EWC 471: Basic Coastal Engineering

Lecture 9 and 10

Topic: Wave Deformation/ Wave Transformation

Wave transformation

When the waves approach the shoreline, they are affected by the seabed through processes such as shoaling, refraction, bottom friction and wave-breaking. However, wave-breaking also occurs in deep water when the waves are too steep. If the waves meet major structures or abrupt changes in the coastline, they will be transformed by diffraction. If waves meet a submerged reef or structure, they will overtop the reef. If the waves meet a permeable structure, partial transmission will take place.

Wave transformations are related to wave phenomena occurring in the natural as well as intervened environment.

Wave transformation

Coastal engineering considers problems near the shoreline normally in water depths of less than 20 m. Project designs usually require knowledge of the wave field over an area of 1-10 km^2 in which the depth may vary significantly. Additionally, study of shoreline changes and beach protection frequently requires analysis of coastal processes over entire littoral cells, which may span 10-100 km in length.

Wave data are generally not available at the site or depths required. Often a coastal engineer will find that data have been collected or hindcast at sites offshore in deeper water or nearby in similar water depths. Present discussion provides procedures for transforming waves from offshore or nearby locations to nearshore locations needed by the engineer.

Wave transformation

Processes that can affect a wave as it propagates from deep into shallow water include:

1. Shoaling
2. Refraction
3. Diffraction
4. Bottom friction
5. Depth-induced wave-breaking
6. Wave set-up, set-down
7. Wave swash
8. Wave run-up
9. White-capping
10. Wave overtopping

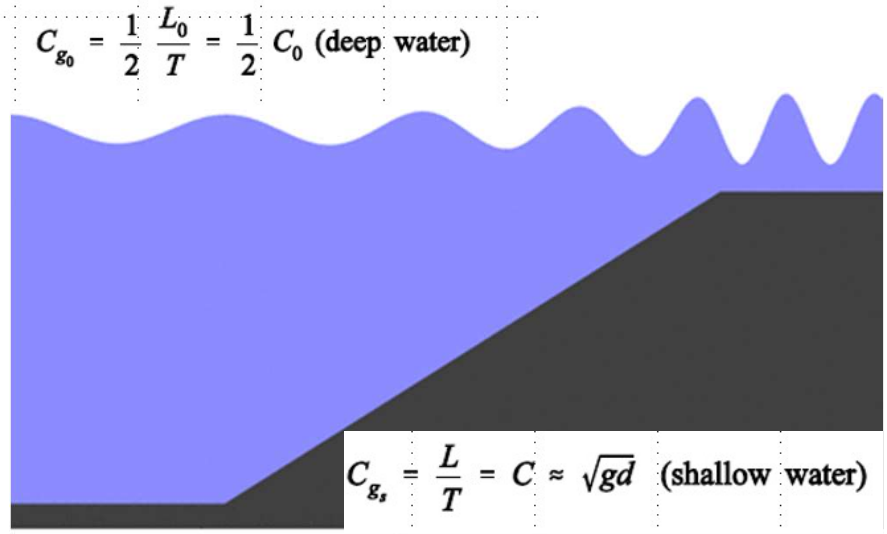
Additionally, the following phenomena may occur, but not due to shallow water effect:

11. Dissipation due to friction.
12. Dissipation due to percolation.
13. Additional growth due to the wind.
14. Wave-current interaction.
15. Wave-wave interactions

Wave Shoaling

Shoaling is the deformation of the waves, which starts when the water depth becomes less than about half the wavelength, i.e. intermediate waves. The shoaling causes a reduction in the wave propagation velocity as well as shortening and steeping of the waves.

Due to wave shoaling surface waves entering shallower water change in wave height. It is caused by the fact that the group velocity, which is also the wave-energy transport velocity, changes with water depth.



Wave Shoaling

From last class:

$$\frac{H_1}{H_2} = \sqrt{\frac{n_2 L_2}{n_1 L_1}} \sqrt{\frac{B_2}{B_1}};$$

If 2 is deep water condition and 1 is unknown:

$$\frac{H}{H_0} = \sqrt{\frac{\frac{1}{2} L_0}{nL}} \sqrt{\frac{B_0}{B}};$$

$$\frac{H}{H_0} = \underbrace{\sqrt{\frac{L_0}{2nL}}}_{K_s} \underbrace{\sqrt{\frac{B_0}{B}}}_{K_r};$$

Ks is shoaling coefficient and Kr is refraction coefficient
If we ignore the refraction (i.e. waves approaching the coast perpendicularly) $B_0/B=1$

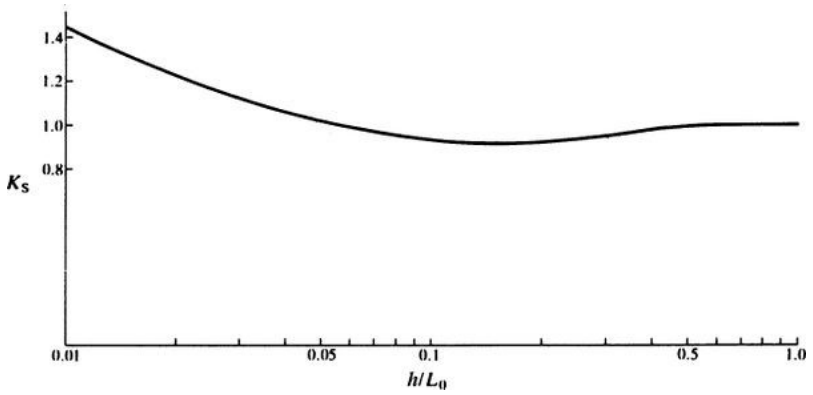
$$\frac{H}{H_0'} = \sqrt{\frac{L_0}{2nL}}$$

