Estimation of Sediment Transport Rates

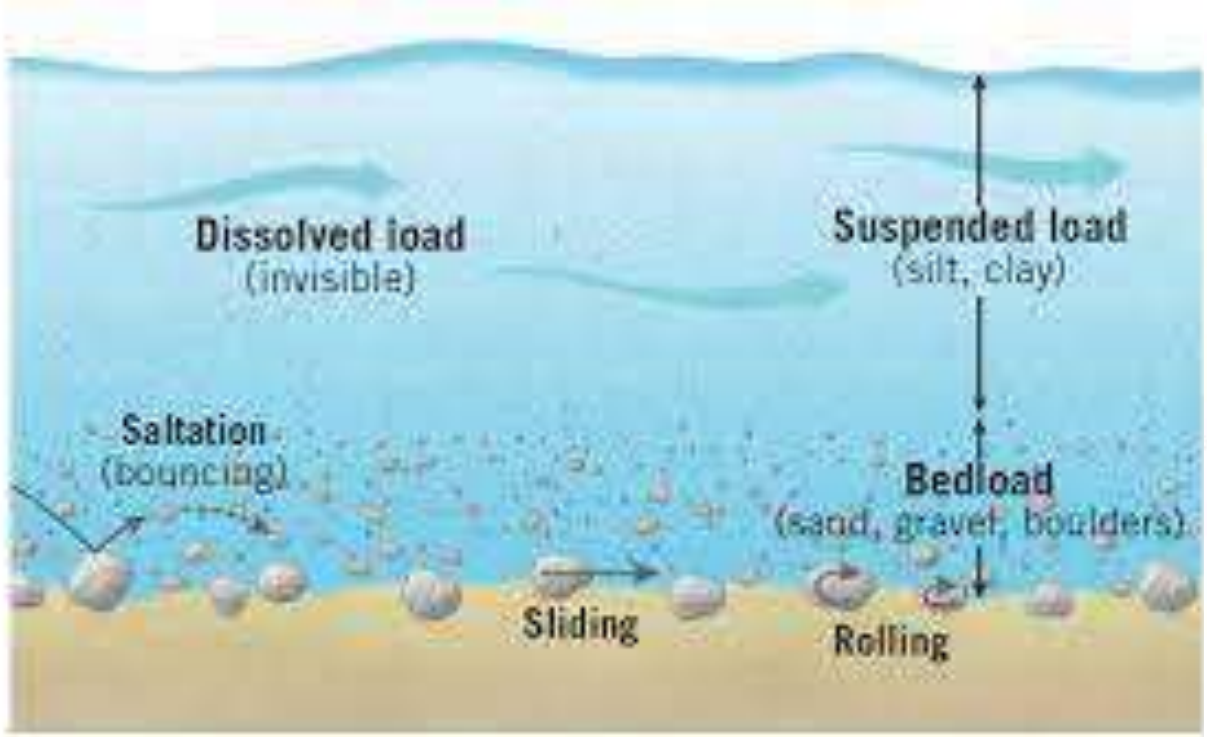
The rate of sediment transport is related to the shear stress on the bed. But, the shear stress (ρghs) is not completely available for the sediment transport. When the bed is covered with bed forms, a part of the shear stress is used for overcoming the form resistance and the remaining is available for grain resistance and sediment transport.

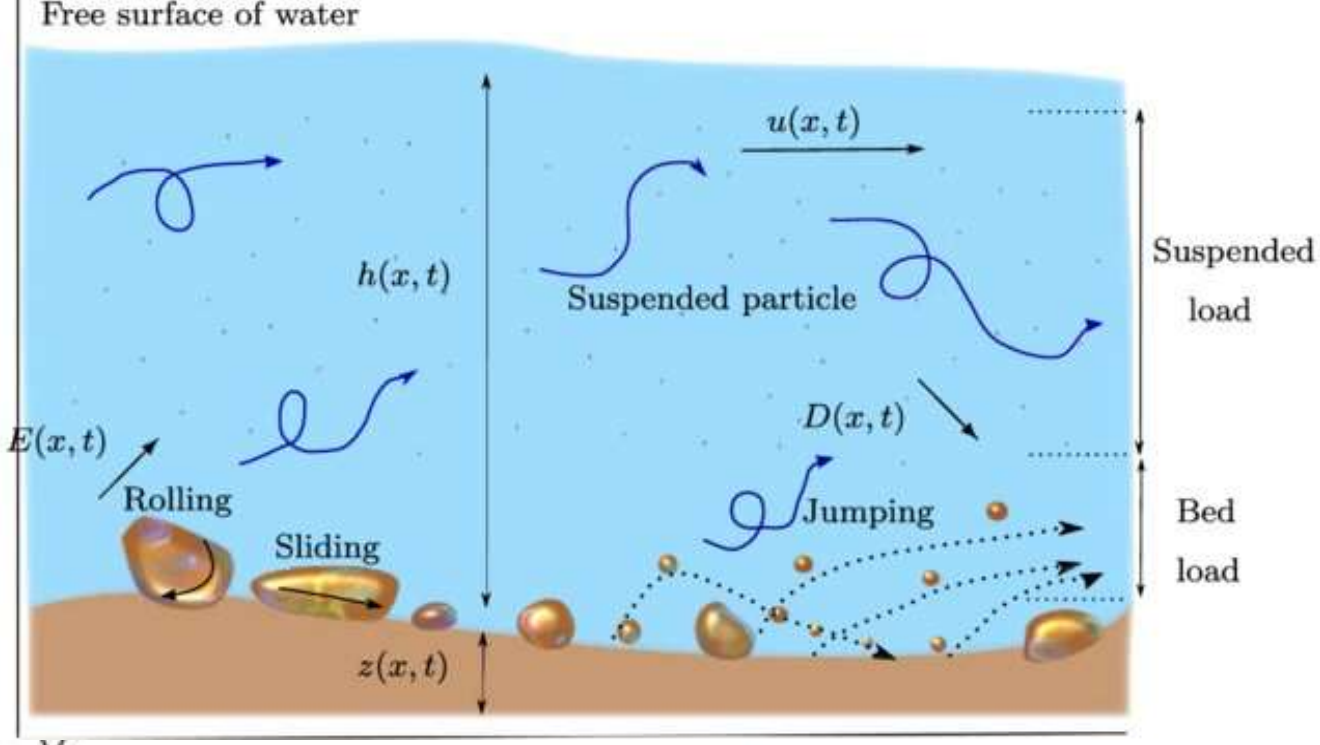
The effects of viscosity, non-uniformity of grains, grain size distribution and shear stress (velocity) make the calculations of sediment load complex and relatively less accurate.

