EWC 471: Basic Coastal Engineering

Lecture 3

Topic: Wave Theory

Small Amplitude Wave theory

From last class:

Coastal landforms:

- Coral reefs
- Mangroves
- Seagrass beds
- Salt marshes
- Sandy beaches and dunes
- Rocky coastlines
- Bays, Gulfs and Estuaries
- Tidal flats











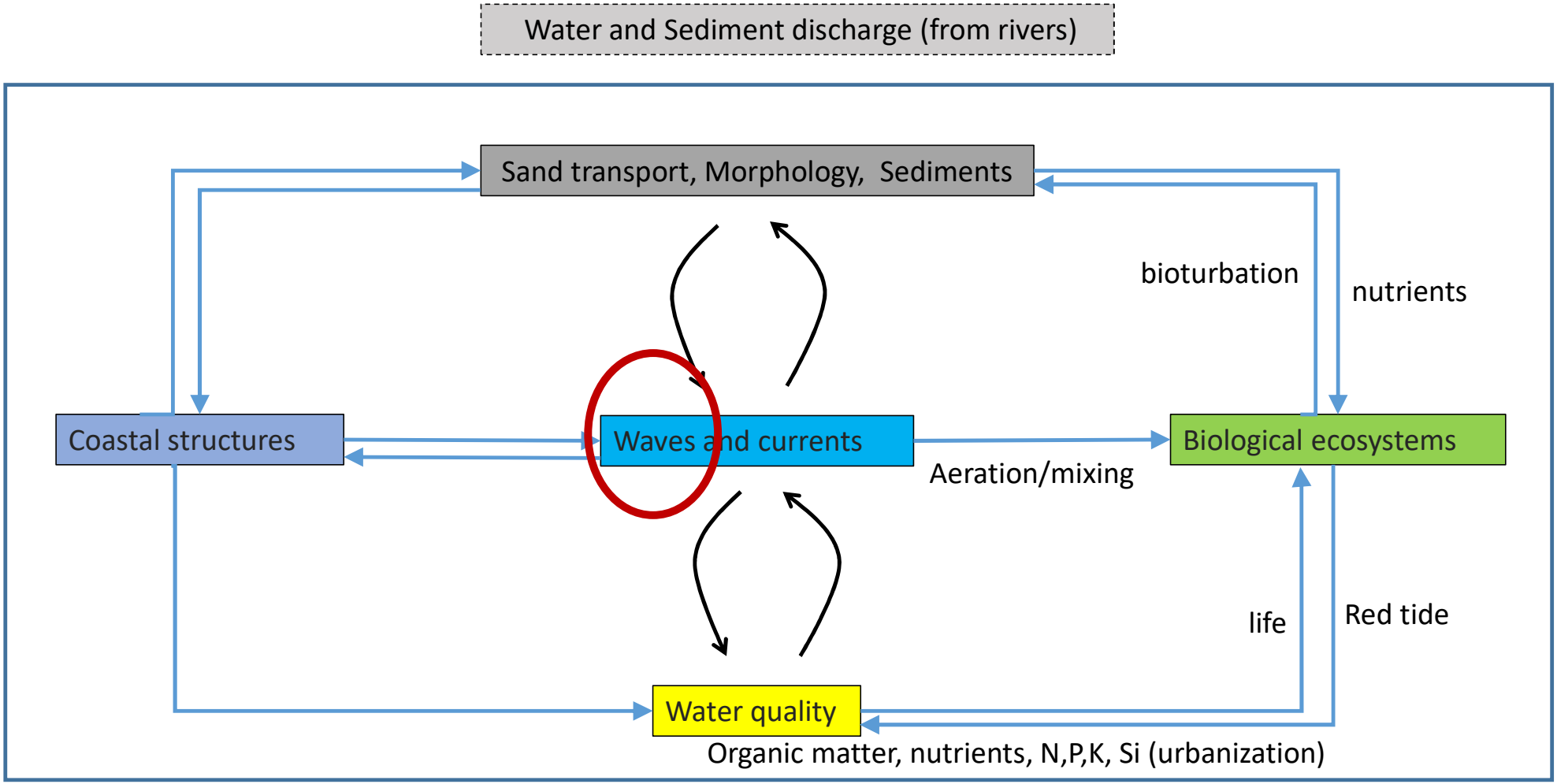






Tidal flat around Urir Char

Putting things in context



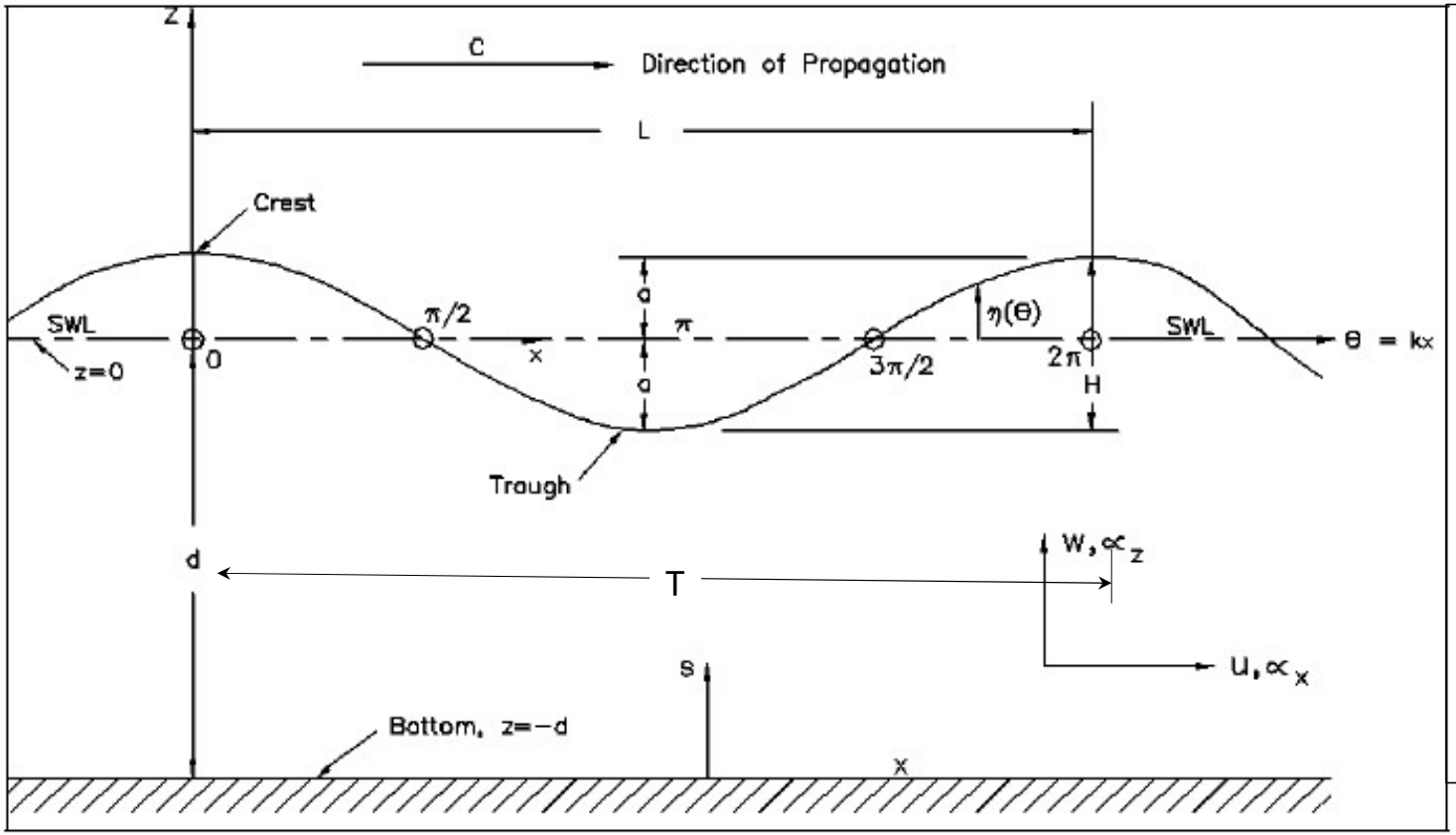
Waves and Wave-Induced Hydrodynamics

Waves are the prime movers for the littoral (coastal or shore region) processes at the shoreline. For the most part, they are generated by the action of the wind over water but also by moving objects such as passing boats and ships. Waves are almost always present at coastal sites owing to the vastness of the ocean's surface area, which serves as a generating site for waves, and the relative smallness of the surf zone, that thin ribbon of area around the ocean basins where the waves break and the wind-derived energy is dissipated.



Source:	Terminology:
Winds	Wind waves
Earthquakes	Tsunami
Ships	Ship waves
Atmospherics processes	Storm surge
Gravitation	Tide

Basic terminology/ Definition of wave parameters



L= wave length
H= wave height
T= wave period
d=water depth
η=surface elevation

Crest = highest point
Trough = lowest point

SWL= still water level
For small amplitude waves
Wave amplitude a=H/2

Angular or radian frequency ω (or σ)=2π/T
Wave number k = 2π/L
Phase velocity or wave celerity C= L/T= ω /k

