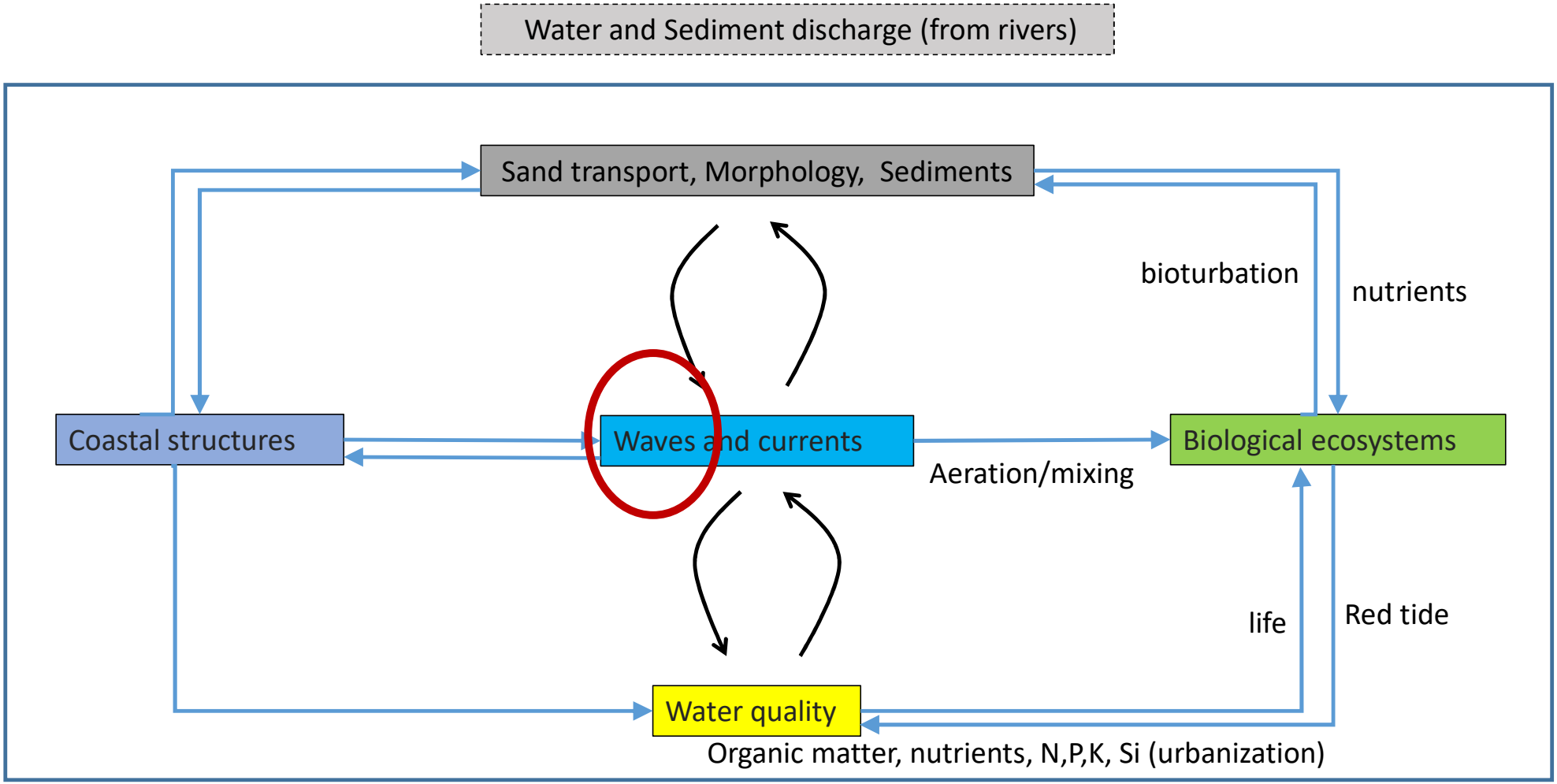
EWC 471: Basic Coastal Engineering

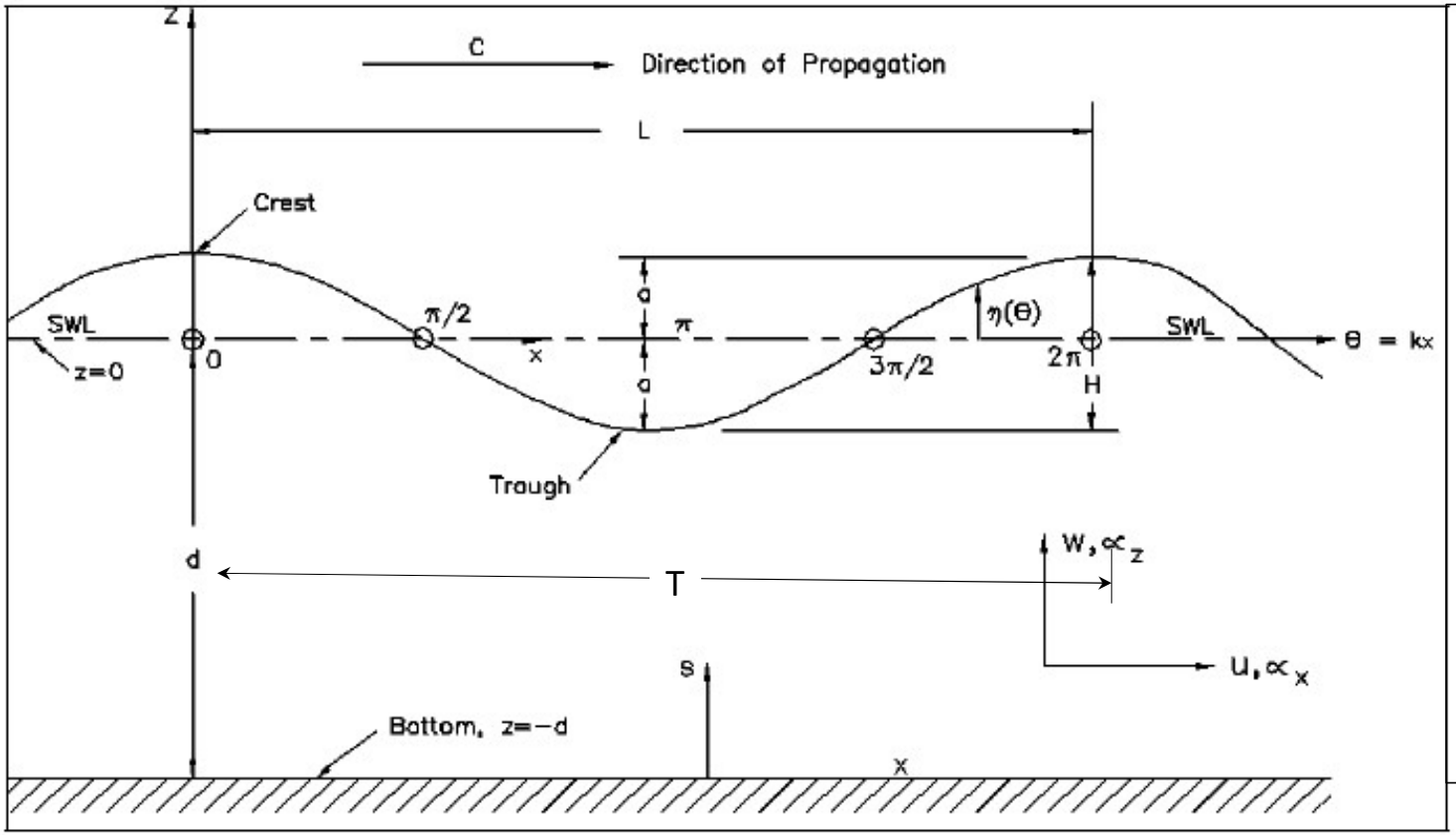
Lecture 4

Topic: Small Amplitude Wave theory and Application Examples

Putting things in context



Basic terminology/ Definition of wave parameters



L= wave length
H= wave height
T= wave period
d=water depth
η=surface elevation

Crest = highest point
Trough = lowest point

SWL= still water level
For small amplitude waves
Wave amplitude a=H/2

