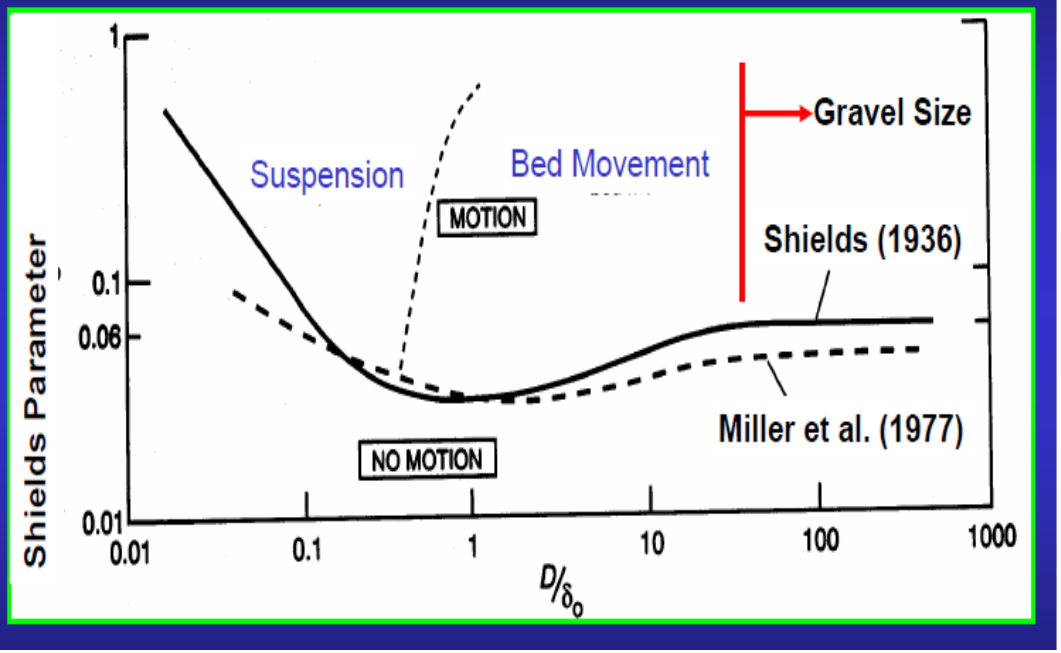
•Cohesive soil requires a very high velocity to erode.
•Sand is easily eroded at much lower velocities (lowest velocity).
•Higher velocity is needed to pick up material than to carry it in suspension.
•Once eroded, wash load never tends to settle (deposit).
•The division between transportation and deposition is small for larger size sediments.

This means that only a small decrease in velocity leads to deposition.





Shield's Parameter vs Particle Size



Particle size classification	Particle size, D_s (mm)	Shield's parameter
very large boulders	>2,048	0.054
large boulders	1,024-2,048	0.054
medium boulders	512-1,024	0.054
small boulders	256-512	0.054
large cobbles	128-256	0.054
small cobbles	64-128	0.052
very coarse gravels	32-64	0.050
coarse gravels	16-32	0.047
medium gravels	8-16	0.044
fine gravels	4-8	0.042
very fine gravels	2-4	0.039
very coarse sands	1-2	0.029
coarse sands	0.5-1.0	0.033
medium sands	0.25-0.5	0.048
fine sands	0.125-0.25	0.072
very fine sands	0.0625-0.125	0.109
coarse silt	0.0313-0.0625	0.165
medium silt	0.0156-0.0313	0.250

The Shield's parameter is highest for fine textured soil, lowest for sand and increases to a constant value for coarse textured soil.