Wave steepness ε = H/L
Relative depth = d/L
Relative wave height = H/d

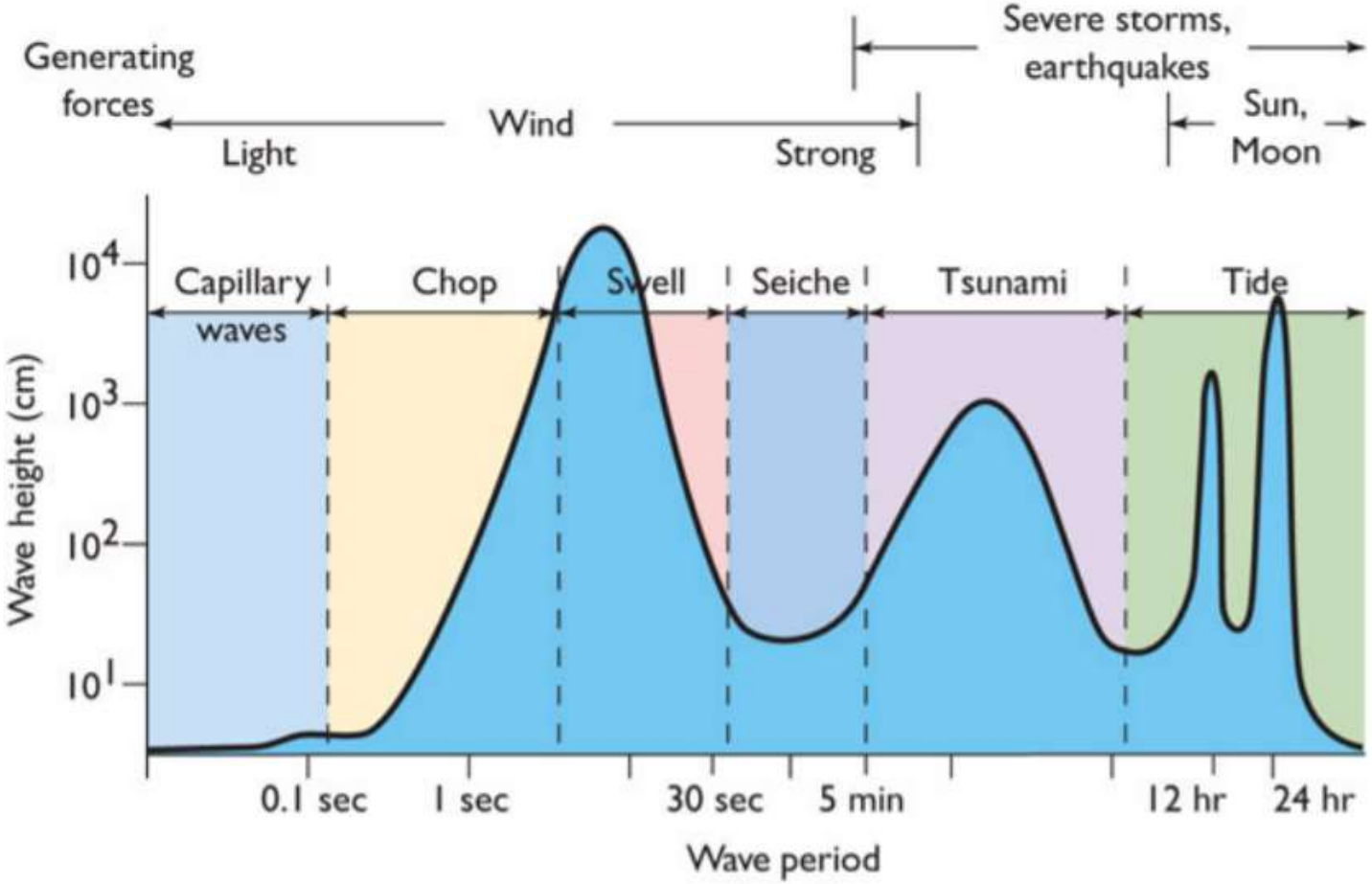
Wave heights, wave periods and wave generating forces:



Capillary waves



Chop waves: rough sea



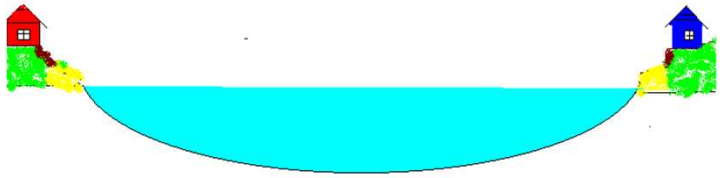
Swell and Seiche



Swell



Swell



Seiche

A **swell**, in the context of an ocean, sea or lake, is a series of mechanical **waves** that propagate along the interface between water and air. Swell are **waves (usually with smooth tops) that have moved beyond the area where they were generated**.

A **seiche** occurs in an enclosed body of water such as a lake, bay or gulf. A **seiche** is a standing **wave** that oscillates in a lake as a result of seismic or atmospheric disturbances creating huge fluctuations of water levels.