prime denotes the change in wave height from deep water to the point of interest considering only two-dimensional shoaling effects. So,

$$\frac{H}{H_o} = \frac{H}{H_o'} \sqrt{\frac{B_o}{B}}$$

Wave Shoaling

Shoaling coefficient: $K_s \equiv \frac{H}{H_0} = \sqrt{\frac{C_0}{2nC}}$

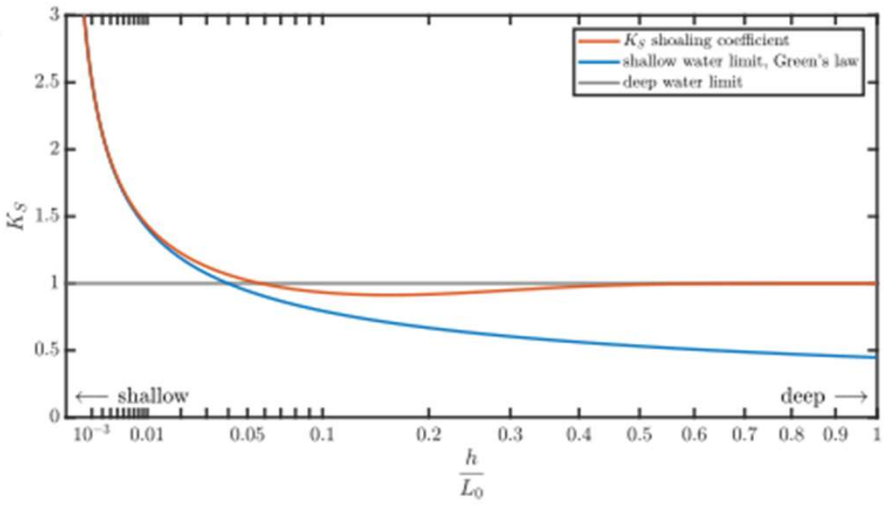


Shoaling of long waves: for very shallow water $\frac{H}{L} \to 0, n \to 1, C \to \sqrt{gd}$

$$K_s \equiv \frac{H}{H_0} = \sqrt{\frac{C_0}{2\sqrt{gd}}}, \quad \mathrm{H} \propto d^{-\frac{1}{4}}, \quad \mathrm{H}d^{\frac{1}{4}} = constant$$

$$\boxed{\mathrm{H}d^{\frac{1}{4}} = constant}$$

'Green's Law'



Theoretically according to Green's Law wave height will reach infinity but in reality breaking occurs.

Wave Shoaling

Apply Green's Law to determine Tsunami height at 1 m depth, if Tsunami height was 1.0 m at 10 m depth.

$$\boxed{\mathrm{H}d^{\frac{1}{4}} = constant}$$

$$\mathrm{H}d^{\frac{1}{4}} = \mathrm{H} * 1^{\frac{1}{4}} = 1 * 10^{\frac{1}{4}}, H = 1.78m$$

This increase in wave height is only due to shoaling.

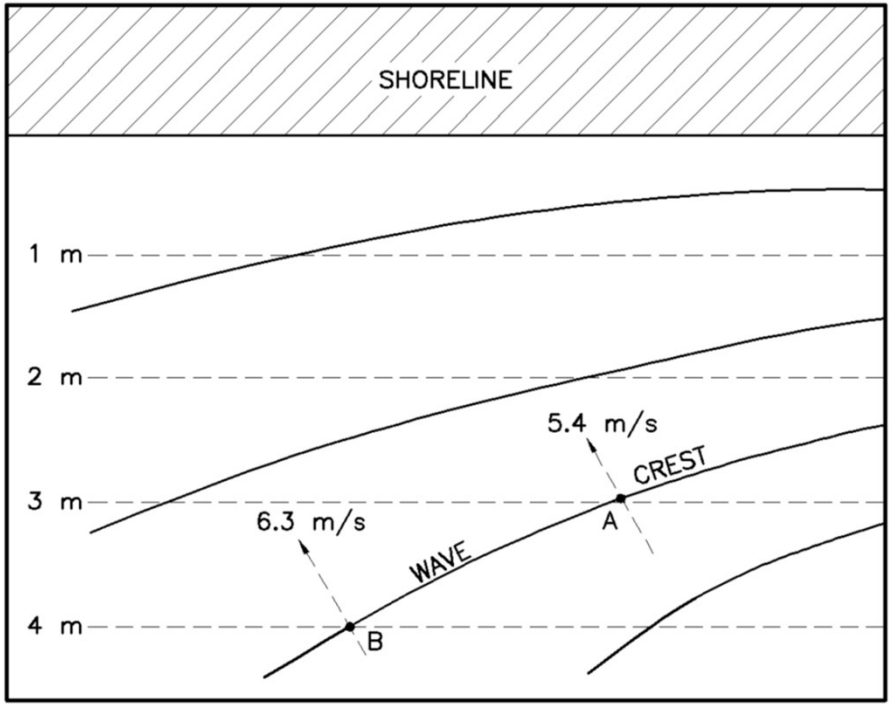
Wave Refraction

Wave refraction is the turning of the direction of wave propagation when the wave fronts travel at an angle with the depth contours at shallow water. The refraction is caused by the fact that the waves propagate more slowly in shallow water than in deep water. A consequence of this is that the wave fronts tend to become aligned with the depth contours. Currents can also result in refraction and this is termed current refraction. For constant depth there is no refraction.