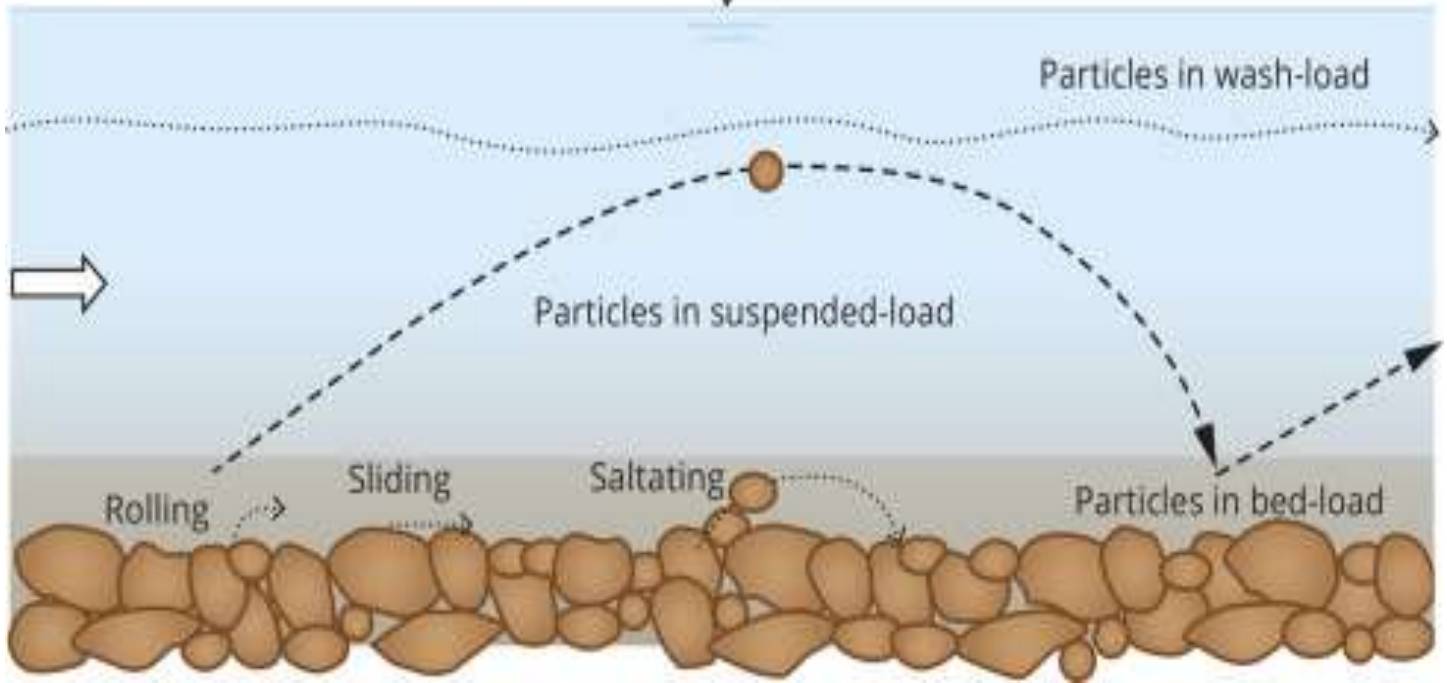
Suspended load

Suspended load transport is an advanced stage of bed load transport. At low values of shear stress only the bed load is transported, while at high shear stress both bed load suspended load are transported. Studies have shown that the concentrations of suspended load decreases with the increase in distance from the bed. The rate of decrease depends on the *ratio of the fall velocity ω and the bed-shear velocity* $u*$.

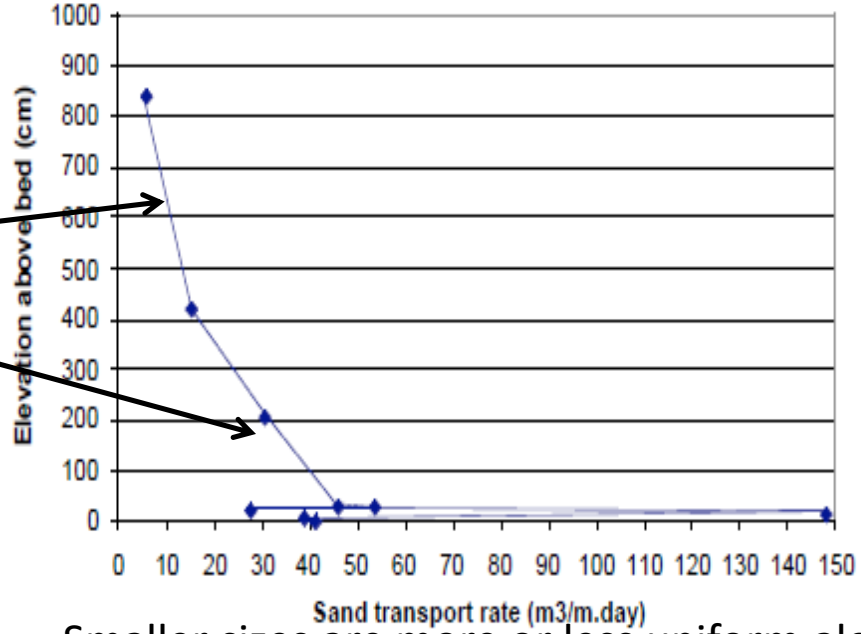
The depth-integrated suspended-load transport q_s,is herein defined as the integration of the product of velocity u and concentration c from the edge of the bed-load layer, z = a, to the water surface, z = h.

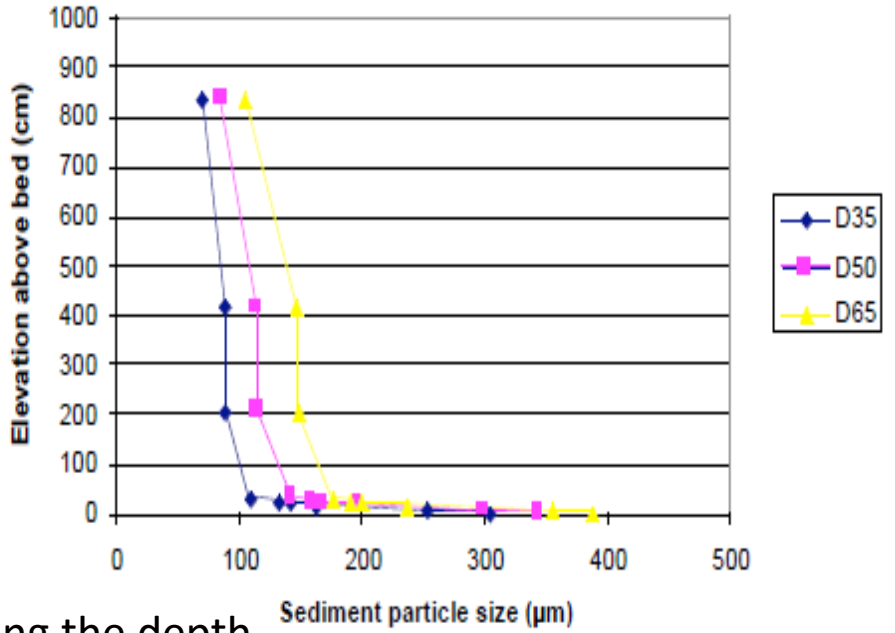

Dissolved load
(invisible)
Suspended load
(silt, clay)
Saltation
(bouncing)
Bedload
(sand, gravel, boulders)
Sliding
Rolling


Free surface of water
$u(x,t)$
$h(x,t)$
Suspended particle
Suspended load
$E(x,t)$
$D(x,t)$
Rolling
Sliding
Jumping
Bed load
$z(x,t)$


Particles in wash-load
Particles in suspended-load
Rolling
Sliding
Saltating
Particles in bed-load

Variation
of concentration
is a function of fall velocity
and shear velocity

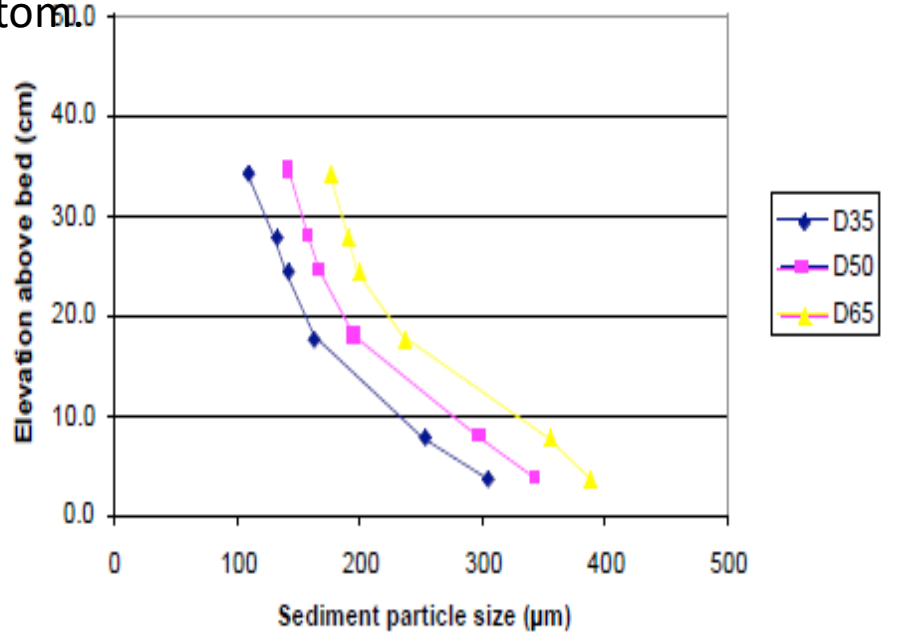




Smaller sizes are more or less uniform along the depth.
Larger sizes are predominantly at the bottom.

Similar field observations in the Jamuna (Brahmapoutra) river, Bangladesh (1995)

Detailed profile close to the bed show a gradual decrease of the sediment size from the bottom upwards, despite the irregular variation in sediment transport rate (figure above)



BED SHEAR STRESS AND SHEAR VELOCITY

Consider a river that is wide and can be approximated as rectangular.