Stages of waves:



1) Sea

- waves in area effected by wind
- tend to be very irregular
- composed of many waves superimposed



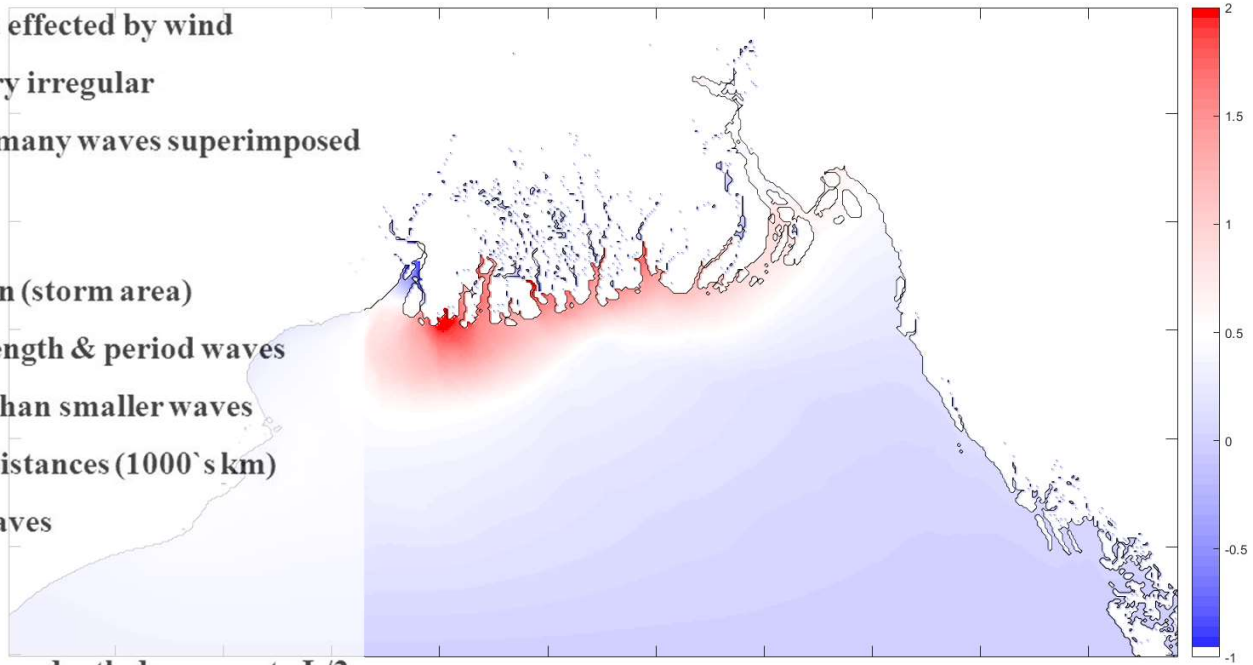
2) Swell

- far from origin (storm area)
- larger wavelength & period waves
- travel faster than smaller waves
- travel great distances (1000`s km)
- deep-water waves



3) Surf

- nearshore where depth decreases to L/2
- waves shoal and break



Linear wave theory/ small amplitude wave theory

- The small-amplitude theory for two-dimensional, freely propagating, periodic gravity waves is developed by linearizing the equations that define the free surface boundary conditions.
- This is the simplest theory to describe the wave motion is also known as the Airy wave theory or the linear wave theory, because of its simplifying assumptions.
- This theory is predicated on the following conditions:
  - an incompressible fluid (a good assumption),
  - Irrotational fluid motion (implying that there is no viscosity in the water, which would seem to be a bad assumption, but works out fairly well),

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \ldots\ldots (1)$$

  - an impermeable flat bottom (not too true in nature),
  - Pressure at air-sea interface is constant, and
  - very small amplitude waves (not a good assumption, but again it seems to work pretty well unless the waves become large).