Wave refraction occurs in transitional and shallow water depths because wave celerity decreases with decreasing water depths to cause the portion of the wave crest that is in shallower water to propagate forward at a slower speed than the portion that is in deeper water. The result is a bending of the wave crests so that they approach the orientation of the bottom contours

Wave refraction and shoaling

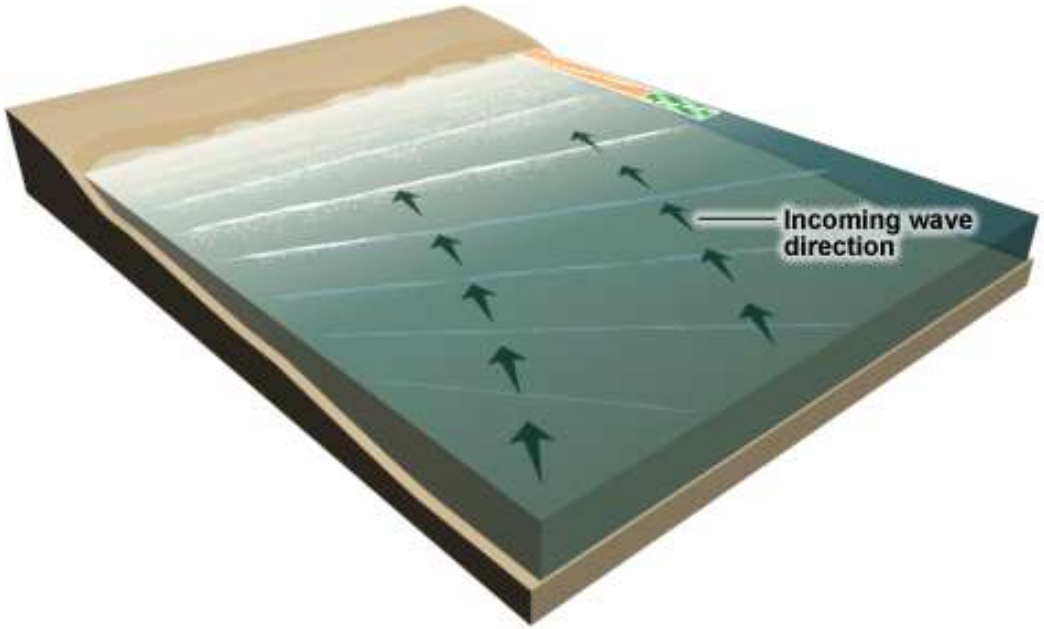
In order to understand wave refraction, consider the case of a steady-state, monochromatic wave propagating across a region in which there is a straight shoreline with all depth contours evenly spaced and parallel to the shoreline. In addition, no current is present. If a wave crest initially has some angle of approach to the shore other than 0 deg, part of the wave (point A) will be in shallower water than another part (point B). Because the depth at *A*, h_A, is less than the depth at *B*, h_B, the speed of the wave at A will be slower than that at B because



$$C_A = \frac{g}{\omega} \tanh k h_A < \frac{g}{\omega} \tanh k h_B = C_B$$

The speed differential along the wave crest causes the crest to turn more parallel to shore. The propagation problem becomes one of plotting the direction of wave approach and calculating its height as the wave propagates from deep to shallow water.

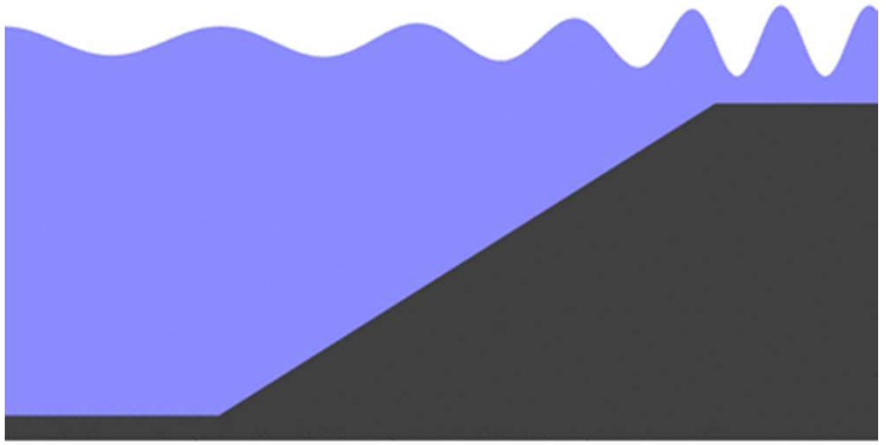
Wave refraction and shoaling



Refraction is the change in direction of waves that occurs when waves travel from one medium to another. Refraction is always accompanied by a wavelength and speed change.

$$C_{g_0} = \frac{1}{2}\frac{L_0}{T} = \frac{1}{2}C_0 \text{ (deep water)}$$

$$C_{g_s} = \frac{L}{T} = C \approx \sqrt{gd} \text{ (shallow water)}$$



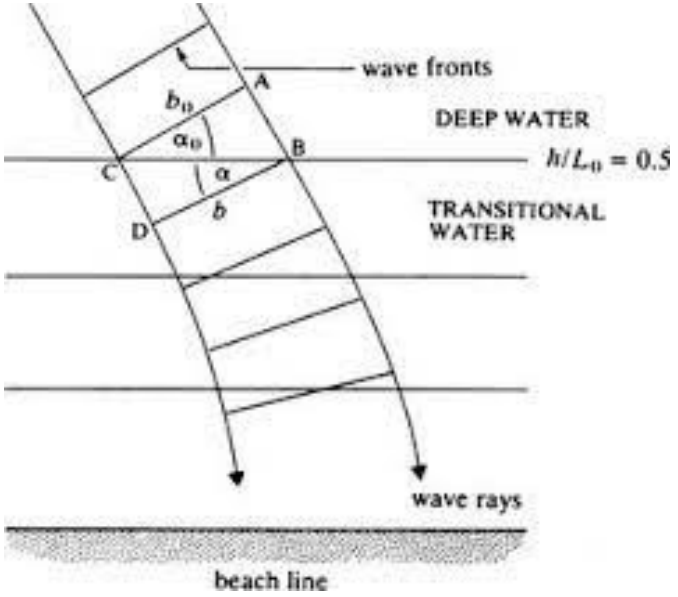
Wave shoaling is the effect by which surface waves entering shallower water change in wave height. It is caused by the fact that the group velocity, which is also the wave-energy transport velocity, changes with water depth.