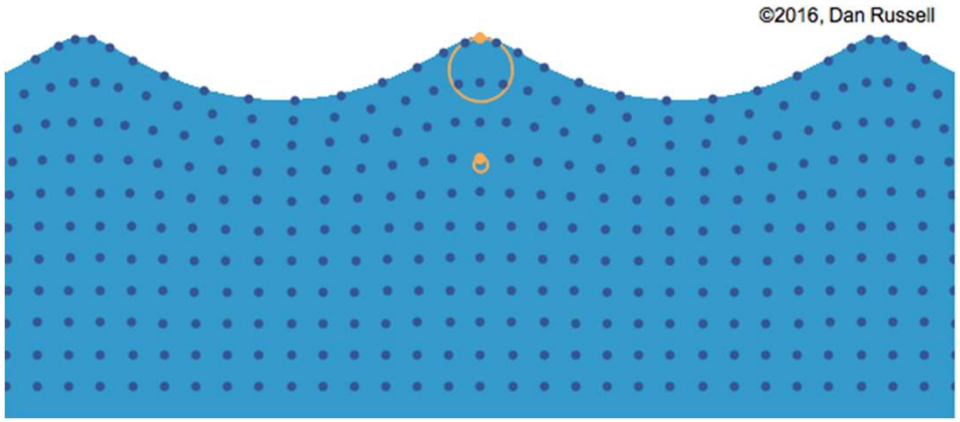
Angular or radian frequency ω (or σ)=2π/T
Wave number k = 2π/L
Phase velocity or wave celerity C= L/T= ω /k

Wave steepness ε = H/L
Relative depth = d/L
Relative wave height = H/d

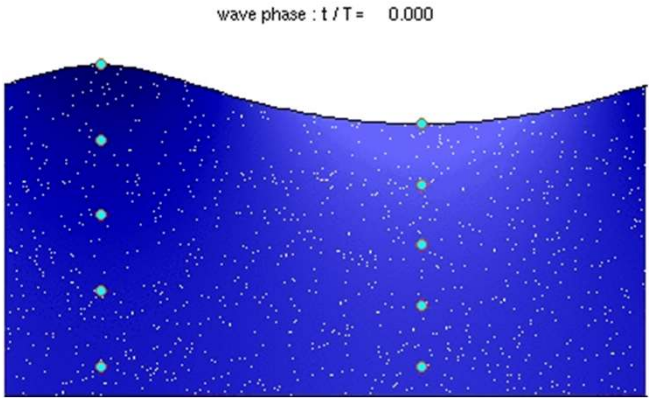
Pure oscillatory waves and finite amplitude waves

Water waves are considered oscillatory or nearly oscillatory if the motion described by the water particles is circular orbits that are closed or nearly closed for each wave period. The linear theory represents pure oscillatory waves.

Waves defined by finite-amplitude wave theories are not pure oscillatory waves but still periodic since the fluid is moved in the direction of wave advance by each successive wave. This motion is termed mass transport of the waves. When water particles advance with the wave and do not return to their original position, the wave is called a wave of translation. A solitary wave is an example of a wave of translation.