Linear wave theory/ small amplitude wave theory

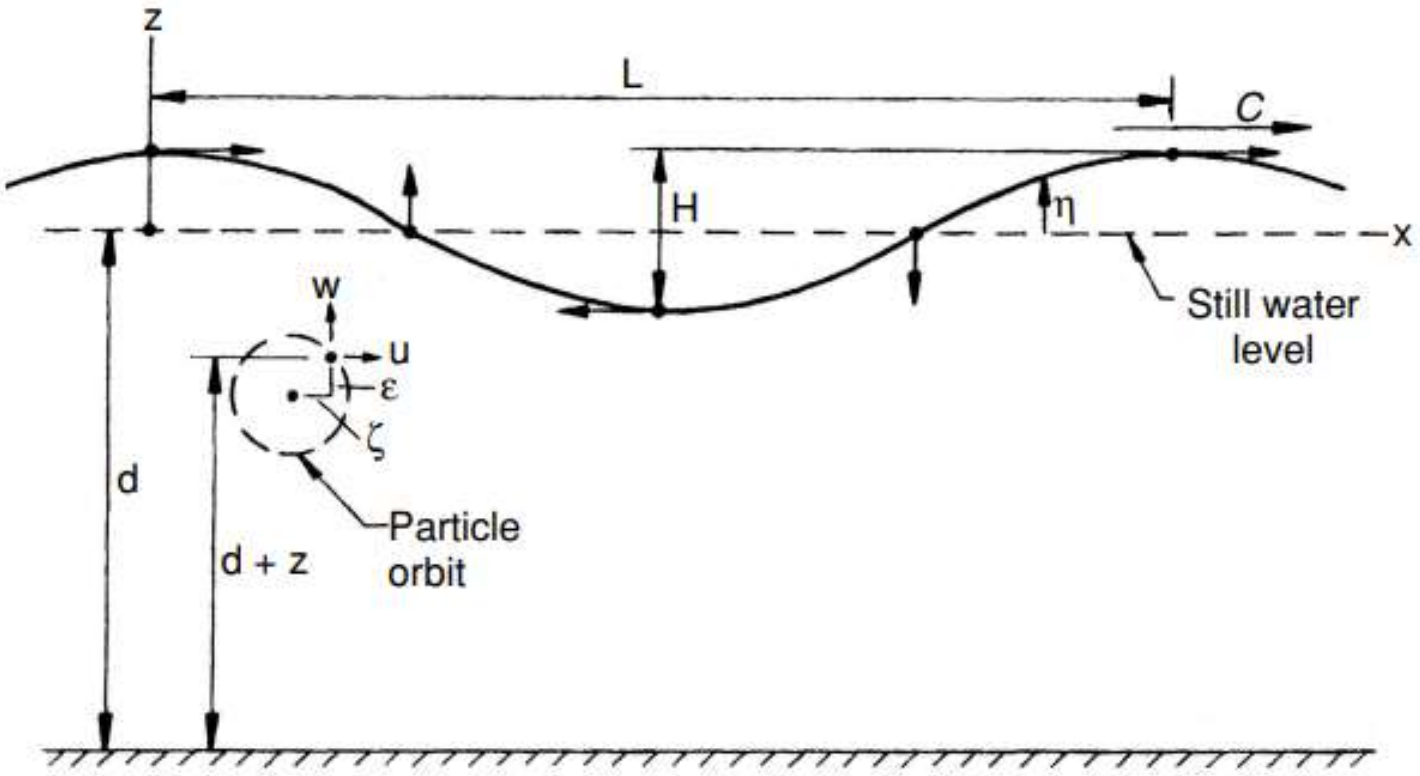


Figure 1 Definition of progressive surface wave parameters.

Linear wave theory/ small amplitude wave theory

Figure 1 depicts a monochromatic wave traveling at a phase celerity C on water of depth d in an x, z coordinate system. The x axis is the still water position and the bottom is at z = -d. The wave surface profile is defined by z = η, where η is a function of x and time t. The wavelength L and height H are as shown in the figure. Since the wave travels a distance L in one period T,

C = L/T ….(2)

The arrows at the wave crest, trough, and still water positions indicate the directions of water particle motion at the surface. As the wave propagates from left to right these motions cause a water particle to move in a clockwise orbit. The water particle velocities and orbit dimensions decrease in size with increasing depth below the still water line.

Linear wave theory/ small amplitude wave theory

The horizontal and vertical components of the water particle velocity at any instant are u and w, respectively.

The horizontal and vertical coordinates of a water particle at any instant are given by ζ and ε, respectively.

The coordinates are referenced to the center of the orbital path that the particle follows.

At any instant, the water particle is located a distance d – (-z) = d+z above the bottom.

The following dimensionless parameters are often used:

wave number $k = 2\Pi/L$

wave angular frequency $\sigma = 2\Pi/T$

We also use the terms

wave steepness defined as the wave height divided by the wave length (i.e., H/L) and

relative depth defined as the water depth divided by the wave length (i.e., d/L) in discussions of wave conditions.

Linear wave theory/ small amplitude wave theory

The small-amplitude wave theory is developed by solving Eq. (1) for the domain depicted in Figure 1, with the appropriate boundary conditions for the free surface (2) and the bottom (1). At the bottom there is no flow perpendicular to the bottom which yields the bottom boundary condition (BBC):

$$w = \frac{\partial \phi}{\partial z} = 0 \text{ at } z = -d \quad \text{..... (3)}$$

At the free surface there is a kinematic boundary condition (KSBC) that relates the vertical component of the water particle velocity at the surface to the surface position:

$$w = \frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} \text{ at } z = \eta \quad \text{..... (4)}$$

Linear wave theory/ small amplitude wave theory

The Bernoulli equation for unsteady irrotational flow may be written

$$\frac{1}{2}(u^2 + w^2) + \frac{p}{\rho} + gz + \frac{\partial \phi}{\partial t} = 0 \qquad \ldots\ldots (5)$$

where g is the acceleration of gravity, p is the pressure, and r is the Xuid density. At the surface where the pressure is zero the dynamic boundary condition (DSBC) becomes

$$\frac{1}{2}(u^2 + w^2) + gz + \frac{\partial \phi}{\partial t} = 0 \text{ at } z = \eta \qquad \ldots\ldots (6)$$