Wave Refraction

For a straight coast with parallel contours:

$$\frac{sin\alpha}{C} = constant = \frac{sin\alpha_0}{C_0}$$

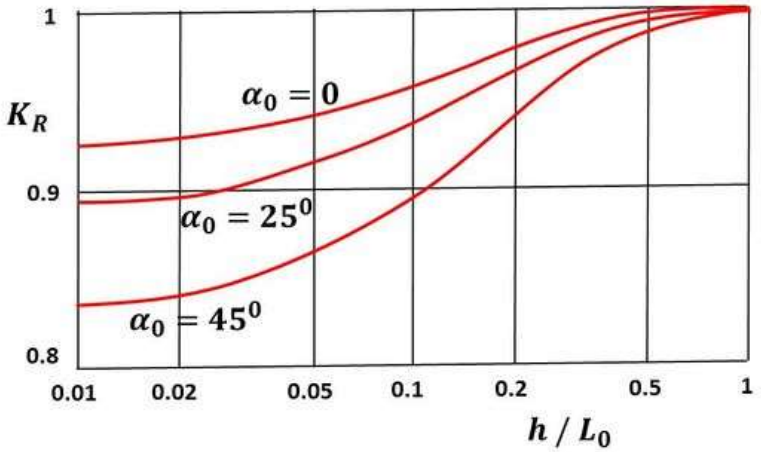
Snell's Law



Refraction coefficient:

$$K_r \equiv \frac{H}{H_0} = \sqrt{\frac{b_0}{b}} = \sqrt{\frac{cos\alpha_0}{cos\alpha}} < 1 \; (always)$$

This means: wave height H will be always smaller than incident wave height H_0



K_R as a function of relative depth h/L_0 for a typical wind wave state and different predominant wave incident angles α_0

Wave Refraction

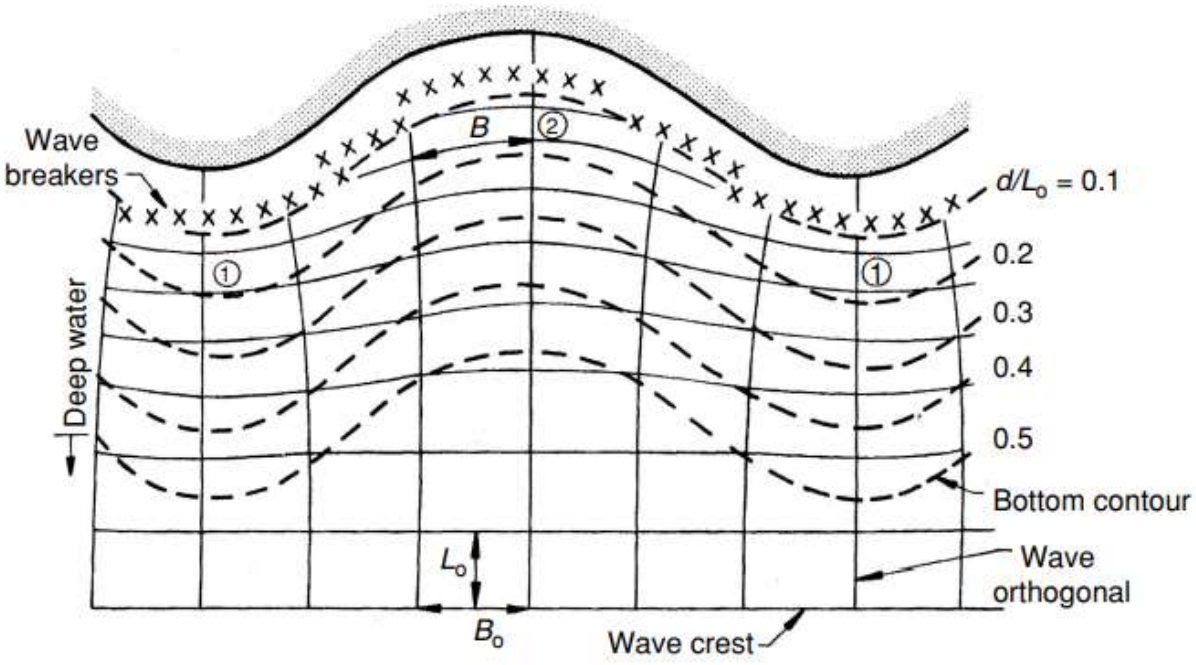
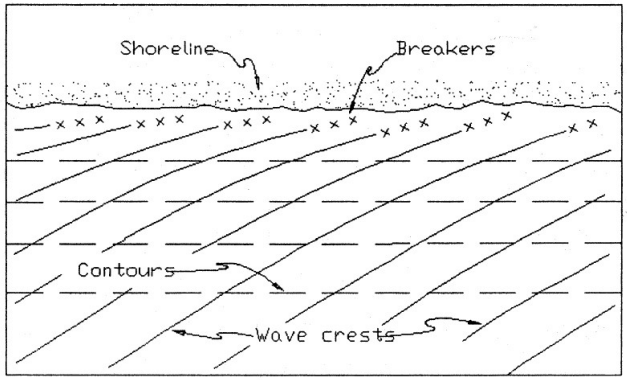
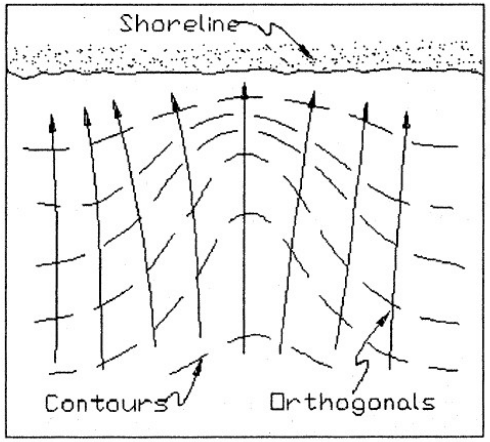