Pure oscillatory waves, particles in circular orbits



Not pure oscillatory waves, particles in nearly closed orbits

Small amplitude wave theory: wave classification

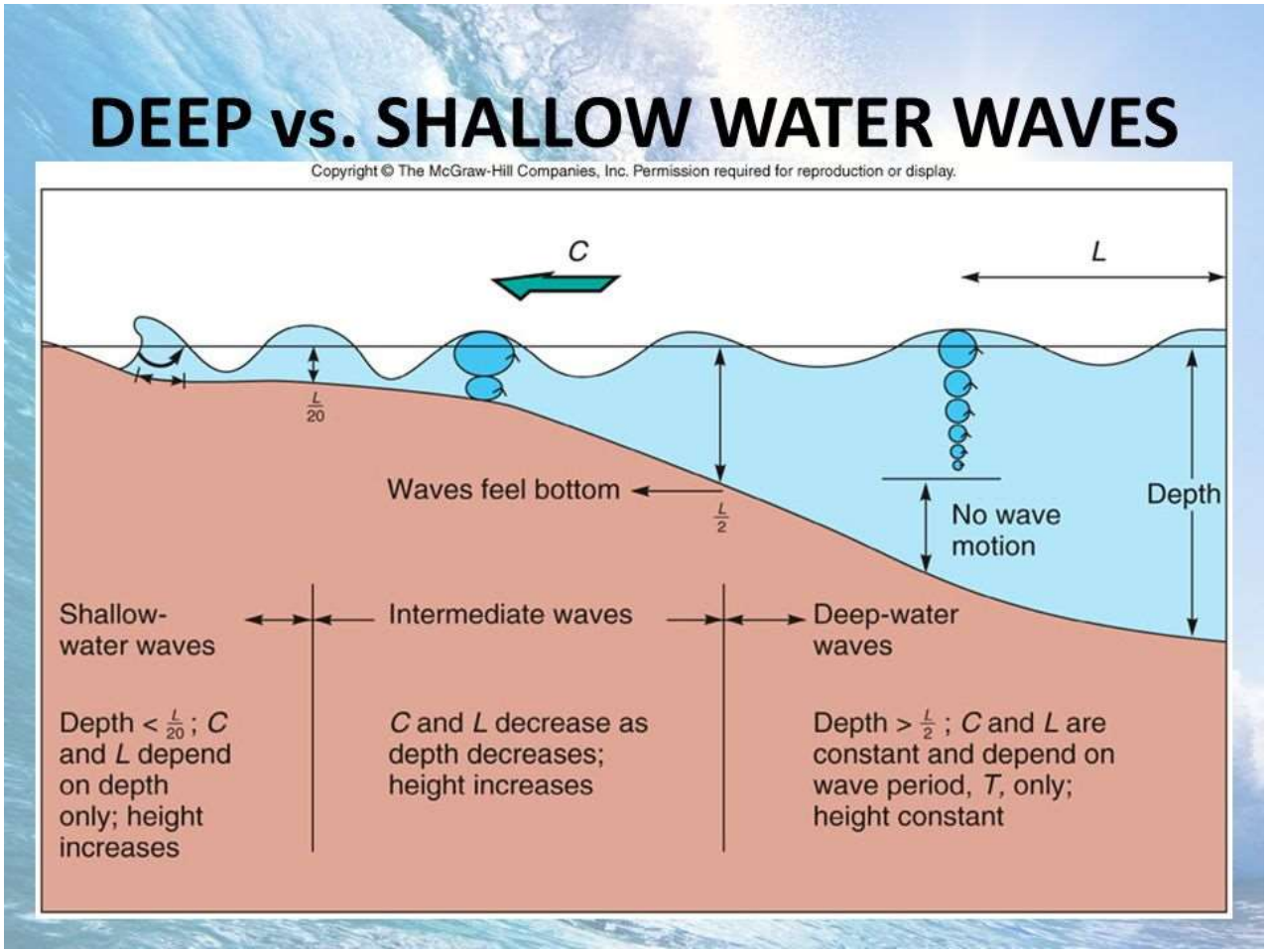
Shallow water waves:

$$C = \sqrt{gd}$$

$$L = \sqrt{gd}T$$

Intermediate waves:

$$\frac{C}{C_o} = \frac{L}{L_o} = \tanh\frac{2\pi d}{L}$$



Deep water waves:

$$C_o = \sqrt{\frac{gL_o}{2\pi}}$$

$$C_o = \frac{gT}{2\pi}$$

$$L_o = \frac{gT^2}{2\pi}$$

Linear wave theory/ small amplitude wave theory

Example 1

A wave in water 100 m deep has a period of 10 s and a height of 2 m. Determine the wave celerity, length, and steepness. What is the water particle speed at the wave crest?