Linear wave theory/ small amplitude wave theory

The KSBC and the DSBC have to be linearized and applied at the still water line rather than at the a priori unknown water surface. This yields for the KSBC

$$w = \frac{\partial \eta}{\partial t} \text{ at } z = 0 \qquad \text{..... (7)}$$

and for the DSBC

$$g\eta + \frac{\partial \phi}{\partial t} = 0 \text{ at } z = 0 \qquad \text{..... (8)}$$

Linear wave theory/ small amplitude wave theory

Employing the Laplace equation, the BBC, and the linearized DSBC, we can derive the velocity potential for the small-amplitude wave theory (Ippen, 1966; Sorensen, 1978; or Dean and Dalrymple, 1984). The most useful form of this velocity potential is:

$$\phi = \frac{gH}{2\sigma} \frac{\cosh k(d+z)}{\cosh\ kd} \sin(kx - \sigma\ t) \quad \text{..... (9)}$$

Since the wavelength (L) or wave number (k) depends on the wave period (T) and water depth (d), when the wave height and period plus the water depth are known the wave is fully defined and all of its characteristics can be calculated.

Linear wave theory/ small amplitude wave theory

We can insert the velocity potential into the linearized DSBC with z = 0 to directly determine the equation for the wave surface profile:

$$\eta = \frac{H}{2}\cos(kx - \sigma\, t) \quad \ldots\ldots (10)$$

which can also be written

$$\eta = \frac{H}{2}\cos 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \quad \ldots\ldots (11)$$

by inserting the wave number and wave angular frequency. Thus, the small amplitude wave theory yields a cosine surface profile. This is reasonable for low amplitude waves, but with increasing wave amplitude the surface profile becomes vertically asymmetric with a more peaked wave crest and a flatter wave trough.

Linear wave theory/ small amplitude wave theory

Combining the KSBC and the DSBC by eliminating the water surface elevation yields:

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0$$

Then, inserting the velocity potential, differentiating, and rearranging we have:

$$\sigma^2 = gk \tanh kd$$

or

$$C = \frac{\sigma}{k} = \sqrt{\frac{g}{k} \tanh kd}$$

and

$$C = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} \quad \ldots\ldots (12)$$

Equation (12) indicates that for small-amplitude waves, the wave celerity is independent of the wave height.

Linear wave theory/ small amplitude wave theory

Equation (12) can also be written as:

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L} \quad \text{..... (13)}$$

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L} \quad \text{..... (14)}$$

From Eq. (14), if the water depth and the wave period are known, the wavelength can be calculated by trial and error. Then the celerity can be determined from C = L/T.

Equations (2.12) to (2.14) collectively are commonly known as the **dispersion equation**.

Linear wave theory/ small amplitude wave theory

Important:

- For a spectrum of waves having different periods (or lengths), the longer waves will propagate at a higher celerity and move ahead while the shorter waves will lag behind.
- It can be demonstrated that as a wave propagates from deep water into the shore, the wave period will remain constant because the number of waves passing sequential points in a given interval of time must be constant.
- Other wave characteristics including the celerity, length, height, surface profile, particle velocity and acceleration, pressure field, and energy will all vary during passage from deep water to the nearshore area.

Small amplitude wave theory: wave classification

An important classification of surface waves is based on the relative depth (d/L). When a wave propagates from deep water offshore into shallower water nearshore the wavelength decreases [see Eq. (14)], but at a slower rate than that at which the depth decreases.

Thus, the relative depth decreases as a wave approaches the shore. When d/L is greater than approximately 0.5, tanh (2Πd/L) is essentially unity and Eqs. (12) to (14) reduce to:

$$C_o = \sqrt{\frac{gL_o}{2\pi}} \quad \text{..... (15)}$$

$$C_o = \frac{gT}{2\pi} \quad \text{..... (16)}$$

$$L_o = \frac{gT^2}{2\pi} \quad \text{..... (17)}$$

Waves in this region are called deep water waves and this condition is commonly denoted by the subscript zero.

Small amplitude wave theory: wave classification

Wave particle velocities and orbit dimensions decrease with increasing distance below the free surface. In deep water at a depth of $-z/L > 0.5$ the particle velocities and orbit dimensions are close to zero. Since for $d/L > 0.5$ the waves do not interact with the bottom, wave characteristics are thus independent of the water depth.

Small amplitude wave theory: wave classification

When the relative depth is less than 0.5 the waves interact with the bottom. Wave characteristics depend on both the water depth and the wave period, and continually change as the depth decreases. The full dispersion equations must be used to calculate wave celerity or length for any given water depth and wave period. Dividing Eq. (13) by Eq. (16) or Eq. (14) by Eq. (17) yields

$$\frac{C}{C_o} = \frac{L}{L_o} = \tanh\frac{2\pi d}{L} \quad \text{..... (18)}$$

Waves propagating in the range of relative depths from 0.5 to 0.05 are called intermediate or transitional water waves.