Figure 4.1. Wave refraction pattern.

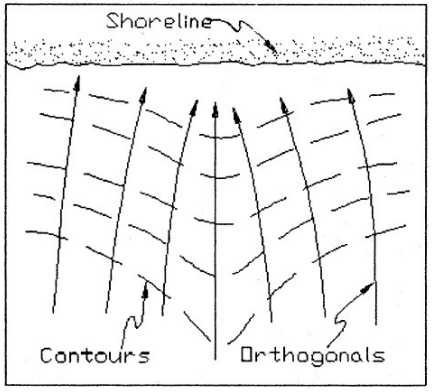
Idealized plots of wave rays for several typical types of bathymetry



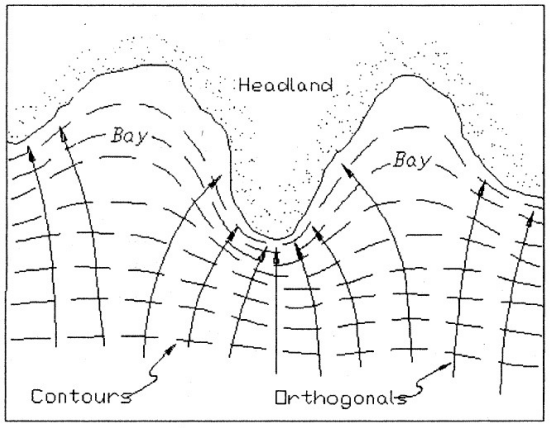
Simple parallel contours tend to reduce the energy of waves inshore if they approach at an angle



Submarine canyons tend to spread the energy away from it



Ridges or shoals and headlands tend to focus rays onto it *and spread energy out to either side*.



The amount of reduction or amplification will depend not only on bathymetry, but on the initial angle of approach and period of the waves.

Wave Refraction

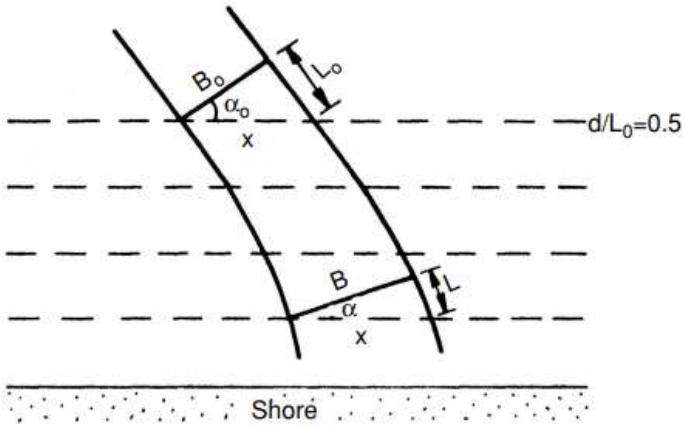


Figure 4.3. Wave refraction over straight parallel bottom contours.

When waves propagate shoreward over bottom contours that are essentially straight and parallel as shown in Figure

$$\frac{\sin\alpha_o}{L_o} = \frac{\sin\alpha}{L} = \frac{1}{x}$$

$$\frac{B_o}{\cos\alpha_o} = x = \frac{B}{\cos\alpha}$$

$$K_r = \sqrt{\frac{B_o}{B}} = \sqrt{\frac{\cos\alpha_o}{\cos\alpha}}$$

Where,

$$\alpha = \sin^{-1}\left(\frac{C}{C_o}\sin\alpha_o\right)$$

Wave Refraction

Consider the wave with deep water height of 2 m and a period of 10 s. Assume the wave crests in deep water are oriented at an angle of 35^0 with the shoreline and that the nearshore bottom contours are essentially straight and parallel to the shoreline. Determine the wave height and crest orientation with respect to the shoreline when the wave propagates into water 2.3 m deep.

Solution:

C_0 = 15.6m/s, and at d=2.3 m C= 4.75m/s

For offshore angle 35^0, $\alpha = \sin^{-1}\left(\frac{C}{C_0} sin\alpha_0\right) = \sin^{-1}\left(\frac{4.75}{15.6} sin(35)\right) = 9.5^0$

$$K_r = \sqrt{\frac{cos35}{cos9.5}} = 0.91$$

As $\frac{C_0}{C} = \frac{L_0}{L}$

Wave Refraction

Solution: (cntd.)

$$\frac{H}{H_0} = \sqrt{\frac{L_0}{2nL}}\sqrt{\frac{B_0}{B}} = \sqrt{\frac{15.6}{2(1)4.75}} * 0.91;$$

$$H = 2 * 1.166 = 2.33m$$

Thus, wave refraction decreases the wave height but the height increase caused by wave shoaling is greater, so there is a net increase in the wave height from deep water to the point where the depth is 2.3 m.