The **bed shear stress** τ_b is the force per unit area with which the flow pulls the bed downstream [$ML^{-1}T^{-2}$]

The bed shear stress is related to the flow velocity through a dimensionless bed resistance coefficient (bed friction coefficient) C_f, where

$$C_f = \frac{\tau_b}{\rho_w \omega^2}$$

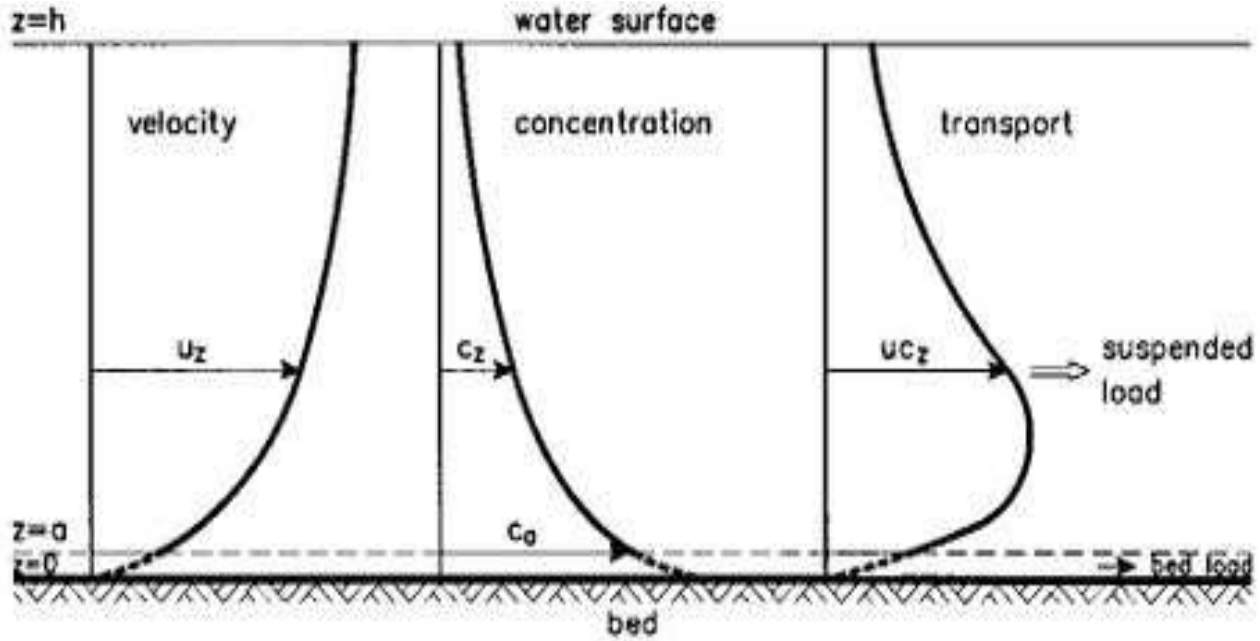
The bed shear velocity u_* [L/T] is defined as:

$$u_* = \sqrt{\frac{\tau_b}{\rho_w}}$$

$C_f = (u_* / \omega)^2$

Between the above two equations,

where C_f = dimensionless resistance coefficient



This definition requires the determination of the velocity profile, concentration profile and a known concentration c_a close to the bed (z=a). These latter parameters are referred to as the reference concentration and the reference level.

Hence, the total suspended load per unit width of the river will be:

$$q_S = \int cu\, dy$$

It is necessary to know how c and u vary with y to compute the suspended load. Because of uncertainties involved in relating both c and u with y, there are no satisfactory relationships.

Hence, attempts have been made to relate suspended sediment load directly with the water discharge, Q.

The approximate suspended sediment concentration C at any depth y is given by:

$$\frac{C}{Ca} = \left[\frac{(D-y) \quad (a)}{(y) \quad (D-a)} \right]^{Rn}$$

where $R_n = \omega / ku_*$ known as Rouse Number (ratio of velocities) and k is the von Karman's constant = 0.40. The above equation is also known as Rouse equation.

d is the depth of flow and C_a and a are the known concentration at a depth a.

Note that in above equation, the term a/(D - a) is less than 1 in the lower ½ of the flow (where, typically, any significant concentration of suspended sediment will be measured). The product with the term (D – y)/y is also less than one over most of the flow depth, and so raising it to an exponential power much *greater* than one will drive the value of Ca towards zero.

Thus the maximum value of the suspended sediment concentration will be determined by the empirically fitting the magnitude of Ca (and where in the flow that value was measured), but the rapidity with which that concentration gradient declines with distance away from the bed will be determined by the magnitude of the exponent, ω/ku* —the larger this number (i.e. the coarser the sediment) the more abruptly will the concentration decline away from the bed.

$$\frac{C(z)}{C_a} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)^P$$

Take the Natural Log (ln) of Both Sides

$$\ln\left(\frac{C(z)}{C_a} \right) = \ln\left[\left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)^P \right]$$

Apply the Log Power Rule

Bring the exponent P to the front using the identity $\ln(x^y) = y \cdot \ln(x)$:

$$\ln\left(\frac{C(z)}{C_a} \right) = P \cdot \ln\left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)$$

$$\ln\left(\frac{C(z)}{C_a} \right) = P \cdot \ln\left(\frac{h-z}{z} \right) + P \cdot \ln\left(\frac{a}{h-a} \right)$$

$$\ln(C(z)) = \ln\left(\frac{h-z}{z} \right) + \underbrace{\ln(C_a) - \ln\left(\frac{h-a}{a} \right)}_{\text{Constant}}$$

Because C_a, h, and a are all fixed boundary values for a specific river, that entire trailing block simplifies into a single numeric **Constant**, leaving you with:

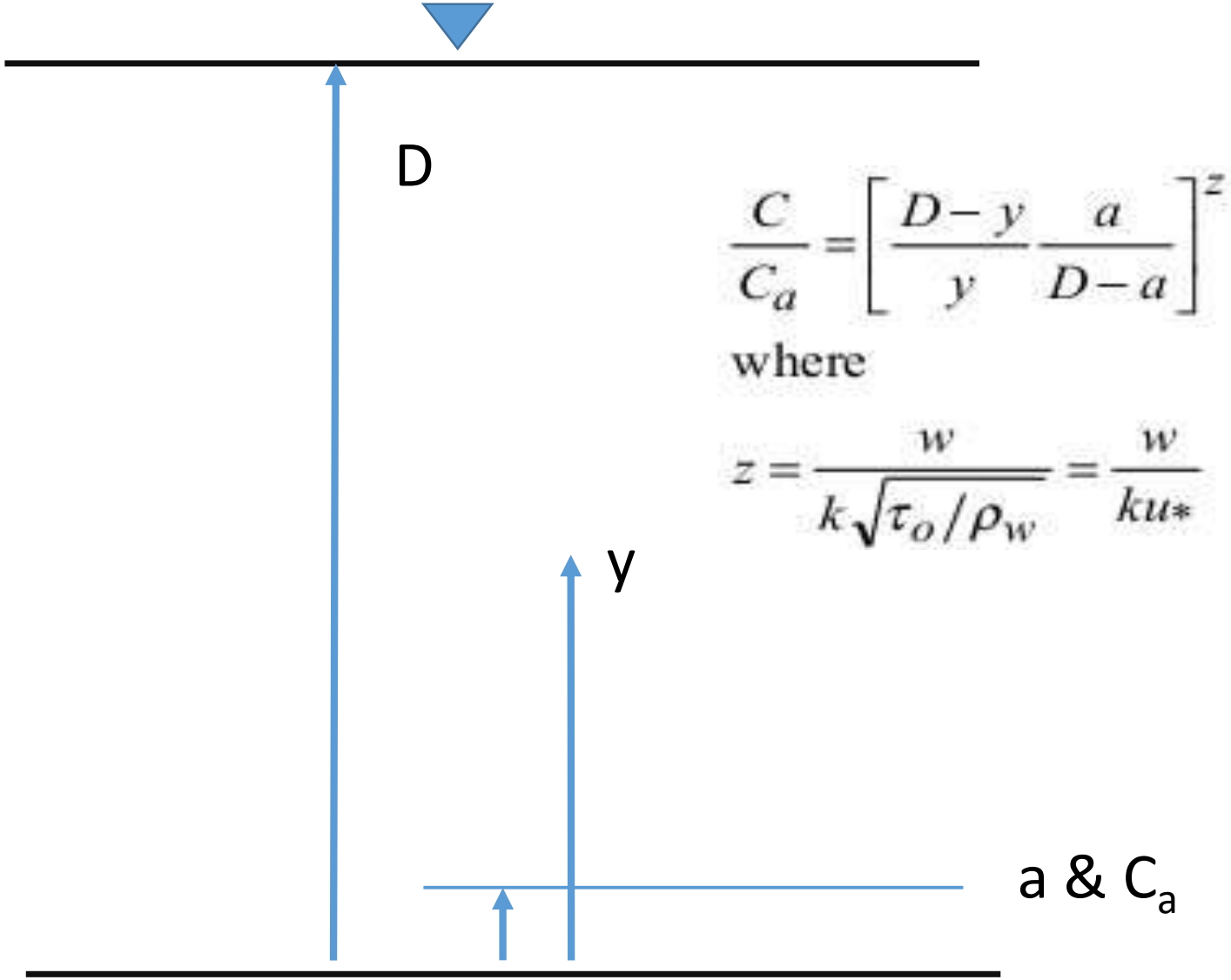
$$\ln(C(z)) = \ln\left(\frac{h-z}{z} \right) + \text{Constant}$$