Small amplitude wave theory: wave classification

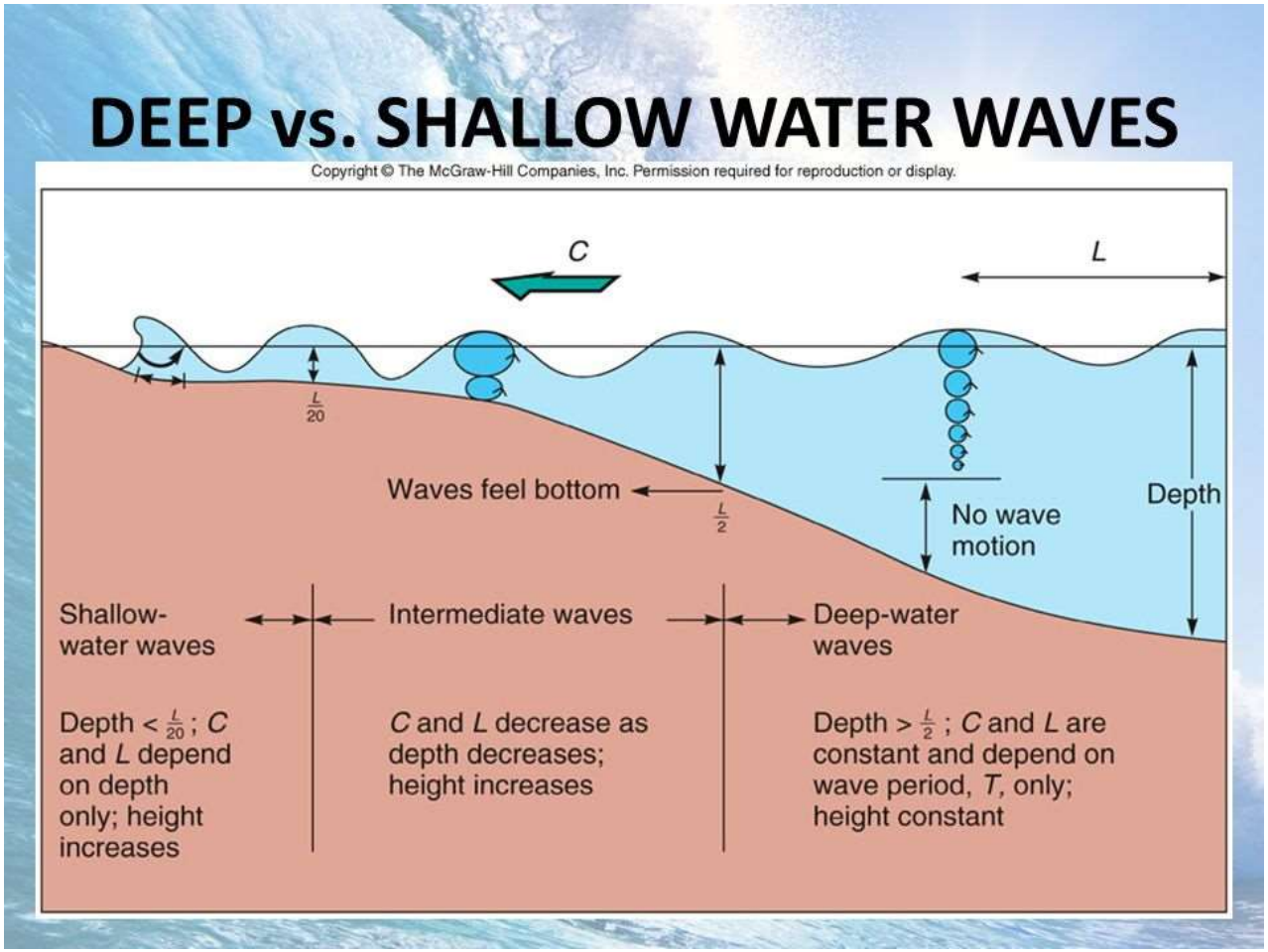
When the relative depth is less than approximately 0.05, tanh (2Πd/L) approximately equals 2Πd/L and the dispersion equation yields

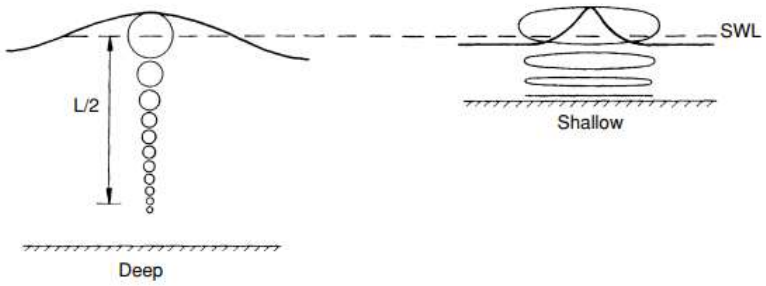
$$C = \sqrt{gd} \quad \text{..... (19)}$$

$$L = \sqrt{gd}T \quad \text{..... (20)}$$

Waves in this region of relative depths are called shallow water waves. In shallow water the small-amplitude wave theory gives a wave celerity that is independent of wave period and dependent only on the water depth (i.e., the waves are not period dispersive).