Wave Diffraction

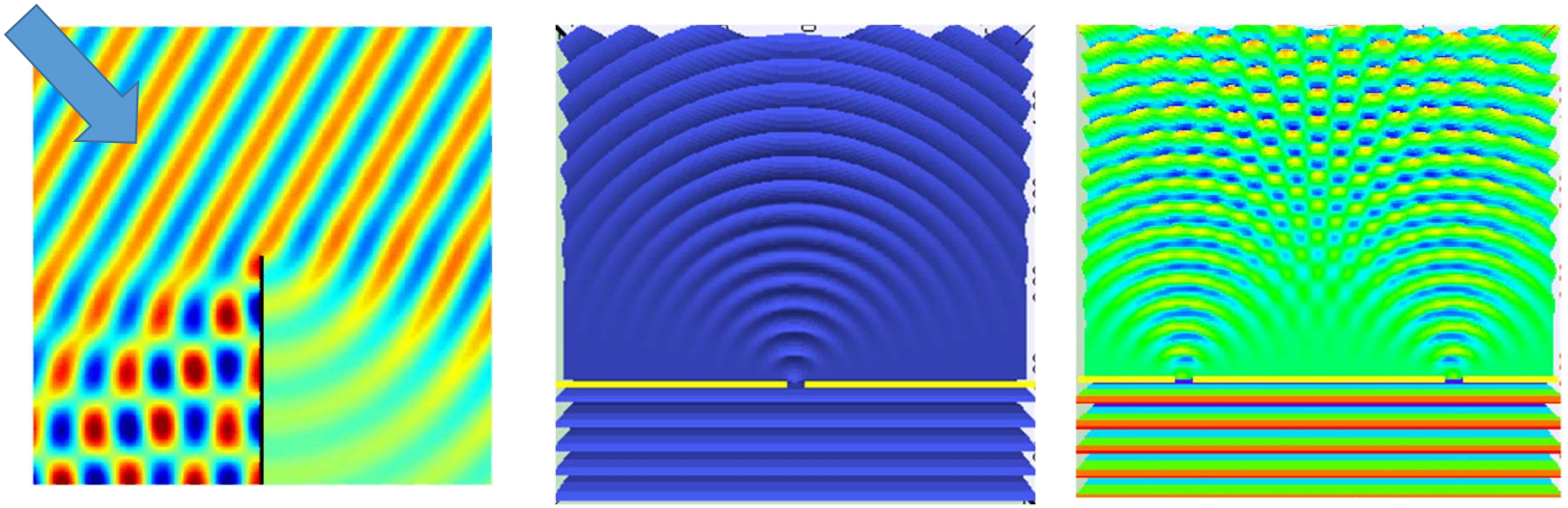
Diffraction can be seen when there are sheltering structures such as breakwaters. Diffraction is the process by which the waves propagate into the lee zone behind the structures by energy transmittance laterally along the wave crests.



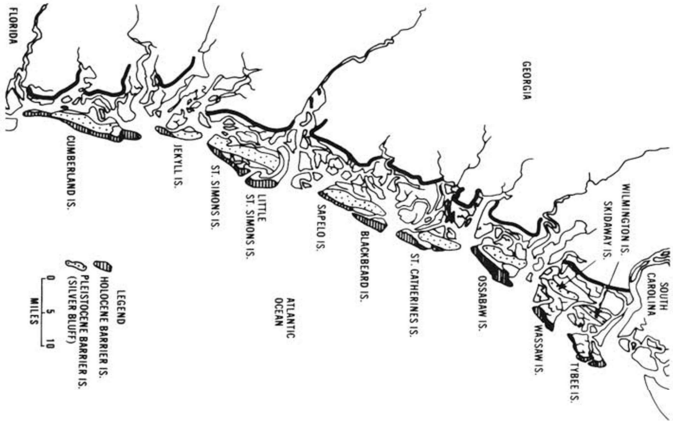


Wave diffraction

The classical introduction to diffraction treats a wave propagating past the tip of a breakwater.



Consider a long-crested wave that has a variable height along its crest. As this wave propagates forward, there will be a lateral transfer of wave energy along the crest (perpendicular to the direction of wave propagation). The energy transfer will be from points of greater to lesser wave height. This process is known as wave diffraction.