The Velocity Profile Equation

$$u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0} \right)$$

Where:

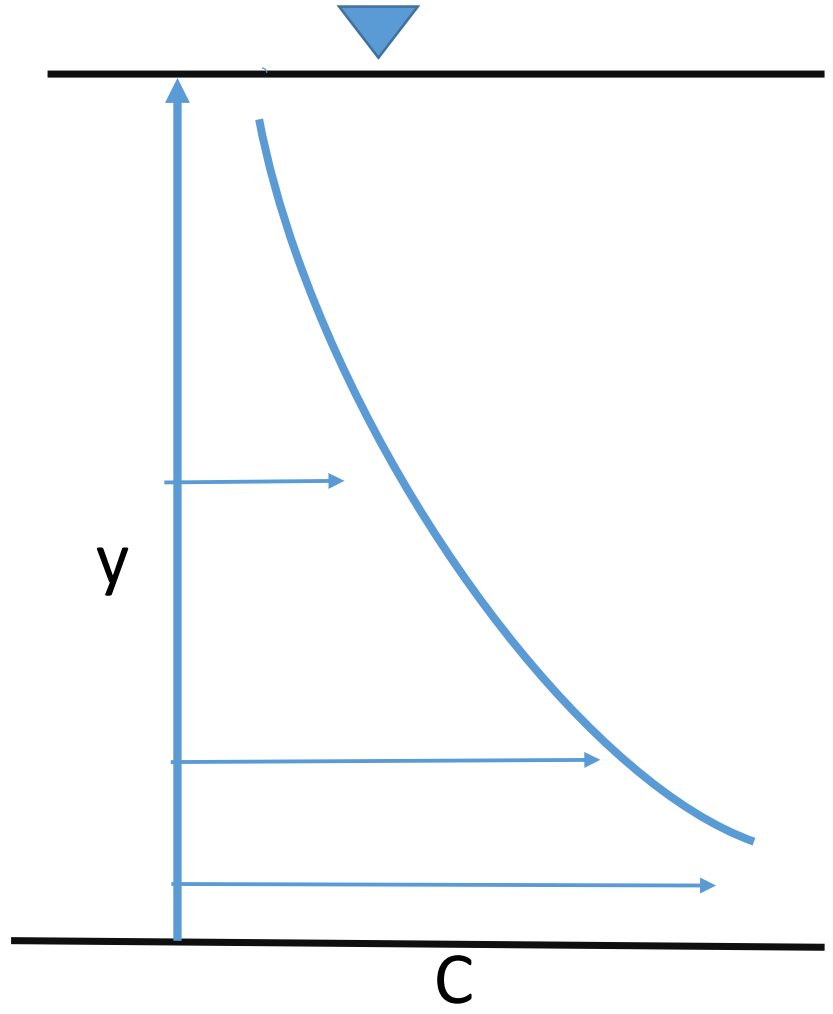
- $u(z)$: Flow velocity (m/s) at a specific height z above the riverbed.
- u_*: Shear velocity (m/s), which represents the friction forces at the bed, calculated as $\sqrt{g \cdot h \cdot S}$ (where g is gravity, h is depth, and S is the energy slope).
- κ: Von Kármán constant (dimensionless, universally accepted as approximately **0.40**).
- z: Vertical height (m) measured upward from the riverbed.



$$\frac{C}{C_a} = \left[\frac{D-y}{y}\frac{a}{D-a}\right]^z$$

where

$$z = \frac{w}{k\sqrt{\tau_o/\rho_w}} = \frac{w}{ku*}$$



Where sediment movement in the flow column is determined by the Rouse number *(*$R_n = \omega/ku^*$*)*, the following conditions are anticipated:

$R_n > 2.5$ Grains travel as bed load

$1.8 < R_n < 2.5$ Grains are trying to go into suspension, but are not quite able to stay up in the flow field

$R_n < 1.8$ Full suspension

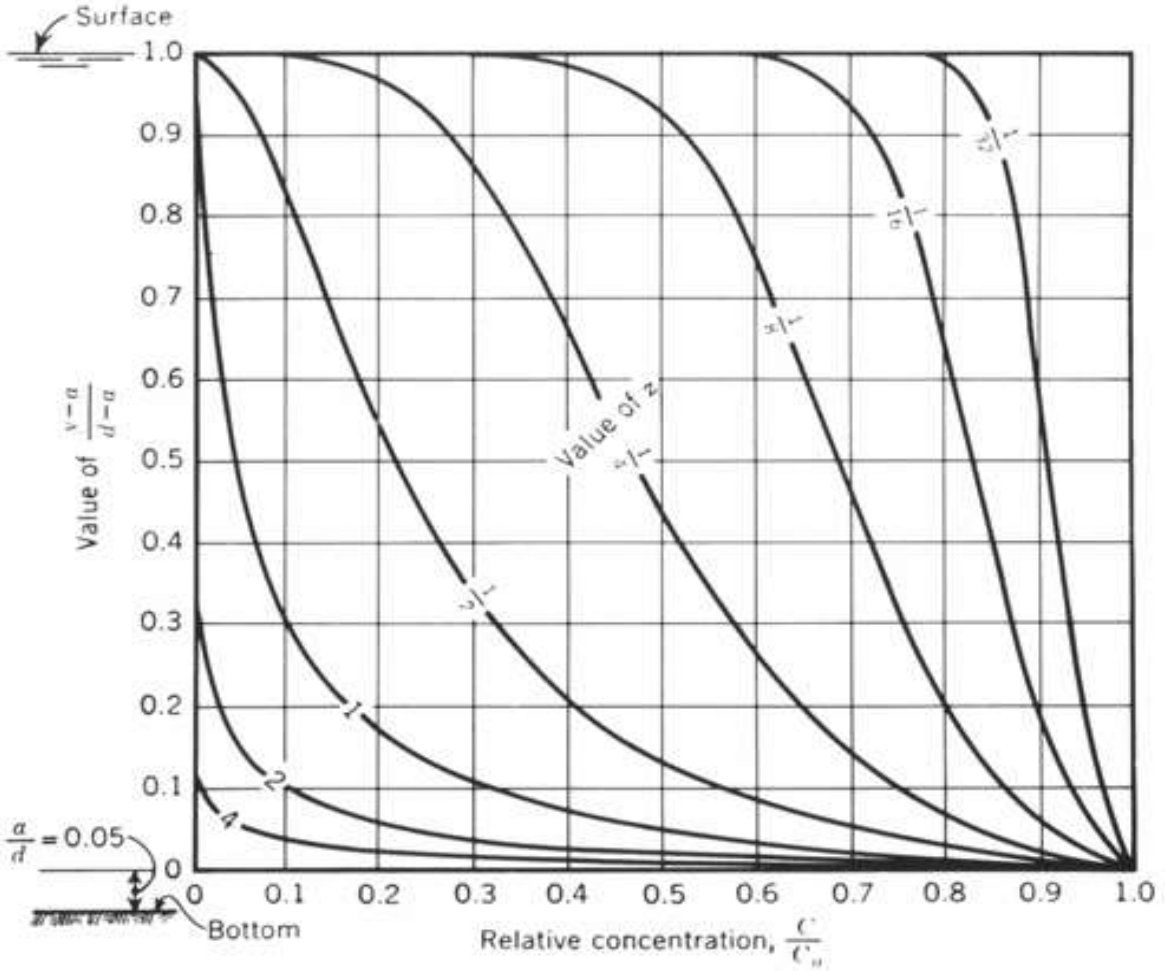
In general, values of the Rouse number greater than about 2.5 lead to a condition of very little to no suspended sediment. In contrast, values of the Rouse number less than about 0.25 predict those grain sizes that will move as wash load, fully supported by the flow.