Small amplitude wave theory: wave classification



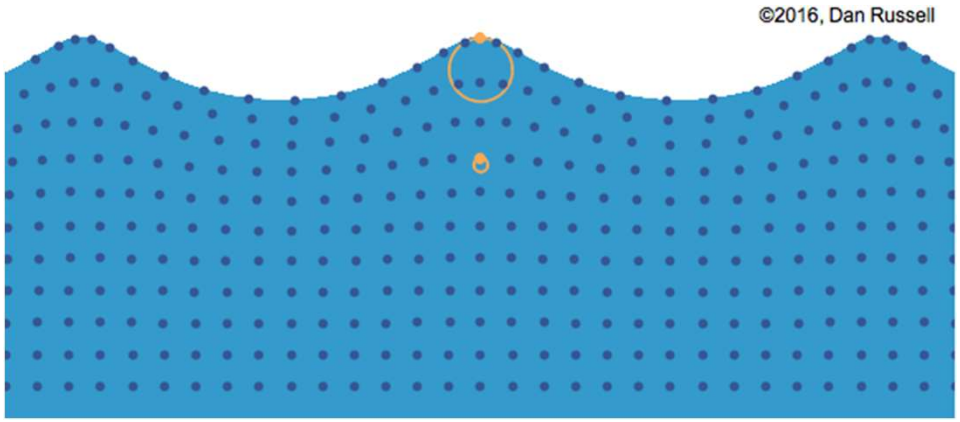


Deep and shallow water surface profiles and particle orbits

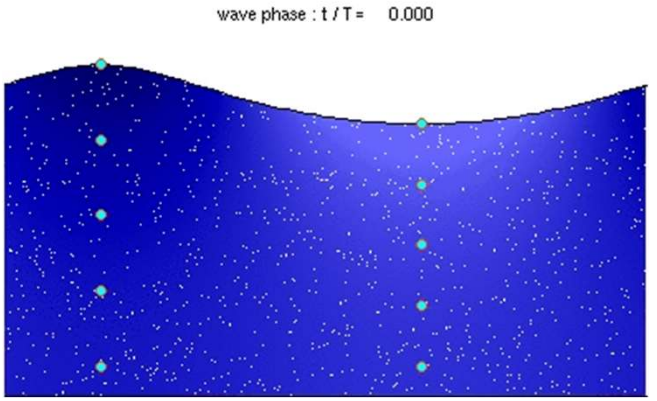
Pure oscillatory waves and finite amplitude waves

Water waves are considered oscillatory or nearly oscillatory if the motion described by the water particles is circular orbits that are closed or nearly closed for each wave period. The linear theory represents pure oscillatory waves.

Waves defined by finite-amplitude wave theories are not pure oscillatory waves but still periodic since the fluid is moved in the direction of wave advance by each successive wave. This motion is termed mass transport of the waves. When water particles advance with the wave and do not return to their original position, the wave is called a wave of translation. A solitary wave is an example of a wave of translation.



Pure oscillatory waves, particles in circular orbits



Not pure oscillatory waves, particles in nearly closed orbits

Linear wave theory/ small amplitude wave theory

Example 1

A wave in water 100 m deep has a period of 10 s and a height of 2 m. Determine the wave celerity, length, and steepness. What is the water particle speed at the wave crest?

Solution:

Assume that this is a deep water wave. Then, from Eq. (2.17)

$$L_o = \frac{9.81(10)^2}{2\pi} = 156\,\text{m}$$

Since the depth is greater than half of the calculated wave length, the wave is in deep water and the wave length is 156 m. [Otherwise, Eq. (2.14) would have to be used to calculate the wave length.] The wave celerity is from Eq. (2.2)

$$C_o = \frac{156}{10} = 15.6\,m/s$$

Linear wave theory/ small amplitude wave theory

Example 1 (continued)

and the steepness is

$$\frac{H_o}{L_o} = \frac{2}{156} = 0.013$$

For deep water the particle orbits are circular having a diameter at the surface equal to the wave height. Since a particle completes one orbit in one wave period, the particle speed at the crest would be the orbit circumference divided by the period or

$$u_c = \frac{\pi H_o}{T} = \frac{3.14(2)}{10} = 0.63\, m/s$$

Note that this is much less than C_o.

Linear wave theory/ small amplitude wave theory

Example 2

Consider the wave from Example 2.3-1 when it has propagated in to a nearshore depth of 2.3 m. Calculate the wave celerity and length.

Solution:

Assuming this is a shallow water wave, Eq. (2.19) yields

$$C = \sqrt{9.81(2.3)} = 4.75\,\text{m/s}$$

and Eq. (2.2) yields

$$L = 4.75(10) = 47.5\,\text{m}$$

So $d/L = 2.3/47.5 = 0.048 < 0.05$ and the assumption of shallow water was correct. Compare these values to the results from previous example.

Thank You