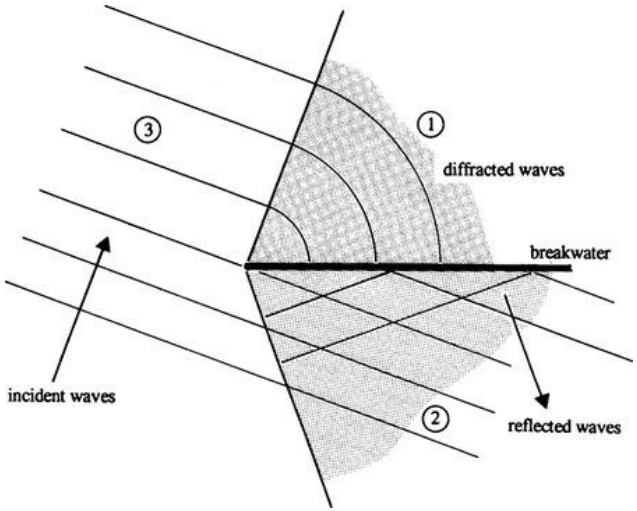


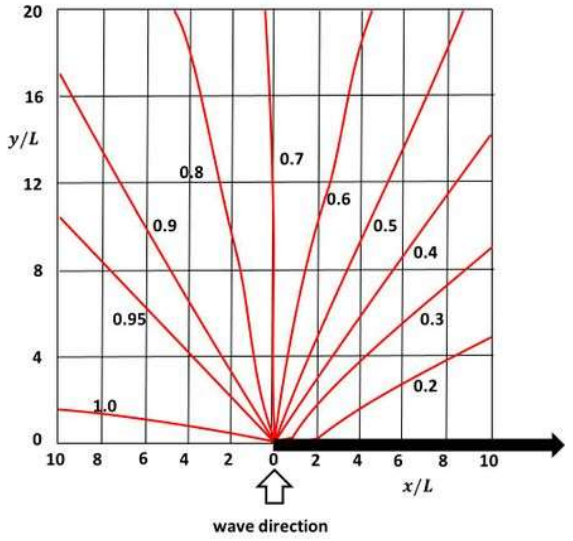
Wave Diffraction

Mathematical solutions for wave diffraction have been developed for the case of constant water depth using linear wave theory. The basic differential equation for wave diffraction is known as the Helmholtz equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$

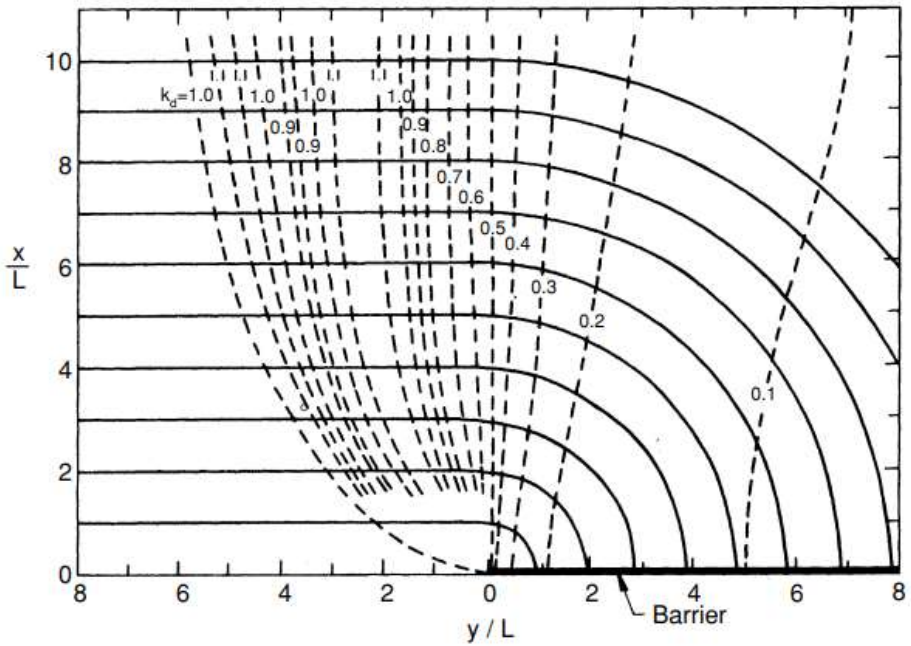
A solution to the Helmholtz equation was first found by Sommerfeld in 1896 who applied it to the diffraction of light.





Wave Diffraction

Semi-infinite Barrier



Wave crest pattern and related Kd values for normal wave incidence. (Wiegel, 1962.)

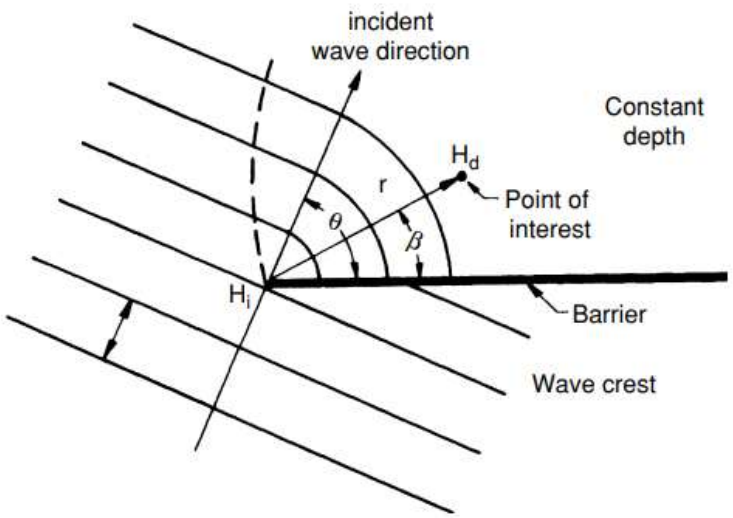


Figure 4.8. Definition sketch for wave diffraction in the lee of a barrier.