In between these values lies the range of partial suspended sediment transport.

A Rouse number less than 1.8 is sometimes suggested at an appropriate functional "threshold" for determining whether a given grain size will move in suspension.

Although the form of sediment concentration equation requires that the suspended sediment concentration be zero at the top of the water column (where y = d) regardless of particle size, we know from observations that this is not always the case. This is a limitation of the equation.

Graph showing the concentration of suspended sediment of a given settling velocity *w* at a height *y* above the bed relative to its concentration C_a at an arbitrarily chosen "reference level" *y* = *a* above the bed.



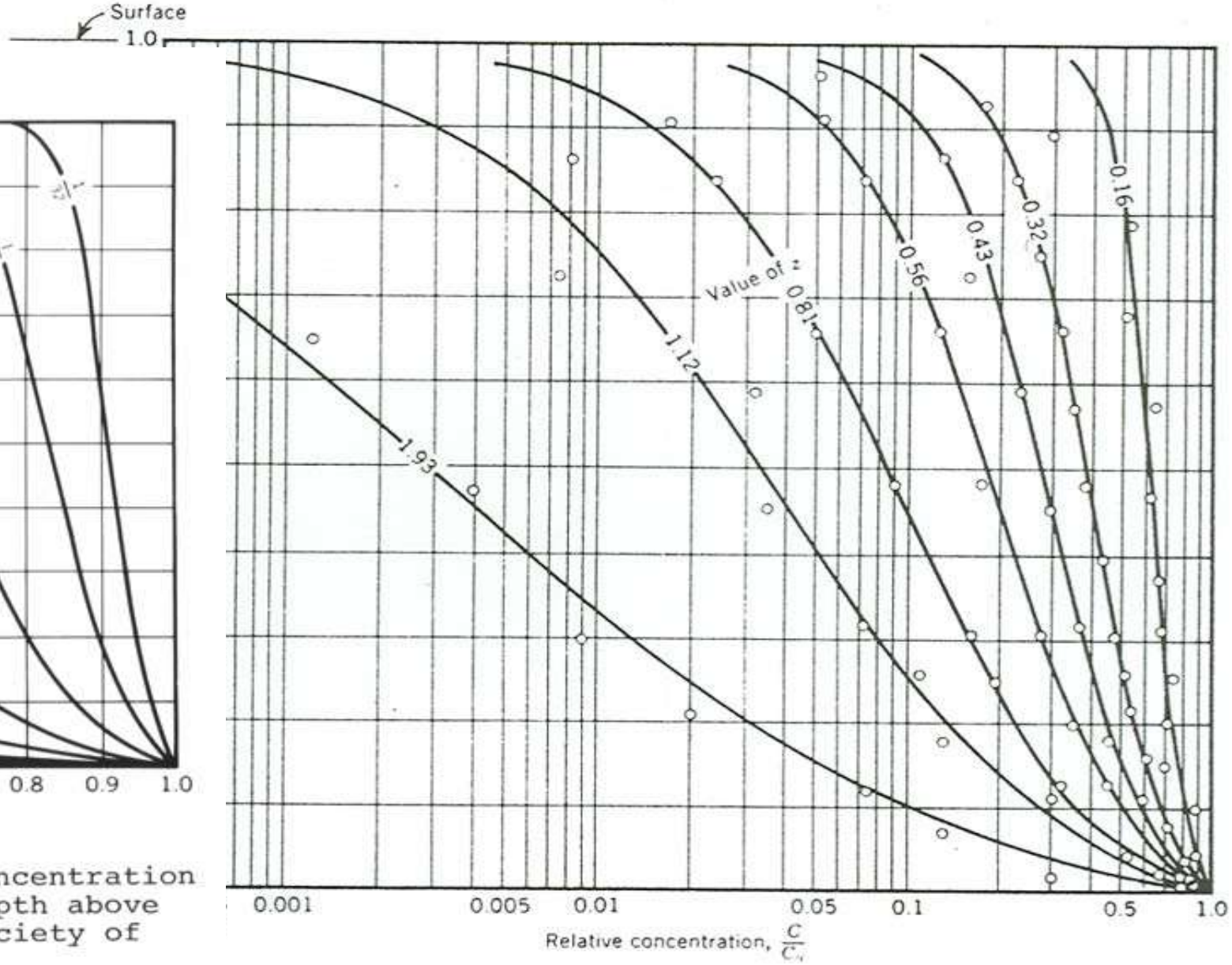


Figure 6.15 Distribution of relative concentration of suspended sediment with relative depth above the datum y = 0.05d (after American Society of Civil Engineers, 1963).

Example

A river with a depth of 4 m and a slope of 2.25 x 10^{-4} carries sediment with a d_{50} of 0.4 mm and a corresponding fall velocity of 0.04 m/sec. If the concentration of sediment at 0.4 m from the bed is 400 ppm, then calculate the sediment concentration at a depth of 1 m and the suspension status. What should be the minimum fall velocity for the sediment to become bed load?

Solution

Given: D = 4m, y = 1m, a = 0.4 m, ω = 0.04 m/sec and C_a = 400 ppm

$$\frac{C}{C_a} = \left[\frac{D-y}{y}\frac{a}{D-a}\right]^z$$

where

$$z = \frac{w}{k\sqrt{\tau_o/\rho_w}} = \frac{w}{ku*}$$

$$u_* = \sqrt{\frac{\tau_b}{\rho_w}} \qquad \tau = \rho_w ghs \text{ and } u_* = (ghs)^{0.5} = (9.81 \times 4 \times 0.000225)^{0.5}$$

Hence u_* = 0.0939 m/sec and z = 0.04 / 0.4 x 0.093 = 1.06

Since z or R_N is less than 1.8, the sediments would be in suspension (check on suspension criteria Using Rouse's Number).

C = C_a [(4 -1)1 x (0.4/(4 – 0.4)]$^{1.06}$ = 0.312 x 400 = 125 ppm

If the particle is to become bed load then: z = ω/ku_* = 2.5

Hence: ω = 2.5 x 0.4 x 0.093 = 0.093 m/sec

Suspended load considering that the suspended-sediment discharge is related to the water discharge:

$Qs = C_0 Q$ (with appropriate units)

where, Qs is the sediment discharge in kg/yr (t/yr), C_0 is the sediment concentration in mg/l or ppm and Q is the discharge.

Rubey (1933) showed that the average sediment concentration, C_{av} is proportional to $R^{1/2} S^{2/3}$.

From Chezy's equation:

$Q = CA R^{1/2} S^{1/2} \sim h^{3/2} S^{1/2}$

Hence: $Qs = C_{av} Q \sim (R^{1/2} S^{2/3}) h^{3/2} S^{1/2} \sim (h^2 S^{7/6})$

Bed Load

Bed-load transport is the transport of sediment particles in a thin layer of 2 particle diameters thick just above the bed by sliding, rolling and saltation.

The bed layer is considered as a layer in which the mixing due to turbulence is so small that it cannot influence the sediment particles, and therefore suspension of particles is difficult.

Unlike suspended load, bed load is always capacity limited – a function of river hydraulics rather than the supply.

Bed load transport q_b can be determined from the measured bed form dimensions and the measured bed form migration velocity (using echo sounding results).

The bed load transport is by definition: mass m_s per unit width of bed times migration velocity v.

Thus:

$q_b = v\, m_s$

The mass (m_s)of a triangular bed form (ripple/dune) with length L, height ΔH, sediment density ρ_s ($\approx 2650 kg/m^3$) and porosity p(≈ 0.4) is:

$m_s = 0.5\, \rho_s (1-p) L\, \Delta H.$

Bed load transport is thus given by:

$q_b = 0.5\, v\, \rho_s (1-p) L \Delta H$ (where v is the migration velocity or velocity near the bed)

As bed forms are not fully triangular, a more general expression as shape/size of bed form is:

$q_b = \alpha\, v\, \rho_s\, (1-p)\, L\, \Delta H$, with $\alpha = 0.5 - 0.7$

In general, bed-load transport rates have been estimated/measured to be between 1% and 10% of the suspended load.