Solution:

Assume that this is a deep water wave. Then, from Eq. (2.17)

$$L_o = \frac{9.81(10)^2}{2\pi} = 156\,\text{m}$$

Since the depth is greater than half of the calculated wave length, the wave is in deep water and the wave length is 156 m. [Otherwise, Eq. (2.14) would have to be used to calculate the wave length.] The wave celerity is from Eq. (2.2)

$$C_o = \frac{156}{10} = 15.6\,m/s$$

Linear wave theory/ small amplitude wave theory

Example 1 (continued)

and the steepness is

$$\frac{H_o}{L_o} = \frac{2}{156} = 0.013$$

For deep water the particle orbits are circular having a diameter at the surface equal to the wave height. Since a particle completes one orbit in one wave period, the particle speed at the crest would be the orbit circumference divided by the period or

$$u_c = \frac{\pi H_o}{T} = \frac{3.14(2)}{10} = 0.63\, m/s$$

Note that this is much less than C_o.

Linear wave theory/ small amplitude wave theory

Example 2

Consider the wave from Example 2.3-1 when it has propagated in to a nearshore depth of 2.3 m. Calculate the wave celerity and length.

Solution:

Assuming this is a shallow water wave, Eq. (2.19) yields

$$C = \sqrt{9.81(2.3)} = 4.75\,\text{m/s}$$

and Eq. (2.2) yields

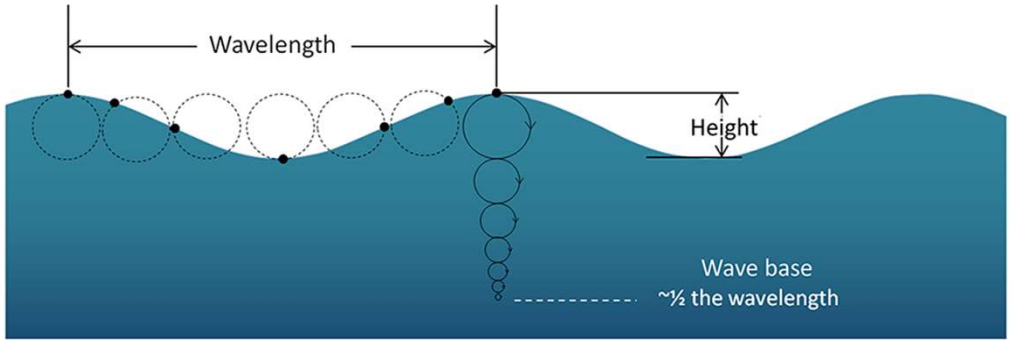
$$L = 4.75(10) = 47.5\,\text{m}$$

So $d/L = 2.3/47.5 = 0.048 < 0.05$ and the assumption of shallow water was correct. Compare these values to the results from previous example.

Linear wave theory/ small amplitude wave theory

Example 3: Consider a depth of 10m and solve for wave length L.

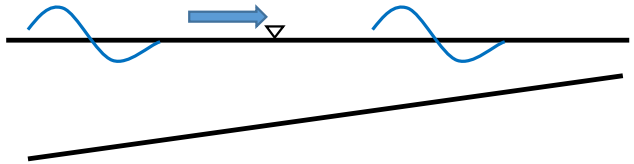
Ans: L=92.37m (through trial and error)



Linear wave theory: Application example

GIVEN: A wave with a period *T* = 10 seconds is propagated shoreward over a uniformly sloping shelf from a depth *d* = 200 m to a depth *d* = 3 m .

FIND: The wave celerities *C* and lengths *L* corresponding to depths *d* = 200 meters and d = 3 m.



Solution:

Deep water wavelength, $L_0 = 1.56\ T^2 = 1.56*(10)^2 = 156$ m

When d = 200m, d/L_0 = 200/156 i.e. >1/2, deep water condition.