Wave Diffraction

Table 4.1. K_d versus θ, β, r/L for Semi-Infinite Breakwater

	β (Degrees)												
r/L	0	15	30	45	60	75	90	105	120	135	150	165	180
						θ = 15°							
1/2	0.49	0.79	0.83	0.90	0.97	1.01	1.03	1.02	1.01	0.99	0.99	1.00	1.00
1	0.38	0.73	0.83	0.95	1.04	1.04	0.99	0.98	1.01	1.01	1.00	.100	1.00
2	0.21	0.68	0.86	1.05	1.03	0.97	1.02	0.99	1.00	1.00	1.00	1.00	1.00
5	0.13	0.63	0.99	1.04	1.03	1.02	0.99	0.99	1.00	1.01	1.00	1.00	1.00
10	0.35	0.58	1.10	1.05	0.98	0.99	1.01	1.00	1.00	1.00	1.00	1.00	1.00
						θ = 30°							
1/2	0.61	0.63	0.68	0.76	0.87	0.97	1.03	1.05	1.03	1.01	0.99	0.95	1.00
1	0.50	0.53	0.63	0.78	0.95	1.06	1.05	0.98	0.98	1.01	1.01	0.97	1.00
2	0.40	0.44	0.59	0.84	1.07	1.03	0.96	1.02	0.98	1.01	0.99	0.95	1.00
5	0.27	0.32	0.55	1.00	1.04	1.04	1.02	0.99	0.99	1.00	1.01	0.97	1.00
10	0.20	0.24	0.54	1.12	1.06	0.97	0.99	1.01	1.00	1.00	1.00	0.98	1.00
						θ = 45°							
1/2	0.49	0.50	0.55	0.63	0.73	0.85	0.96	1.04	1.06	1.04	1.00	0.99	1.00
1	0.38	0.40	0.47	0.59	0.76	0.95	1.07	1.06	0.98	0.97	1.01	1.01	1.00
2	0.29	0.31	0.39	0.56	0.83	1.08	1.04	0.96	1.03	0.98	1.01	1.00	1.00
5	0.18	0.20	0.29	0.54	1.01	1.04	1.05	1.03	1.00	0.99	1.01	1.00	1.00
10	0.13	0.15	0.22	0.53	1.13	1.07	0.96	0.98	1.02	0.99	1.00	1.00	1.00

Wave Diffraction

(Table 4.1. continued.)

	β (Degrees)												
r/L	0	15	30	45	60	75	90	105	120	135	150	165	180
						$\theta = 60°$							
1/2	0.40	0.41	0.45	0.52	0.60	0.72	0.85	1.13	1.04	1.06	1.03	1.01	1.00
1	0.31	0.32	0.36	0.44	0.57	0.75	0.96	1.08	1.06	0.98	0.98	1.01	1.00
2	0.22	0.23	0.28	0.37	0.55	0.83	1.08	1.04	0.96	1.03	0.98	1.01	1.00
5	0.14	0.15	0.18	0.28	0.53	1.01	1.04	1.05	1.03	0.99	0.99	1.00	1.00
10	0.10	0.11	0.13	0.21	0.52	1.14	1.07	0.96	0.98	1.01	1.00	1.00	1.00
						$\theta = 75°$							
1/2	0.34	0.35	0.38	0.42	0.50	0.59	0.71	0.85	0.97	1.04	1.05	1.02	1.00
1	0.25	0.26	0.29	0.34	0.43	0.56	0.75	0.95	1.02	1.06	0.98	0.98	1.00
2	0.18	0.19	0.22	0.26	0.36	0.54	0.83	1.09	1.04	0.96	1.03	0.99	1.00
5	0.12	0.12	0.13	0.17	0.27	0.52	1.01	1.04	1.05	1.03	0.99	0.99	1.00
10	0.08	0.08	0.10	0.13	0.20	0.52	1.14	1.07	0.96	0.98	1.01	1.00	1.00
						$\theta = 90°$							
1/2	0.31	0.31	0.33	0.36	0.41	0.49	0.59	0.71	0.85	0.96	1.03	1.03	1.00
1	0.22	0.23	0.24	0.28	0.33	0.42	0.56	0.75	0.96	1.07	1.05	0.99	1.00
2	0.16	0.16	0.18	0.20	0.26	0.35	0.54	0.69	1.08	1.04	0.96	1.02	1.00
5	0.10	0.10	0.11	0.13	0.16	0.27	0.53	1.01	1.04	1.05	1.02	0.99	1.00
10	0.07	0.07	0.08	0.09	0.13	0.20	0.52	1.14	1.07	0.96	0.99	1.01	1.00

Wave Diffraction

Consider a train of 6 s period waves approaching a breakwater so that the angle of approach at the breakwater head (θ) is 60^0. The water depth in the lee of the breakwater is 10 m. Determine the wave height at an angle (β) of 30^0 from the breakwater and a distance of 96.6 m (=r) from the breakwater head if the incident wave height at the head is 2.2 m.

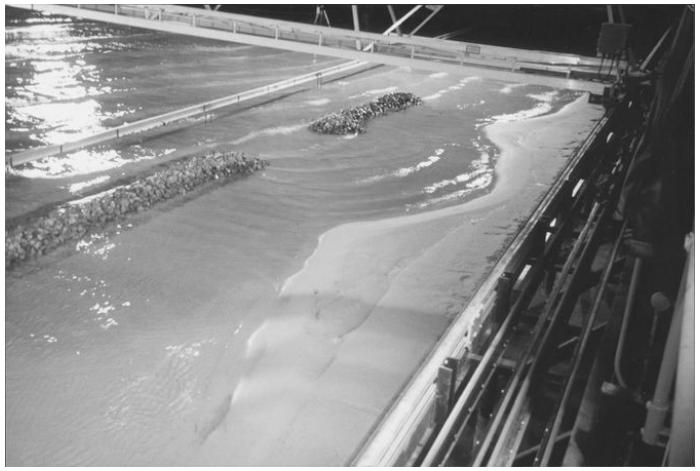
Solution: for T=6s and d=10m, we get L = 48.3m through trial.
So r/L= 96.6/48.3= 2
From Table for θ=60^0 and β=30^0, diffraction coefficient K_d=0.28
So, the wave height at the point of interest would be 0.28*2.2 = 0.62 m.
At the point of interest the wave would be propagating in the direction of the 30^0 radial line.

Combined Refraction-Diffraction

Refraction and diffraction often occur together. For example: around offshore breakwaters; here diffraction is often predominant close to the structure with refraction becoming more important further away from the structure.



Photograph of real wave diffraction at the Elmer breakwater scheme, Sussex, England.



Physical model study in the UK Coastal Research Facility at HR Wallingford.



Aerial photograph of wave diffraction at the Happisburgh to Winterton scheme, Norfolk, England

Thank You