Example

A 160 m wide river (n = 0.023) with a depth of 2.2 m and slope of 0.08% has sand dune bed forms. If the length and the height of the dunes are 80 and 15 cm respectively, then estimate the bed load. The velocity near the bed is 30% of the mean velocity (migration velocity) and the porosity as 50%. Assume any other data as necessary

Solution

The bed load $q_b = 0.5\ v\ \rho_s\ (1-p)\ L\ \Delta H$

The velocity $V = [\ (1/0.023) \times 2.2^{2/3} \times 0.0008^{1/2} = 2.08$ m/sec
The bed velocity (migration velocity) is $0.3 \times 2.08 = 0.624$ m/sec

Putting values: $q_b = 0.5 \times 0.624 \times 2650 \times (1-0.5) \times 0.8 \times 0.15 = 54.57$ kg/sec/width
The total bed load is: $q_b \times W = 54.57 \times 160 = 8731$ kg/sec

Estimation of Brd Load Transport Rates

An equation which has found wide application for bed load transport in the gravel-bed rivers is the Meyer-Peter & Muller (M-P&M) equation, a simplified version of which is:

$$q_b = 0.253(\tau_0 - \tau_{cr})^{3/2}$$

where q_b is the mass rate of transport per unit width of channel (kg/s m) and τ_0 and τ_{cr} are bed shear stress and critical shear stress (from Shield's parameter).

Example: A 50 m-wide channel has a depth of 2.5 m at a slope of 0.0015 over a gravel bed in which d_{50} = 3.2 cm. How much bed load the river may transport?

Solution:

The bed load transport rate is given by equation: $q_b = 0.253(\tau_0 - \tau_{cr})^{3/2}$

We need to determine τ_0 - thus: $\tau_0 = (1000)(9.81)(2.5)(0.0015) = 36.79\ Nm\text{-}2$

τ_{cr} is estimated by the Shields stress for a normal bed:

$\tau_{cr} = 970D = (970)(0.032) = 31.04 Nm{-}2$

So, substituting in bed load equation:

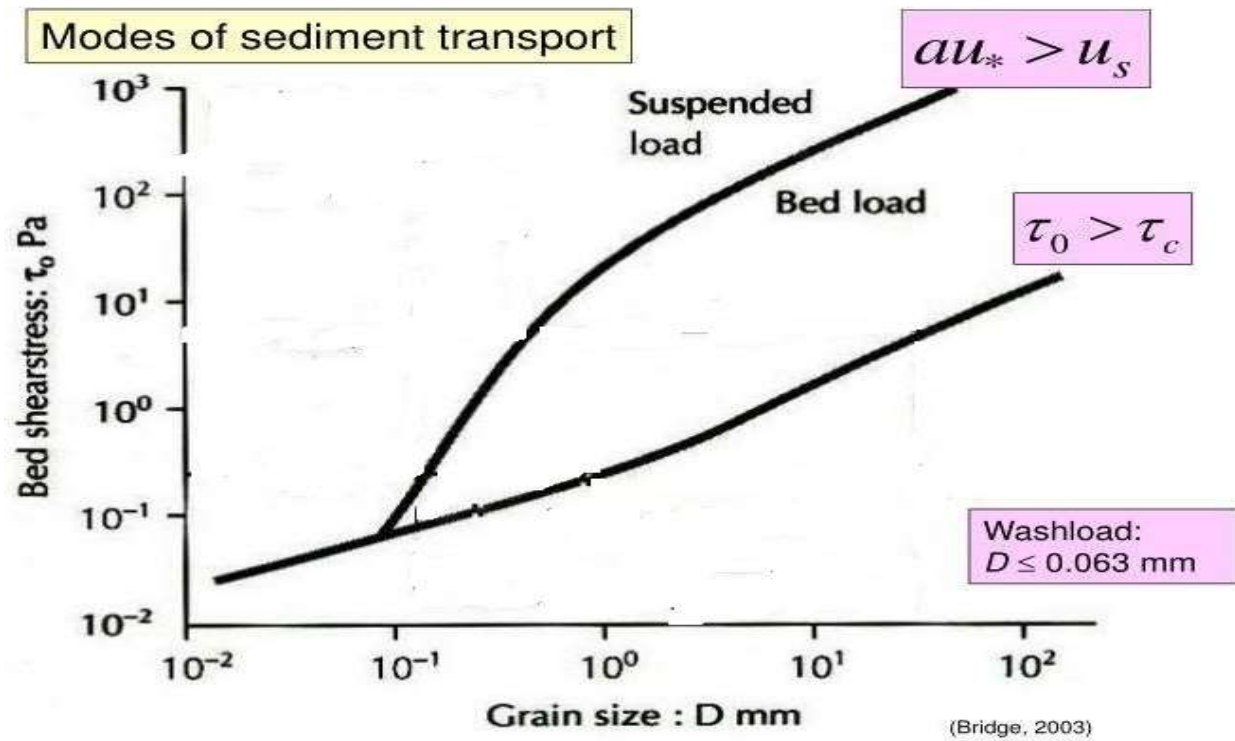
$q_b = 0.253(36.79 - 31.04)^{3/2} = 0.253(5.75)^{3/2} = 3.49$ kg/s m-1

The total sediment transport rate $Q_s = q_b w = (3.49)(50.0) = 174.5$ kg/s.

Criteria for Sediment Transport Modes

- Bedload: $\tau_0 > \tau_c$
- Suspended bed material: $au_* > u_s$
- Washload: $D \leq 0.063$ mm

τ_0 = Bed shear stress
τ_C = Critical shear stress
U_* = Shear velocity
U_S = Fall velocity



Estimation of Sediment Transport Rates

There are various formulae for predicting the amount of sediment transported as:

- Bed load
- Suspended load
- Total load

From water resources engineering point of view one generally requires the sediment transport rate of total load, and hence, the methods for predicting this would be discussed here.

In many practical situations, the suspended load is measured or estimated, through the available equations, as this constitutes about 80 to 90 percent of the total load. After estimating suspended load a certain percentage of it is added to estimate total load.

There are probably over a dozen of total load estimation relationships, some of the methods may be considered semi-empirical, most of them are based on dimensional analysis and graphical plotting or regression analysis.

Hence the basis for the choice of an appropriate sediment transport relation in practice can only be the relative accuracy of these methods. Yang (1996) has shown through examples that the prediction of total load by different formulae may vary by as much as four times of one another.

According to Garde and Ranga Raju (2000), the methods proposed by the following researchers give better results than other methods:

- Ackers-White (1973)
- Engelund-Hansen (1967)
- Brownlie (1981)
- Yang (1973)
- Karim-Kennedy (1990)

There are other methods like that proposed by Van Rijn (1984) which estimate the bed load and suspended load components of the total load separately.

Some of the methods to calculate total load are mentioned in the following sections.

The formulas that have been developed for predicting the total bed material transport have proven extremely useful. The formulas predict the total bed material transport under most conditions, including those in which the transport is predominantly as bed load.

The one disadvantage of the total bed material formulae is that they do not explicitly distinguish between the different modes of transport and hence it is not possible to assess the distribution of the sediments, that is, the amounts travelling close to the bed and the amounts in suspension.

Two formulas are recommended for the prediction of total bed material transport.

The Ackers and White (1973 and 1992) technique is reliable and of general application. It has been shown to agree well with laboratory, including lightweight sediments, and field data over a wide range of conditions.

The theory assumed that the quantity of fine, cohesive bed sediments travelling mainly in suspension are *correlated with total boundary shear stresses* and that the quantity of coarse, non-cohesive bed sediments travelling mainly as bed load are correlated with grain shear stresses.

The predictive technique of Engelund and Hansen (1967) is extremely simple to use and gives good results over a wide range of conditions. It does suffer the one disadvantage that fluid viscosity is not taken into account and hence predictions of low transport rates close to the threshold of movement can be of suspect.