The low roughness (Reynolds number) part of the curve shows that fine grains, hidden partly within the laminar sublayer, would require a larger shear stress relative to their size than would somewhat larger, more exposed particles.
For most rivers, we are concerned with the right-hand side of the graph, where Shields data roughly defines a dimensionless critical shear stress of 0.06.
Hence the initiation of bed movement by turbulent open channel flow is traditionally evaluated using:

$$\tau^* = \tau_c / [d\, g\, (\rho s - \rho)] = 0.06 \qquad \text{or} \quad \tau_c = 0.06\, d\, g\, (\rho s - \rho)$$

Substitutions for gravitational acceleration g = 9.81ms^{-2} and the densities are respectively 2650 kgm^{-3} (for quartz, the most commonly occurring mineral in nature) and 1000 kgm^{-3} (for water), the Shields stress for quartz becomes:

$$\tau_c = \frac{9.81(2650 - 1000)d}{} \times 0.06 \text{ and reduces to } \tau c \cong 970d$$

The dimensionless shear stress ($\tau^* = \tau_c / g(\rho s - \rho)d$) has also been plotted against the dimensionless particle Reynolds number ($Re^* = u^* d/\nu$ or $Re^* = d/\delta 0$) where $\delta 0$ is the thickness of the laminar sublayer. The particle Reynolds number can be taken as an approximate measure of grain size. The dimensionless shear stress in the Shields diagram is commonly termed the *Shields parameter.*

Several aspects of the Shields diagram are particularly noteworthy:

1. The lowest Shields stress occurs in the sand range (0.06-2.00 mm). Sand is small enough to have small mass but too large for cohesive forces to come into play.
2. Silt/clay, in spite of the smaller size, requires a higher shear stress for motion than sand. Here cohesive forces become overwhelmingly large and bind the sediment together into a mass that is very resistant to erosion.
3. The Shields parameter for gravel (and higher order particles) is constant at 0.06, implying that Shields stress here becomes a simple function of grain size. This is a quite remarkable finding and allows to derive simple relationship between the size of gravel and the shear stress required to move it.

Thus, the lowest velocity for eroding sediment is in the sand range and higher velocity is required to erode silt/clay sediments and materials in the gravel range and the size of gravel eroded is a linear function of velocity.

Furthermore, the velocity required to erode sediment is higher than that required to keep the same-size material in motion.

Application of Shield's Diagram

Example

The depth of flow and the slope of a wide channel are 0.3 m and 0.1%. If the mean grain size of the sediment is 1 mm and the Shield's parameter is 0.035 then determine whether the sediment would be eroded or not?

Solution

$\tau^* = \tau_c / [d\ g\ (\rho s - \rho)] = 0.035$

Hence: $\tau_c = 0.035 \times [d\ g\ (\rho s - \rho)] = 0.035 \times 0.001 \times 9.81 \times 1650 = 0.656$ N/m^2

The shear stress on the bed is: $\tau = \rho g h s$

Hence: $\tau = 1000 \times 9.81 \times 0.3 \times 0.001 = 2.943$ $N/m^2 s$

Since the shear stress on the bed is greater than the critical shear stress ($\tau > \tau_c$):

The sediment particle would move/erode.

Application of Shield's Diagram

Problem:

A 55 m wide channel with 1.0 cm gravel bed has a slope of 0.005. What is the minimum required flow to initiate particle motion on the bed (Chezy's C is 32)?

Solution:

The Shield's parameter for gravel is 0.06. Hence:

$\tau_c = 970D = (970)(0.01) = 9.7\ Nm{-}2$

The shear stress in this case:

$\tau_c = \rho ghs = (1000)(9.81)(h)(0.005) = 49.05h$

Setting the actual shear stress (τ) equal to the critical shear stress (τ_c) yields:

$49.05h = 9.7$ and so $h = 0.242$ m

Minimum required discharge: $Q = whC\,(hs)^{0.5}$

$Q = 55 \times 0.242 \times 32 \times (0.242 \times 0.005)^{0.5} = 14.8\ m^3/sec$

That is, for the given channel conditions, sediment on the bed will begin to move once the flow exceeds 14.8 m^3/sec.