So, $C = C_0 = \frac{gT}{2\pi} = \frac{9.81*10}{2*3.14} = 15.6\ m/s$ (Ans.) (or simply, C = L/T = 156/10 = 15.6m/s)

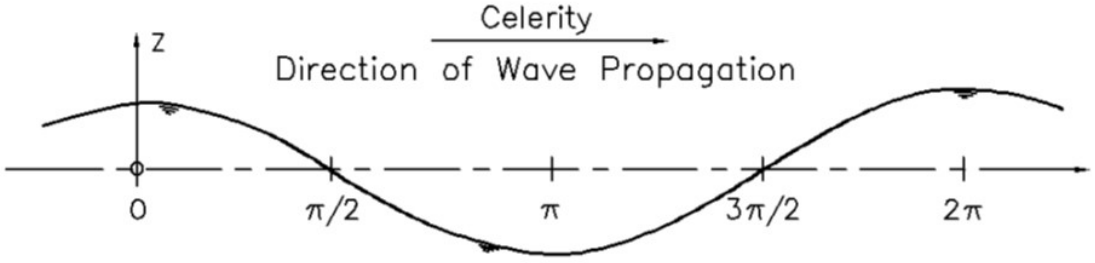
And deep-water wavelength, L_0 = 156m (Ans.)

When d= 3m, d/L_0 = 3/156 = 0.0192 i.e. < 1/20 or 0.05, shallow water condition.

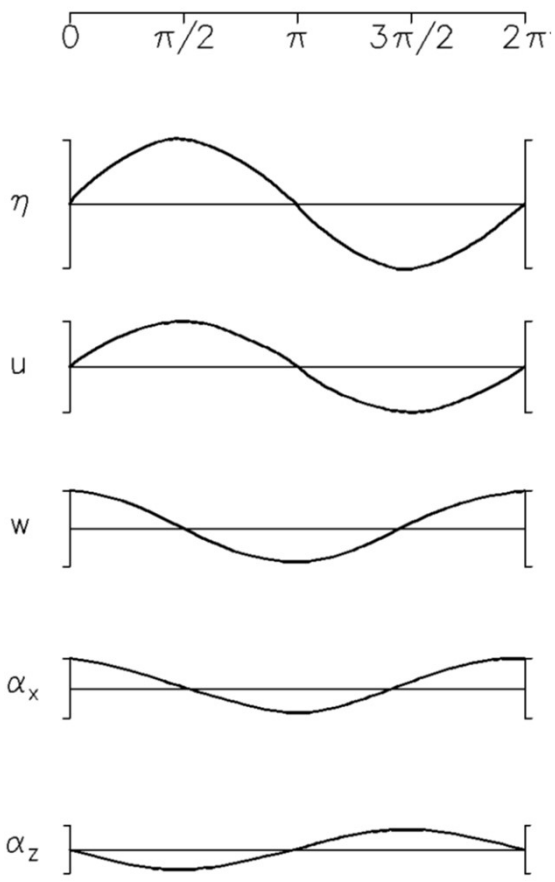
So, from L = $\sqrt{gdT}$ = $\sqrt{9.81 * 3}$ $* 10$ = = 54.25 m (Ans.)

and C = L/T = 54.25/10 = 5.43 m/s (Ans.)

Linear wave theory/ small amplitude wave theory



Velocity	u=+; w=0	u=0; w=−	u=−; w=0	u=0; w=+	u=+; w=0
Acceleration	α_x=0; α_z=−	α_x=−; α_z=0	α_x=0; α_z=+	α_x=+; α_z=0	α_x=0; α_z=−
θ	0	π/2	π	3π/2	2π



Profiles of particle velocity and acceleration by Linear wave theory in relation to the surface elevation

Linear wave theory/ small amplitude wave theory

- In wave force studies, the local fluid velocities and accelerations for various values of *z* and *t* during the passage of a wave must often be found. The horizontal component *u* and the vertical component *w* of the local fluid velocity are given by the following equations

$$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta \qquad w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$$

- The local fluid particle accelerations are:

$$\alpha_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta = \frac{\partial u}{\partial t}$$

$$\alpha_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta = \frac{\partial w}{\partial t}$$