Acker's White Formula

The original development of the theory was given by Ackers and it can be summarized in the following way. At first is necessary compute the value of the non dimensional diameter Dgr, and a number of parameters (n,A,m e C) related with the application of the formula.

$$D_{gr} = D35\left[\frac{g(\gamma_r - 1)}{\upsilon^2}\right]^{1/3}$$

v being the kinematic viscosity (m2 s-1)

For values of Dgr between 1 and 60,

n = 1 - 0.56 log Dgr

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \qquad m = \frac{9.66}{D_{gr}} + 1.34$$

$\log C = 2.86 \log D_{gr} - (\log D_{gr})^2 - 3.53$

for values of D_{gr} greater then 60,

n = 0

A = 0.17

m =1.5

C = 0.025

Once the former parameters are computed its possible to write an equation that is a function of sediment transport:

$$G_{gr} = C\left(\frac{F_{gr}}{A} - 1\right)^m$$

where F_{gr} denotes the sediments mobility and is defined as:

$$F_{gr} = \frac{V^{*n}}{\sqrt{g\, D35\, (\gamma_r - 1)}} \left[\frac{V}{\sqrt{32}\, \log(\alpha\, h / D35)} \right]^{1-n}$$

where,

α = 1/Ks

h = local depth
V* = shear velocity
V = velocity

The transformation of the function G_{gr} in volume transported by unit of time can then be made through the following formula:

$$q_{sv} = \frac{G_{gr}\, D35}{h} \left(\frac{V^*}{V} \right)$$

Online_ackers_white: Calculation of sediment discharge by the Ackers-White formula

INPUT DATA: [Sample input file]

Select:

Hydraulic depth D :	m	2
Channel width B :	m	50
Mean velocity V :	m/s	2
Bottom slope S_o :	m/m	0.0003
Mean diameter d_{50} :	mm	0.2
Temperature T :	°C	23
Specific gravity S_s : [Leave blank to choose default S_s = 2.65]		

SI units (metric)
U.S. Customary units

OUTPUT:

Sediment discharge Q_s : 3313.168 Newtons/sec

Sediment discharge Q_s : 29180.20 M.Tons/day

Calculate

Press button to

or recalculate

http://onlinecalc.sdsu.edu/onlineackerswhite.php

Engelund and Hansen Formula

Based on the shear approach and experimental data from the series of tests on a large flume this formula is developed.

$$q_s = 0.05\gamma_s v^2 \sqrt{\frac{D_{50}}{g\left(\frac{\gamma_s}{\gamma}-1\right)}} \left[\frac{\zeta_o}{(\gamma_s-\gamma)D_{50}}\right]^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

where, qs is sediment transport per unit time per unit width, ζo is bed shear stress, V is average flow velocity, $D50$ is median diameter of bed material, γs is specific weight of sediment particles and γ is specific weight of water and g is acceleration due to gravity.

Estimation of Sediment Transport Rates

Sediment load of Bangladesh rivers

The rivers in Bangladesh carry a large amount of suspended load.
As estimation of suspended load is rather complex, estimation of total load has been carried out by a number of methods.

Such estimation for the **Ganges, Brahmaputra-Jamuna, Kushiyara, Monu and Teesta** are shown in the following slides:

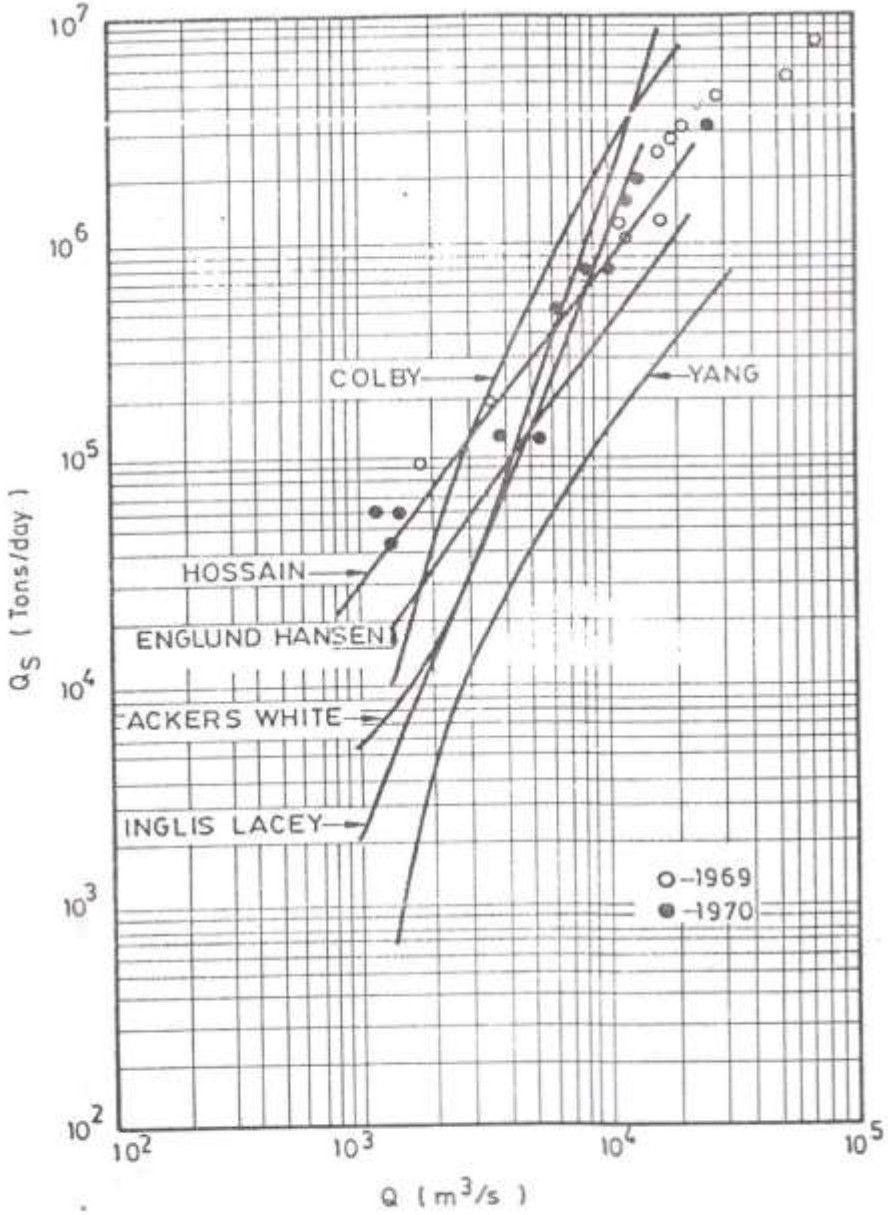
Ackers and White
Engelund and Hansen
Colby
Inglis and Lacey

Table 4.2.12: Annual Sediment Transport of the Kushiyara River at Sherpur

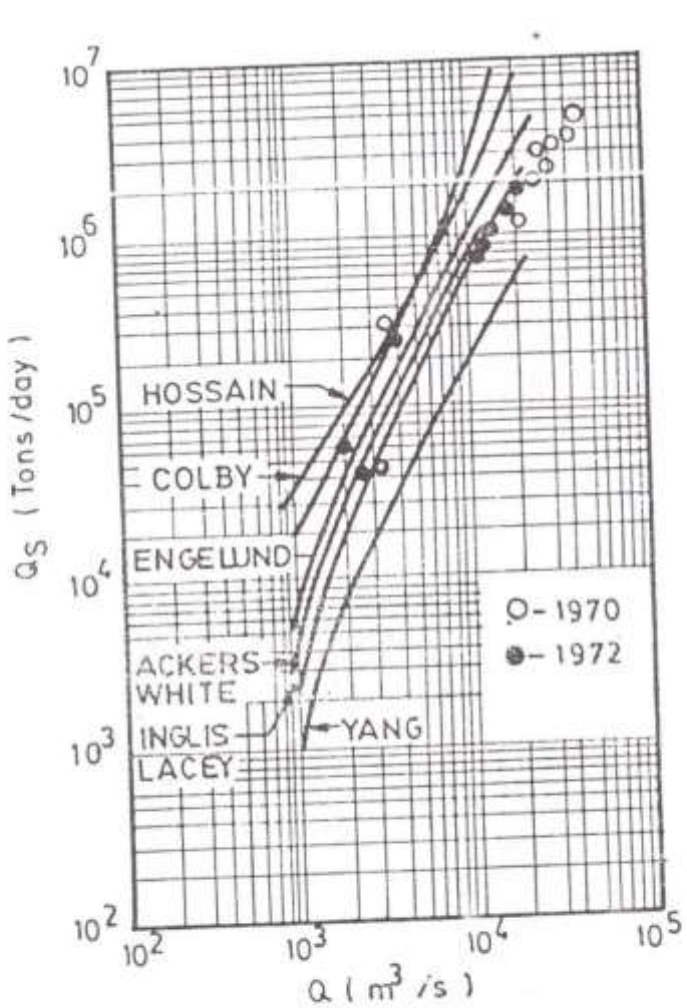
Year	Annual Average Discharge(m^3/s)	Observed Sediment Load (MT)	Computed Sediment Load (MT)	
			Engelund-Hansen Equation	Ackers-White Equation
2008	855.88	9500.5	35886.33	23326.11
1998	847.76	11780.4	30852.26	26224.42
1983	62.97	347.43	681.19	504.08

Table 4.3.7: Annual Sediment Transport of Monu River at Monu Rly. Bridge

Year	Annual Average Discharge(m^3/s)	Observed Sediment Load (MT)	Computed Sediment Load (MT)	
			Engelund-Hansen Equation	Ackers-White Equation
1983	14.4	179.61	112.33	82.03



Comparison of sediment transport formulae for the Jamuna at Nakfajarchar



Comparison of sediment transport formulae for the Jamuna at Nayarbari

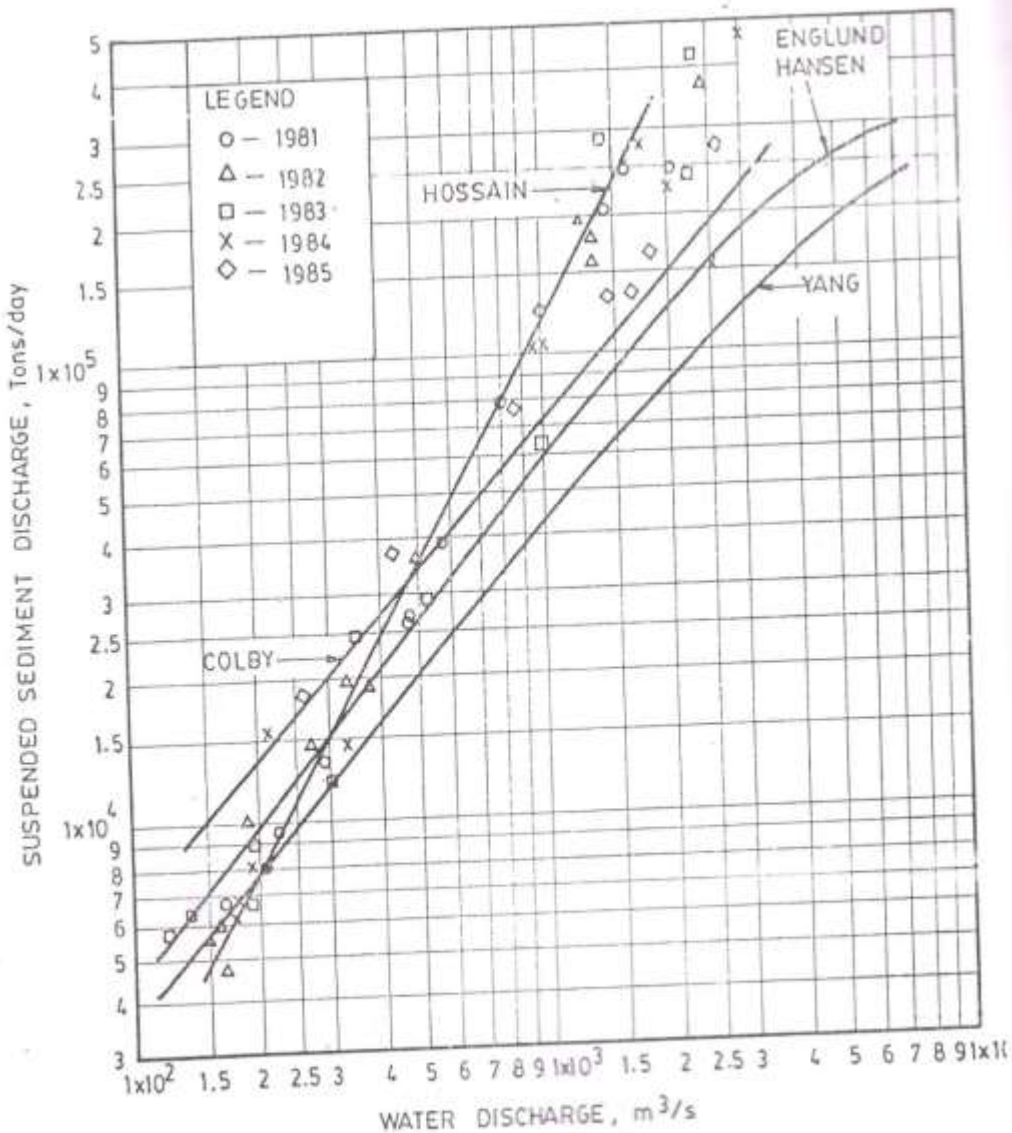
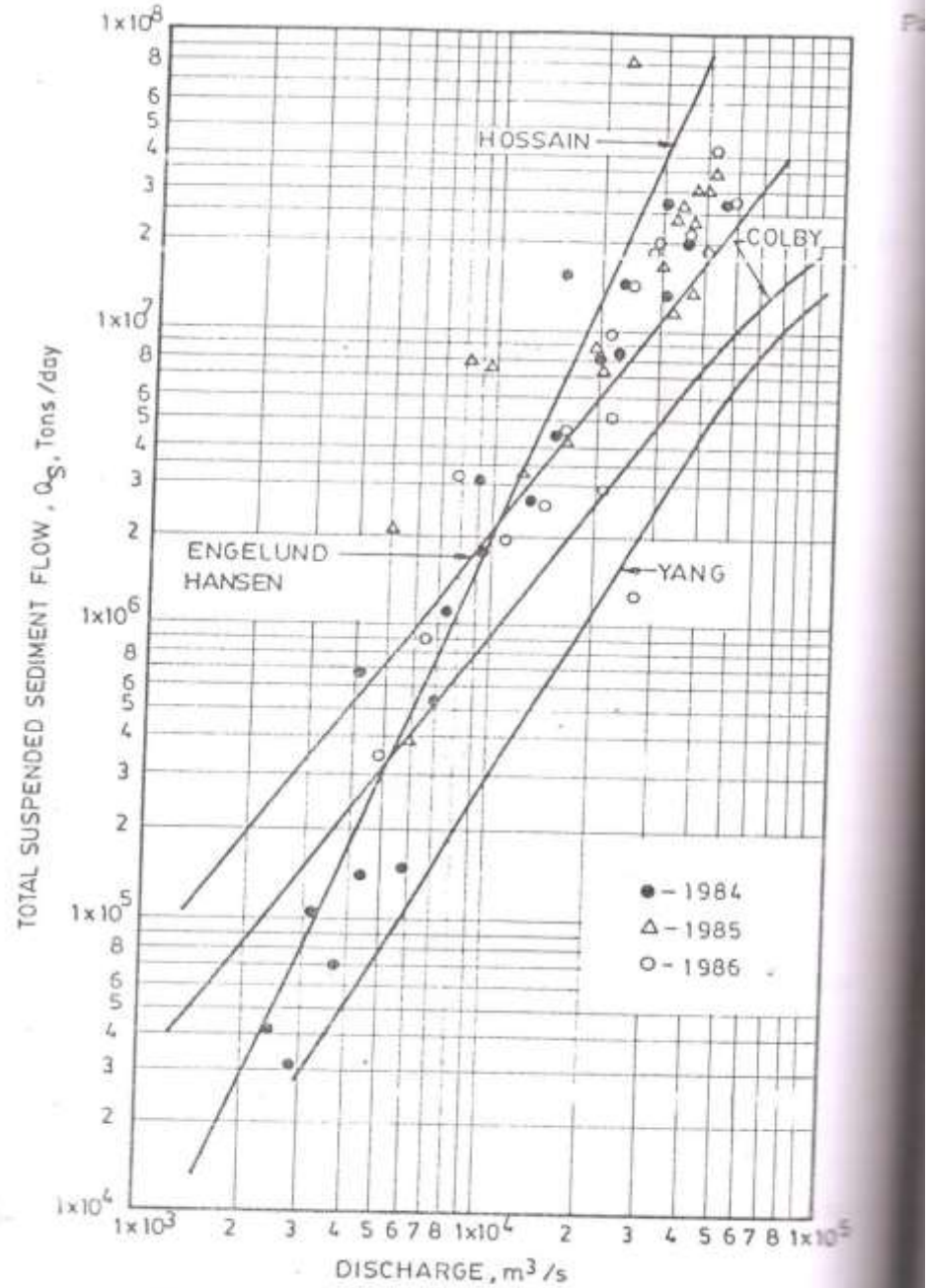


Fig. 6.6 Comparison of sediment transport formulae for the Teesta at Dalia.



6.7. Comparison of sediment transport formulae for the Ganges at Hard